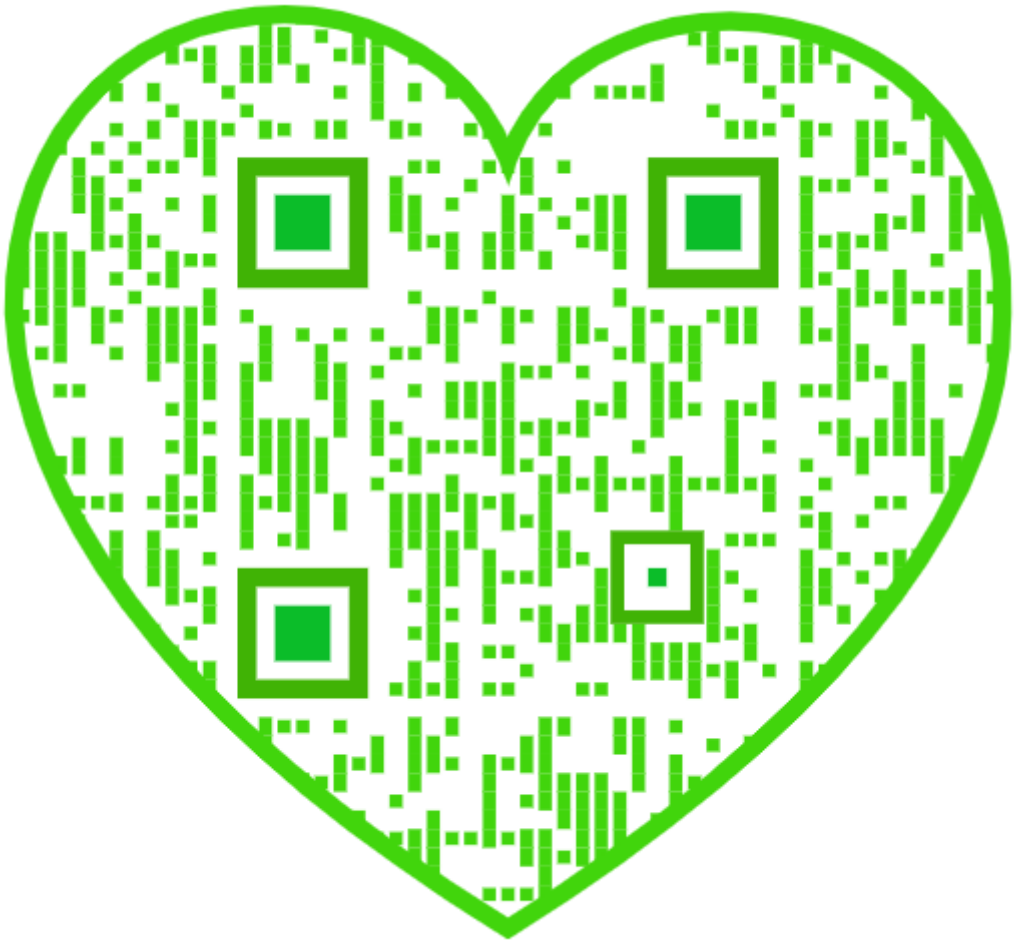

WALTON AND HOLMES'S

ARITHMETICS

FOURTH BOOK

BY

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W. & H. ARITH. FOURTH BOOK.

W. P. 3

PREFACE

THIS, the fourth and final book of the Walton and Holmes series of arithmetics, is intended for use in the eighth year of an eight-grade elementary course, and in either the eighth or the eighth and ninth years of a nine-grade course.

It gives to the grammar school pupil a final view of the subject, and provides for him a reference arithmetic that will be of use to him after leaving the elementary grades.

The treatment of the subject of arithmetic herein given is neither exhaustive nor extremely theoretical. The aim in the minds of the authors has been to produce a useful and usable text-book adapted to the needs and capabilities of eighth and ninth grade pupils. For this reason, the plan of the book is strictly topical rather than spiral, in order that each topic may be given such intensive study as is possible and desirable for grammar school pupils, not only for a mastery of the subject, but as a direct preparation for the mathematics of the secondary school where the topical treatment prevails.

Each topic is treated as fully as the limits of a text-book will permit, and is presented with full appreciation of its practical applications in the problems of the workshop, the factory, the counting room, and everyday experience.

This is an arithmetic pure and simple, and there has been no attempt at the introduction of novelty either in the presentation of topics or in the selection of problems. The authors have selected reasonable problems, and have avoided the use of those which would be unlikely to occur in ordinary business experience.

Attention is called to the unusually large number of problems contained in this book as well as in the other three books of this series.

It has been thought best to present the subjects of Longitude and Time, Standard Time, Square Root, Partial Payments, Foreign Exchange, and the Metric System in the body of the book rather than to reserve them for an appendix. This does not, of course, prevent the teacher from omitting these subjects if it seems desirable to do so, as the question of their omission is one that any capable teacher of grammar grades should be fully competent to decide.

In the preparation of the manuscript, the authors desire to acknowledge the helpful assistance of many teachers who have kindly offered suggestions and criticisms of great value and supplied many of the problems that the book contains. Particular acknowledgment should be made of the valuable assistance of Miss A. J. Meadowcroft, Principal of the Burnham School, Haverhill, Mass., Miss Gertrude E. Bigelow, Supervisor of Practice, of the Boston Normal School, Mr. John D. Billings, Principal of the Webster School, Cambridge, Mass., Mr. James W. Applebee, Ingalls School, Lynn, Mass., and Mr. Thomas W. Davis, formerly Principal of the Howard School, Cambridge, Mass.

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FOURTH BOOK

INTRODUCTION

1. To the Pupil. — **Arithmetic** is the science that treats of numbers and their use.

Every business transaction, no matter if it is but the purchase or sale of a paper of pins, a pound of nails, or a half yard of tape, requires the use of arithmetic. To be able to apply it is a great satisfaction and convenience. In order to use it successfully, you must study and practice it long enough and carefully enough to fix its facts and principles firmly in mind.

2. The occasion for the use of arithmetic is the measurement of quantities of things of various kinds.

3. A single thing, or a single quantity taken as a standard for measuring quantities of the same kind, is called a **unit.**

A boy, a cent, a yard, a pailful, a dozen, are units.

4. To show how great any quantity is when compared with a unit of its kind, we use **numbers.**

5. A number that refers to some particular kind of object or unit of measure is called a **concrete number.**

10 bottles, 6 yards, 3 pens, 4 cents, $2, are concrete numbers.

6. A number that does not refer to any particular kind of object or unit of measure is called an **abstract number.**

Forty-five, six hundred eleven, 347, are abstract numbers.

7. The knowledge of numbers includes a knowledge of their expression, combination, and relations.

8. The oral expression of numbers is called **numeration.**

9. The expression of numbers by written symbols — letters or figures — is called **notation.**

10. A single unit is expressed by the word *one* or the symbol (figure) 1.

One with one is expressed by the word *two*, or the figure 2.

The simplest groups of numbers are expressed in words and in figures as follows:

One, two, three, four, five, six, seven, eight, nine.
1, 2, 3, 4, 5, 6, 7, 8, 9.

The numbers from one to nine are called **digits.**

One with nine is *ten;* ten is expressed in figures thus, 10.

The figure 0 is called **zero** or **naught;** it stands for no number.

Groups of tens are expressed in words and figures as follows:

Ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.
10, 20, 30, 40, 50, 60, 70, 80, 90.

Ten tens are *one hundred*, expressed in figures thus, 100.

Ten hundreds are *one thousand*, expressed in figures thus, 1000.

11. By taking tens of smaller groups, we form larger groups, *hundreds*, *thousands*, *millions*, and so on.

This system of numbering by tens is called the **decimal system of notation.**

1 *unit* is called a unit of the first order; 1 *ten* is called a unit of the second order; 1 *hundred* a unit of the third order, etc.

12. Units with tens are expressed as follows:

Ten 10	Thirteen 13	Sixteen 16
Eleven 11	Fourteen 14	Seventeen 17
Twelve 12	Fifteen 15	Eighteen 18
Nineteen 19		Twenty 20

Twenty-one	21	Thirty-one	31	Forty-one	41
Twenty-two	22	Thirty-two	32	Forty-two	42
Twenty-three	23	Thirty-three	33	Forty-three	43
Twenty-four	24	Thirty-four	34	Forty-four	44
Twenty-five	25	Thirty-five	35	Forty-five	45
etc.		etc.		etc.	

13. The names of the first ten numbers and the names *unit*, *hundred*, *thousand*, *million*, etc., are simple words. By combining these names, other number names are formed. Thus, four hundred, three thousand, seven million, four hundred thousand.

14. The decimal system originated in India. But it came to us through the Arabs. Hence, it is called the **Arabic system.**

15. By means of the nine digit figures, the 0, and a dot (.), called the **decimal point**, any number can be expressed.

16. Illustrative Examples. (1) Express as one number 6 ten-thousands, 4 thousands, 3 hundreds, 7 tens, 2 units.

ILLUSTRATION

ten-thousands	thousands	hundreds	tens	units	decimal point
6	4	3	7	2	.

EXPLANATION. In writing numbers, to express units we write a figure in the first place at the left of the position of the decimal point, to express tens we write a figure in the second place at the left, to express hundreds we write a figure in the third place, to express thousands we write a figure in the fourth place, to express ten-thousands we write a figure in the fifth place, and so on. Hence, the above is written as one number thus, 64,372. 64,372 is read sixty-four thousand three hundred seventy-two.

(2) Write four hundred and six units.

EXPLANATION. We write thus, 406, putting a zero in the tens' place to show that there are no tens to be expressed. Whenever there is no number of a certain place to be expressed, we supply the place with a **zero.**

17. By the Arabic notation each figure always expresses the same number of units.

18. Written with other figures, each figure in any place expresses units of the place it occupies. This is called the **local value** of the figure.

19. Each figure in any place expresses ten times what it would represent if written in the next place to the right, and one tenth what it would represent if written in the next place to the left.

20. Illustrative Example. Write 6437 and 2 tenths; 643 and 72 hundredths; 64 and 372 thousandths.

ILLUSTRATION

ten-thousands	thousands	hundreds	units	tenths	hundredths	thousandths
6	4	3	7 .	2		
	6	4	3 .	7	2	
		6	4 .	3	7	2

EXPLANATION. If we place a decimal point in 64,372 at the left of 2, thus, 6437.2, each figure expresses one tenth as much as before; the 2 (units) express 2 tenths, the 7 (tens) express 7 units, and so on.

If we place the decimal point in 6437.2 at the left of 7, thus, 643.72, the 2 expresses 2 hundredths, the 7 expresses 7 tenths, and so on.

21. Whole numbers are called **integral numbers** or **integers.** Thus, 5, 29, 4329050 are integers.

Parts of whole numbers are called **fractions.**

We read 6437.2, six thousand four hundred thirty-seven and two tenths; 643.72, six hundred forty-three and seventy-two hundredths; 64.372, sixty-four and three hundred seventy-two thousandths.

Taken in order, figures to the right of thousandths express ten-thousandths, hundred-thousandths, millionths, ten-millionths, and so on.

22. The following table shows how the groups and orders of units are written by the Arabic notation:

Trillions			*Billions*			*Millions*			*Thousands*			*Units (Ones)*			*Fractions*			Group Names
Hundred-trillions	Ten-trillions	Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units (Ones)	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Order Names
480	,	297	,	034	,	508	,	672	.	8	3	4	5							
		443	,	820	,	227	,	967	.	8	0	0	5							Figures
						345	,	000	.	7	0	9								
								570	.	0	9	8								

The uppermost number above is read thus, 480 trillion, 297 billion, 34 million, 508 thousand, 672, and 8 thousand 345 ten-thousandths.

Oral Exercise

23. 1. Read the other numbers written beneath the table.

2. Read the number 345,345,345.345.

In reading the above numbers, notice that each group takes its name from the right-hand place in the group. Notice, too, that no name is read for the ones' group.

When whole numbers and fractions are written together, in reading the expression, we use *and* before the fractional part.

3. What name is given to the 2d group at the left of the decimal point? What name is given to the 3d group at the left? to the 4th? to the 5th?

4. What is the position of the thousands' group? the millions' group? the trillions' group?

Oral Exercise

24. Beginning at the left, give the name of the place of each figure in the following: thus (1) 4 thousands, 8 hundreds, 7 tens, 5 units (ones).

1. 4875	**3.** 604.2	**5.** 705,412	**7.** 1.4234
2. 1209	**4.** 75.83	**6.** 9705.412	**8.** 5.0625

9. What is the first place at the left of units called? What is the first place at the right of units called?

10. Give alternately the names of the places from the 1st to the 7th, at the right and left of units.

11. What is the name of the 3d place at the left of the decimal point? at the right?

12. What is the position with reference to the decimal point of the thousands' place? the millions'? the billions'? the thousandths'? the millionths'?

25. Illustrative Example. Read 1504670389.

EXPLANATION. Beginning with units' group and counting the groups, we find 1 in the 4th, or billions', group. From left to right, the expression reads 1 billion, 504 million, 670 thousand, 389.

Read:

1. 48790	**9.** 38904918	**17.** .862	**25.** 92.3046
2. 9078	**10.** 1760480	**18.** .704	**26.** 501.501
3. 84000	**11.** 300708	**19.** .067	**27.** 200.007
4. 40762	**12.** 447516300	**20.** .840	**28.** .207
5. 11094	**13.** 80040704	**21.** 981.2	**29.** .8483
6. 304091	**14.** 2321987650	**22.** 5.558	**30.** 8000.0483
7. 460203	**15.** 9876004381	**23.** .1476	**31.** .00116
8. 510510	**16.** 18073063918	**24.** 40.08	**32.** 100.00016

Written Exercise

26. Illustrative Example. Write in figures nineteen million one hundred thirty thousand three hundred sixteen and fourteen thousandths.

EXPLANATION. Beginning with the highest, or millions', group, we write the groups in order from left to right, thus, 19,130,316.014.

Write in figures and point off:

1. Six hundred seventy and sixty-seven thousandths.
2. Two thousand one hundred fourteen and thirteen hundredths.
3. Fourteen thousand five hundred and five tenths.
4. Five million thirty-six thousand six hundred ten.
5. One hundred sixty million one hundred thousand forty-one.
6. Two hundred seven and one hundred seventeen ten-thousandths.
7. Forty-three million seven hundred thousand two hundred sixty-seven.
8. Nine hundred one million two hundred eighteen thousand five hundred twenty-two.
9. Three and one thousand four hundred sixteen ten-thousandths.
10. Sixteen and six thousand six hundred sixty-six ten-thousandths.
11. Eighty-three and three hundred thirty-four thousandths.
12. Fifty-six thousand four hundred thirty-seven millionths.
13. Seven hundred and thirty-six thousandths.
14. Seven hundred thirty-six thousandths.
15. Four hundred and fourteen millionths.
16. Four hundred fourteen millionths.
17. Five hundred sixty-eight hundred-thousandths.

27. The money used in the United States is reckoned on a scale of tens, and thus applies the decimal system.

TABLE

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.

The sign for cent or cents is ¢ or ct.; the sign for dollar or dollars is $; the sign for eagle or eagles is E.

Since United States money is reckoned decimally, a unit of one order being 1 tenth of a unit of the next higher order, it is written in figures, with dollars as integers or whole numbers, dimes as tenths of a dollar, cents as hundredths of a dollar, and mills as thousandths of a dollar.

We reckon and express eagles as dollars, and dimes as cents.

Thus, the number, 2 eagles, 1 dollar, 43 cents, 8 mills is written $21.438. It is read 21 dollars, 43 cents, 8 mills. 21 dollars, 3 cents is written $21.03.

Oral and Written Exercise

28. Read as dollars and cents:

1. $8.17	**4.** $0.72	**7.** $50.39	**10.** $406.80
2. $1.35	**5.** $0.04	**8.** $94.90	**11.** $1909.55
3. $2.02	**6.** $9.50	**9.** $168.03	**12.** $3000.40

Using the decimal point and dollar sign, write:

13. 15 dollars, 36 cents.

14. 90 dollars, 50 cents.

15. 500 dollars, 7 cents.

16. Nine dollars, five cents, 2 mills.

17. 93 dollars, 62 cents, 5 mills.

18. 66 thousand dollars, 60 cents, 8 mills.

Read as dollars, cents, and mills:

19. $2.532, $61.013, $602.503, $24.003, $5204.209.

29. The Roman notation employs seven letters: I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, M for 1000.

By combining these letters any number can be expressed.

Thus, III = 3; IV = 4; VIII = 8; IX = 9; XIV = 14.

Principles to be observed in writing by letters are: —

1. *When a given letter is* **preceded** *by a letter representing a value less than the number to be written, the difference of the values of the letters is expressed.*

Thus, IV = 4; IX = 9; XL = 40; XC = 90.

2. *When a given letter is* **followed** *by a letter or a combination of letters representing equal or less value than the given letter, the sum of the values of the letters is expressed.*

Thus, XX = 20; CC = 200; XXXIV = 34; LXIX = 69.

3. *A bar placed over any letter multiplies it by a thousand.*

Thus, V = 5, $\overline{\text{V}}$ = 5000.

Oral and Written Exercise

30. Read the following:

	a	*b*	*c*	*d*	*e*	*f*
1.	XI	XIV	XVII	XX	XXIX	XL
2.	XII	XV	XVIII	XXI	XXX	XLV
3.	XIII	XVI	XIX	XXV	XXXIV	L

Write in figures:

4.	M	7.	CCC	10.	DXVIII	13.	CCCXCIV		
5.	D	8.	MMCC	11.	DCXLI	14.	$\overline{\text{M}}$		
6.	C	9.	CCVI	12.	MDXCIV	15.	$\overline{\text{D}}$		

Write in Roman notation:

16.	16	19.	95	22.	84	25.	1850	28.	7016
17.	45	20.	411	23.	6001	26.	1519	29.	9640
18.	104	21.	517	24.	1710	27.	4630	30.	693

31. A book rack has 5 red books on the first shelf, 3 red books on another shelf, and 4 red books on a third shelf.

To find the whole number of red books on the three shelves we combine or unite the numbers 5 and 3 and 4 and find that there are 12 red books on all the shelves.

The process of combining or uniting two or more numbers in one number is called **addition.**

The numbers to be united (or added) are called **addends.**

The result obtained by adding two or more numbers is called the **sum.**

32. Illustrative Example. In a common year the months of 31 da. contain 217 da., the months of 30 da. contain 120 da., and the remaining month has 28 da. Find the number of days in the year.

WORK

217
120 } addends
28
365, sum

EXPLANATION. For convenience in combining the numbers, we write the units of the same order in the same column. We put together first the units (ones); 8 and 7 are 15. We write 5 in the units' (ones') place below the line and reserve one ten to add with the tens. 1, 2, 2, and 1 (tens), are six tens. We write 6 at the left of the 5. 1 hundred and 2 hundreds are 3 hundreds. We write 3 at the left of the 6. In practice, to combine these numbers, we say or think, "8, 15" and write 5: "1, 3, 5, 6" and write 6: "1, 3" and write 3. The last result, 365, is called the *sum.* *Ans.* 365 days.

The addition of 217, 120, and 28 may be expressed as in the columns above, or thus, $217 + 120 + 28 = 365$.

The sign + means *and* or *with*, and is called *plus.*

The sign = means *equal* or *equals.*

Only numbers of the same kind and units of the same order can be added together.

Numbers of the same kind are called **like numbers.**

Thus, \$8 and \$3 are *like numbers;* also 5 pounds and 7 pounds; also 6 and 4; but \$8 and 5 pounds are *unlike numbers.*

Oral and Sight Exercise

33. 1. Add 2 to each of the following numbers, giving only sums; 2, 5, 7, 9, 1, 0, 3, 4, 6, 8.

Add each of the other eight digits to each of the above numbers.

Add the following till the sum is 70 or more:

2. By 2's beginning with 2; with 1.

3. By 10's beginning with 3; by 5's beginning with 4; with 3.

4. By 4's beginning with 3; with 2; with 1.

5. By 8's beginning with 7; with 6; with 5; with 4; with 3.

Add till the sum is 60 or more:

6. By 3's beginning with 2; with 1.

7. By 6's beginning with 5; with 0.

8. By 9's beginning with 8; with 7; with 6; with 5; with 4.

Add till the sum is 90 or more:

9. By 7's beginning with 6; with 5; with 4; with 3; with 2; with 1.

Extend each of the following combinations:

10.	3	13	23	7	17	27	27	17
	2	2	2	3	3	3	13	33
11.	6	60	60	60	160	260	260	360
	5	50	150	250	50	50	150	250
12.	6	60	60	8	80	180	90	190
	7	70	170	5	50	50	60	160

Written Exercise

34. Illustrative Example. Add 1824, 2571, 278, 3636.

WORK

1824
2571
278
3636
8309, sum

EXPLANATION. For convenience, we write the numbers with units of the same order in the same column. We add first the units; thus, 6, 14, 15, 19. We write 9 under the units' column, and add the 1 (ten) with the tens; thus, 1, 4, 11, 18, 20 (tens). We write 0 under the tens' column, and add 2 (hundreds) with the hundreds; thus, 2, 8, 10, 15, 23 (hundreds). We write 3 under the hundreds' column, and add 2 (thousands) with the thousands; thus, 2, 5, 7, 8 (thousands). We write 8 under the thousands' column. *Ans.* 8309.

To prove the work in addition, we add downwards and compare the results.

Add and prove:

1.	2.	3.	4.	5.	6.	7.	
5073	7667	14853	679	86	9794	958	
5932	6569	2485	3568	1190	5752	566	13
4430	9478	250	8746	2074	321	747	
243	909	9745	7566	6238	7589	789	
14201	4798	2333	868	5343	5245	1	15
9867	8959	849	5055	249	5631	35	

8. Add Example 7 by groups.

EXPLANATION. In units, the groups 15 and 13 = 28, with 8 = 36. Carrying 3 (0), we have in tens the groups 14 and 15, 29 (0). Carrying 2 (00), we have in hundreds the groups 16 and 14 = 30 (00).

Add by groups:

9.	10.	11.	12.	13.	14.	15.
5087	3505	1234	5632	15238	96531	97865
4639	6477	8765	7814	6349	87420	89724
9870	8738	9009	9069	7458	35900	31016
9008	7942	6882	310	28569	47025	34567
7569	7843	7214	9356	39677	60810	90888

Written Exercise

35. Illustrative Example. Find the sum of 564.18, 329.45, 207.136, 81.95, 165.029.

Work
564.18
329.45
207.136
81.95
165.029
1347.745

Explanation. To add decimals, we write them with units of the same order in the same column, and, beginning with the lowest order, add them as we add whole numbers.

Thus, 9, 15 (thousandths). We write 5 (thousandths), and add the 1 with the hundredths. Thus, 1, 3, 8, 11, 16, 24 (hundredths). We write 4 (hundredths), and add the 2 with the tenths. Thus, 2, 11, 12, 16, 17 (tenths). We write 7 (tenths), and add the one with the units. Thus, 1, 6, 7, 14, 23, 27. We write 7 (units), and add 2 with the tens, and so on. *Ans.* 1347.745.

Arrange in columns, add, and prove:

1. 361.06 + 68.569 + 0.25 + 607.176 + 98.329
2. 508.7 + 0.994 + 75.832 + 9.47 + 11.976
3. 0.675 + 18.5 + 18.435 + 819.04 + 665.007
4. 10.025 + 8.89 + 325.019 + 67.709 + 684.64
5. 7.907 + 22.02 + .0826 + 24.25 + 38.1
6. 6.78 + 68.90 + 73.683 + 132.333 + 1.457
7. $425.57 + $9.61 + $78.32 + $54.18 + $126.73 + $45.98
8. $8.76 + $53.68 + $7896.50 + $4072.31 + $8.65 + $32.19
9. $5786.54 + $621.09 + $408.19 + $3.48 + $50.72 + $0.38
10. $7958.93 + $0.95 + $7.16 + $24.68 + $753.10 + $38.70
11. $185.07 + $406.504 + $27.16 + $0.50 + $1.25 + $75.336
12. $0.75 + $56.394 + $84.005 + $50.07 + $100.004 + $77.77
13. $5.09 + $.054 + $108.042 + $1.875 + $0.625 + $15.29
14. $9.005 + $1.99 + $43.621 + $25.09 + $50.712 + $63.25
15. $7.046 + $8.888 + $250.006 + $66.666 + $140.009 + $9.89
16. $6.548 + $7.654 + $75.07 + $13.335 + $75.872 + $14.67

Written Exercise

36. **1.** It is 142 mi. from New York to Albany and 882 mi. from New York to Atlanta. How far is it from Atlanta to Albany *via* New York?

2. A man goes from St. Petersburg to Rome, a distance of 1395 mi., thence to London, 1195 mi., and from London to Liverpool, 202 mi. How far does he travel in all?

3. A letter to Hongkong, China, from New York, goes to San Francisco, 3250 mi., thence to Honolulu, 2395 mi., thence to Hongkong, 4945 mi. What distance does it travel?

4. In 1800 the area of the United States was 827,844 sq. mi. In 1909 the total area including all its possessions was 2,977,130 sq. mi. greater than in 1800. What was the area in 1909?

5. In 1800 the population of the United States per square mile was 6.41. In 1905 the population per square mile was 21.07 greater. What was the population per square mile in 1905?

6. In 1905 the money of various kinds in circulation in the United States was as follows:

Gold	$651,063,589	Greenbacks	$332,420,697
Silver	175,022,043	National Bank Notes	480,028,849
Gold Certificates . .	485,210,749	Miscellaneous Currency	9,272,018
Silver Certificates . .	454,864,708		

What was the total?

7. Mr. E. J. Brown's estate was inventoried as follows: Homestead, $12,000.00; half interest in firm of Brown & Root, $100,000.00; 10 shares Phœnix National Bank, $1200.00; 5 shares Chicago, Milwaukee, & St. Paul R.R., common stock, $710.00; furniture, $300.00; cash $1116.15. What was the total value?

Written Exercise

37. 1. The areas of the Great Lakes are as follows: Lake Superior, 32,000 sq. mi.; Lake Michigan, 22,450; Lake Huron, 23,800; Lake Erie, 9960; Lake Ontario, 7240. What is the total area?

2. The United States shore line on the Great Lakes is as follows: Lake Superior, 955 mi.; Lake Michigan, 1320 mi.; Lake Huron, 510 mi.; Lake Erie, 370 mi.; Lake Ontario, 230 mi. What is the total length of our shore line on the Great Lakes?

3. Find the totals of the following:

NEW ENGLAND CENSUS STATISTICS, 1900

STATES	POPULATION	MALES	FEMALES	NATIVE-BORN	FOREIGN-BORN
Maine	694,466	350,995	343,471	601,136	93,330
New Hampshire .	411,588	205,379	206,209	323,481	88,107
Vermont . . .	343,641	175,138	168,503	298,894	44,747
Massachusetts .	2,805,346	1,367,474	1,437,872	1,959,022	846,324
Rhode Island . .	428,556	210,516	218,040	294,037	134,519
Connecticut . .	908,420	454,294	454,126	670,210	238,210

4. Find the total population of the following New England cities according to the 1900 census:

Boston . .	560,892	Bridgeport .	70,996	Waterbury .	45,859
Providence .	175,597	Lynn . . .	68,513	Holyoke . .	45,712
Worcester .	118,421	Lawrence . .	62,559	Brockton . .	40,063
New Haven .	108,027	New Bedford	62,442	Pawtucket .	39,321
Fall River .	104,863	Springfield .	62,059	Haverhill . .	37,175
Lowell . .	94,969	Somerville .	61,643	Salem . . .	35,956
Cambridge .	91,886	Manchester .	56,987	Chelsea . .	34,072
Hartford . .	79,850	Portland . .	50,145	Malden . .	33,664

Written Exercise

38. 1. The following shows the number of newspapers published in the United States and Canada in 1908. Find the total number.

Alabama	237	Montana	112
Alaska	23	Nebraska	590
Arizona	69	Nevada	58
Arkansas	313	New Hampshire	87
California	736	New Jersey	363
Canada	1,167	New Mexico	83
Colorado	370	New York	1,835
Connecticut	159	North Carolina	255
Delaware	30	North Dakota	296
District of Columbia	63	Ohio	1,119
Florida	157	Oklahoma	559
Georgia	373	Oregon	229
Idaho	116	Pennsylvania	1,381
Illinois	1,619	Rhode Island	48
Indiana	784	South Carolina	146
Iowa	1,047	South Dakota	353
Kansas	732	Tennessee	309
Kentucky	307	Texas	902
Louisiana	202	Utah	88
Maine	143	Vermont	71
Maryland	181	Virginia	245
Massachusetts	569	Washington	321
Michigan	765	West Virginia	217
Minnesota	739	Wisconsin	687
Mississippi	212	Wyoming	56
Missouri	1,000	Total	

2. The total number of newspapers published in the world in 1908, in addition to those of the United States and Canada (see Ex. 1), was as follows: Great Britain, 9500; Germany, 8049; France, 6681; Austria-Hungary, 2958; Italy, 2757; Switzerland, 1005; Japan, 1000; Asia, exclusive of Japan, 1000; Spain, 1000; Russia, 1000; Australia, 1000; Holland, 980; Belgium, 956; Greece, 130; all others, 1000. What was the total number of newspapers published in the world in 1908?

Written Exercise

39. 1. The total railway mileage in the United States in 1896 was 181,153.77 mi. The increase in 1897 was 1651.84 mi.; in 1898, 1967.85 mi.; in 1899, 2898.34 mi.; in 1900, 4051.12 mi.; in 1901, 3891.66 mi.; in 1902, 5234.41 mi.; in 1903, 5505.37 mi.; in 1904, 5927.12 mi.; in 1905, 4196.70 mi.; in 1906, 6262.13 mi.; in 1907, 5588.02 mi. What was the total mileage in 1907?

2. During the year 1905 the number of persons killed and injured by railroad accidents in the United States was as follows: employees killed 3361, injured 66,833; passengers killed 537, injured 10,457; other persons killed 5305, injured 8718. Find the total number of persons killed and injured.

3. The number of immigrants entering the United States in 1905 from countries of Asia was as follows: China, 2166; Japan, 10,331; India, 190; Turkey in Asia, 6157; other countries of Asia, 5081. What was the total Asiatic immigration?

4. In 1906 the steamships of the Hamburg-American line had the following net tonnage; what was the total tonnage of the entire fleet? *Deutschland*, 16,000; *Prinzessin Victoria Louise*, 15,000; *Pennsylvania*, 13,265; *Pretoria*, 13,050; *Graf Waldersee*, 13,080; *Patricia*, 13,273; *Bulgaria*, 10,417; *Batavia*, 11,046; *Moltke*, 12,000; *Blücher*, 12,000; *Hamburg*, 10,600; *Prinz Oskar*, 11,000; *Prinz Adelbert*, 11,000; *Amerika*, 23,500; *Kaiserin Auguste Victoria*, 25,000.

5. The copper production of the United States in 1907 was as follows: Alaska, 3305 tons; Arizona, 128,433 tons; California, 17199.4 tons; Colorado, 6672 tons; Idaho, 5735.5 tons; Michigan, 110,158.5 tons; Montana, 113,145.4 tons; New Mexico, 4326.4 tons; Utah, 34,166.5 tons; Wyoming, 1459.5 tons; Southern States, 11,204.3 tons; other states, 3814.3 tons. What was the total production?

Written Exercise

40. A Record of Expenses for the Month of June. Orders Drawn. Park Dept. City of N.

Date		Order Drawn in Favor of	Order No.	No. 1 Total Amount of Order		No. 2 Station-ery		No. 3 Repairs		No. 4 Printing		No. 5 Miscel-laneous		No. 6 General Supplies		No. 7 Improve-ments	
1910																	
May	2	Clark & Co. . . .	11	$1	30									1	30		
"	5	A. E. Abbe . . .	12	9	00	6	50					2	50				
"	11	H. P. Corbin & Co.	13	10	00			1	25							8	75
"	11	J. A. Church . .	14	76	00	26	00			50	00						
"	12	C. B. Brown . .	15	61	58							31	00	30	58		
"	17	N. E. Electric Co. .	16	67	93			45	00					22	93		
"	17	O'Connor & Parker	17	49	15	18	00					31	15				
"	17	Doolittle Bros. . .	18	332	80			332	80								
"	18	Alton Rubber Co. .	19	40	50							40	50				
"	20	B. W. Thomas .	20	31	10	1	10	20	00	10	00						
"	20	D. L. Doyle & Co. .	21	25	38			16	32							9	06
"	20	T. B. Smith . .	22	74	59											74	59

Written Exercise (*Continued*)

				No. 1		No. 2		No. 3		No. 4		No. 5		No. 6		No. 7	
"	21	W. A. Coyne . .	23	57	82							50	25	7	57		
"	23	C. J. Moore & Co. .	24	20	11					20	11						
"	23	J. A. French . .	25	12	10			6	00	6	10						
"	24	B. A. Robbins . .	26	273	00			95	00			37	00	100	00	41	00
"	25	L. B. Ray & Co. .	27	780	00			80	00			224	00	476	00		
"	26	F. Miller	28	100	00							50	00			50	00
"	26	D. P. Mills . . .	29	162	50	162	50										
"	26	A. G. Farwell . .	30	130	00					130	00						
"	26	G. B. Hooker . .	31	250	00			250	00								
"	27	James Files . . .	32	272	25	72	25					200	00				
"	30	John Boyle Co. .	33	70	00	32	00			16	00			22	00		
"	30	F. E. Brown . .	34	280	30	200	30			40	00	40	00				
"	30	C. F. Dummer . .	35	562	77	250	00			162	77			150	00		

1. Find the total footing for column No. 1.

2. Find the total footings for columns No. 2, 3, 4, 5, 6, 7.

3. Add the footings for columns 2, 3, 4, 5, 6, 7.

If your addition has been correct, the total of column 1 will equal the sum of the totals of columns 2, 3, 4, 5, 6, 7.

Written Exercise

41. Add the following ledger columns in lines and in columns:

	24.	25.	26.	27.	28.
1.	$10 82	$22 14	$53 43	$849 80	$787 49 =
2.	31 94	40 56	81 71	26 51	201 16 =
3.	69 30	78 95	37 20	8 49	90 75 =
4.	81 68	42 12	57 69	169 12	87 12 =
5.	53 09	45 08	26 75	95 04	445 73 =
6.	34 70	13 03	56 07	219 20	34 24 =
7.	64 13	56 30	83 96	454 66	93 76 =
8.	17 62	92 06	60 40	32 33	21 07 =
9.	5 46	68 47	47 05	378 74	150 81 =
10.	29 55	86 54	63 88	77 63	257 24 =
11.	31 37	65 45	20 69	996 84	78 13 =
12.	67 26	33 87	22 97	545 37	399 25 =
13.	68 91	93 69	71 99	65 98	40 45 =
14.	25 94	17 16	22 61	618 23	73 34 =
15.	30 18	84 88	65 41	135 75	480 56 =
16.	74 28	6 13	36 54	417 82	102 68 =
17.	28 37	83 54	76 82	51 19	227 17 =
18.	70 93	3 26	71 91	769 86	62 43 =
19.	12 25	31 42	47 12	191 50	553 34 =
20.	85 81	96 64	17 35	59 08	76 46 =
21.	69 85	87 30	6 89	75 88	9 78 =
22.	7 68	64 84	99 99	87 50	897 62 =
23.	88 72	68 83	33 67	9 98	709 62 =
29.					=

42. The selling price of a pair of shoes is $5. If their cost to the maker is $3, how much is the profit?

To find the profit we take $3 out of $5, which leaves $2.

The process of taking part of a number away to find what is left is **subtraction.**

The number of which a part is to be taken is called the **minuend.**

The part taken or subtracted is called the **subtrahend.**

The part left is called the **remainder** or the **difference.**

We find the difference of two numbers by subtracting the less from the greater.

The Panama canal is about 49 mi. long. The Suez canal is about 88 mi. long. What is the difference of their lengths? The difference of their lengths is 39 mi. The subtraction of 49 from 88 is expressed thus: $88 - 49 = 39$. The expression is read 88 less 49 equals 39, or 88 minus 49 equals 39.

The sign $-$ is called the *minus* sign.

Only numbers of the same kind and units of the same order can be subtracted.

Oral Exercise

43. **1.** Beginning with 2, subtract each of the nine digits from each of the following numbers, giving only remainders:

9, 10, 12, 14, 13, 11, 15, 18, 16, 19, 17.

2. Subtract by 2's from 30; from 29: by 10's and 5's from 54.

3. Subtract by 4's from 48; from 47; from 46: by 8's from 96; from 95; from 94; from 93; from 92; from 91; from 90; from 89.

4. Subtract by 3's, by 6's, and by 9's from 90: by 7's from 84; from 83; from 82; from 81; from 80; from 79; from 78.

Written Exercise

44. Illustrative Example. From 50,258 take 7187.

WORK

50258, minuend
7187, subtrahend
43071, remainder

EXPLANATION. We write the smaller number under the larger, with units of the same order in the same column. We begin with the lowest order to subtract. 7 from 8 leaves 1. We write 1 under the units' column. We cannot take 8 tens out of 5 tens, so we think of one of the 2 hundreds as united with the 5 tens, making 15 tens. 8(0) from 15(0) leaves 7(0). We write 7 under the tens' column. 1(00) from the remaining 1(00) leaves no hundreds. We write 0 under the hundreds' column. We cannot take 7(000) out of 0 thousands, so we think of 1 of the 5 ten thousands as changed to 10(000); 7(000) from 10(000) leaves 3(000). We write 3 under the thousands' column. Nothing from the remaining 4(0000) leaves 4(0000). We write 4 under the 5 ten thousand. *Ans.* 43,071.

In practice, we think, 7 from 8 = 1, and write 1; 8 from 15 = 7, and write 7; 1 from 1 = 0, and write 0; 7 from 10 = 3, and write 3; 0 from 4 = 4, and write 4.

Or we may think 7 and 1 are 8, and write 1; 8 and 7 are 15, and write 7; 1 and 0 are 1, and write 0; 7 and 3 are 10 and write 3; 0 and 4 are 4, and write 4.

PROOF. The parts when added should equal the whole; then the remainder 43,071 plus the subtrahend 7187 = the minuend 50,258.

To prove the work in subtraction, we add the remainder or difference to the subtrahend. The sum should equal the minuend.

Subtract and prove:

1. 1407 − 765
2. 4006 − 192
3. 9081 − 473
4. 2441 − 1597
5. 5015 − 364
6. 7026 − 857
7. 2003 − 648
8. 5003 − 922
9. 6507 − 5028
10. 8040 − 1816
11. 3005 − 2907
12. 6000 − 5654

13. 4000 − 2567
14. 3009 − 1098
15. 2304 − 1999
16. 7680 − 6789
17. 9052 − 8665
18. 7803 − 5976
19. 9630 − 7548
20. 4105 − 3299
21. 9180 − 2498
22. 60,320 − 16,807
23. 70,067 − 14,789
24. 26,431 − 16,446
25. 718,482 − 4666
26. 80,000 − 29,055
27. 100,000 − 57,035
28. 500,000 − 302,009
29. 648,305 − 57,397
30. 440,073 − 155,986
31. 632,504 − 285,279
32. 750,024 − 668,187
33. 1,639,533 − 946,879
34. 1,022,798 − 979,999
35. 5,332,263 − 964,064
36. 20,752,981 − 3,495,382

Written Exercise

45. Illustrative Example. From 12.475 subtract 5.0294.

Work
12.475
5.0294
7.4456

Explanation. We begin with the lowest order to subtract. We cannot take .0004 out of 0 ten-thousandths, so we think of one of the .005 as changed to 10 ten-thousandths; .0004 from .0010 leaves .0006. We write 6 under the ten-thousandths. We cannot take .009 from the remaining .004, so we think of one of the .07 as united with the .004 making .014; .009 from .014 leaves .005. We write 5 under the thousandths. .02 from the remaining .06 leaves .04. We write 4 under the hundredths, and so on. *Ans.* 7.4456.

Subtract and prove:

1. 32.05 − 17.125
2. 12.083 − 10.5
3. 16 − .016
4. 80.50 − 79.48
5. 71.07 − 6.019
6. 6.26 − 5.178
7. 530 − 4.685
8. 167.76 − 7.584
9. 56.328 − 48.09
10. 902.68 − 10.728
11. 820.05 − 9.876
12. 45.09 − 32.59
13. 7.005 − 6.438
14. 9000 − .999
15. 4.156 − .787
16. 74.5 − 5.69
17. 202.02 − 20.2
18. 7601 − 6.59
19. 8090 − .875
20. 42.50 − 2.625
21. 24.95 − 9.375

Written Exercise

46. Illustrative Example. A traveler in Europe had $1874. After spending $1576.34, how much had he left?

Work	Explanation. As there are no cents in the minuend, we think of 1 of the $4 as changed to 100¢. 34¢ from 100¢ leaves 66¢; we write 66 for the cents. $1576 from the remaining $1873 leaves $297. *Ans.* $297.66.
$1874.	
1576.34	
$ 297.66	

Subtract and prove:

1. $71.28 − $1.78
2. $161.35 − $54.18
3. $61.86 − $18.79
4. $869.07 − $735.10
5. $24.87 − $1.35
6. $867.70 − $15.23
7. $187.60 − $26.17
8. $406.57 − $23.72
9. $7.16 − $4.90
10. $713.34 − $.88
11. $538 − $408.37
12. $7818.05 − $342.03
13. $1957.90 − $691.49
14. $18.70 − $8.23
15. $3355.35 − $392.42
16. $94.60 − $54.66

17. From $94.60 subtract $54.66.

18. How many more are 9460 men than 5466 men?

19. What number added to 5466 makes 9460?

20. The sum of two numbers is 9460, and the greater one is 5466. What is the smaller one?

21. Find the difference between 9460 and 5466.

22. 9460 − 5466 = ?

23. 9460 − ? = 5466.

24. 9460 less 5466 equals what?

25. Subtract 5466 from 9460.

26. 9460 is the minuend, 5466 is the subtrahend. What is the remainder?

Written Exercise

47. **1.** The roadway over the Natural Bridge in Virginia is 122 ft. above the creek below, and is 40 ft. above the top of the arch. How far above the creek is the top of the arch?

2. Benjamin Franklin was born in 1706 and died in 1790. What was his age at the time of his death?

3. Sound travels 4768 ft. a second in salt water, and 4714 ft. a second in fresh water. How many more feet per second does it travel in salt water than in fresh water?

4. The first savings bank was established in 1778 at Hamburg. In 1816 the first savings bank in New England was established. What time elapsed between the dates?

5. The earnings of a coöperative bank were $1376.34 for the first six months, and $1874.53 for the second six months. What was the increase?

6. The Amazon River is 3750 mi. long. The Mississippi River is 4300 mi. long. Which is the longer and how much longer is it?

7. The sailing distance from New York to Queenstown is 2890 mi. How much farther has a steamer to go when she has covered 1368 mi. of the distance at the end of the third day out?

8. Iron melts at 2890° Fahrenheit, and gold at 2590°. How many degrees higher is the melting point of iron than of gold?

9. The first constitution of the state of Indiana was adopted in 1816. In 1852 the University of Indiana was established. How many years intervened?

10. The Concord Fire Insurance Company has assets of $620,759.39, and liabilities of $342,089.31. How much do its assets exceed its liabilities?

Written Exercise

48. **1.** In a certain state there were 582,125 native-born and 78,961 foreign-born persons. There were how many more native-born than foreign-born persons?

2. What is the difference in value between a $5 note and an English pound sterling valued at $4.8665 of United States money?

3. In a certain summer the rainfall amounted to 15.5 in. For June and July it was 9.785 in. What was it for August?

4. The polar diameter of the earth is 7898.8809 mi. The equatorial diameter is 7924.9111 mi. Which is the greater and how much?

SOME MOUNTAIN HEIGHTS IN FEET

Mt. Blanc, France, 15,744.	Mt. Olympus, Greece, 9745.
Mt. Everest, Nepal, 29,002.	Pike's Peak, Colorado, 14,111.
Mt. Vesuvius, Italy, 4205.	

5. Mt. Washington is 9465 ft. lower than Mt. Blanc. How high is Mt. Washington?

6. How much does the height of Olympus exceed a mile (5280 ft.)?

7. How much does the height of Vesuvius fall short of a mile?

8. How much does the height of Pike's Peak exceed 2 mi.?

9. Compare the heights of Mt. Blanc and Pike's Peak; of Mt. Blanc and Mt. Olympus.

10. How much higher than Mt. Everest is a balloon at a height of 32,786 ft.?

11. The estimated height attained by Glaisher in 1862 was 37,000 ft. How much does this exceed the height of Mt. Everest?

49. The following is taken from the account book of the City Coal & Wood Co., and is a record of transactions between the company and John McBride for the month of March, 1909. The left-hand side shows the amount of his purchases from the company, or debits, for which he is the **debtor** (Dr.). The right-hand side shows the money he paid the company during the month, or credits, for which he is the **creditor** (Cr.).

A record of business transactions is called an **account.**

Dr. JOHN McBRIDE Cr.

1909					1909				
Mar.	2	8 Tons Lehigh Coal @ $6.25	50	00	Mar.	20	Cash .	60	00
Mar.	8	4 Cords hd. Wood . @ 8.50	34	00	Mar.	23	Cash .	25	00
Mar.	15	5 bbl. Port. Cement @ 2.20	11	00					
							Balance	10	00
			95	00				95	00
Apr.	1	Balance due	10	00					

In the above account, the last item on the credit side is the sum required to make the two sides of the account equal. It is called the **balance.**

The balance is found by subtracting the less from the greater side of the account. The balance is entered on the less side and thus makes the footing of the two sides equal.

Entering the balance and the footing, and drawing the proper lines is called **closing the account.**

Had the amount of cash paid exceeded Mr. McBride's purchases, the balance would have been on the left or Dr. side of his account.

Add to the above account the following transactions:

Bought, Mar. 18, 5 cords hd. wood at $8.50; Paid, Mar. 28, $75. On which side of the account is the balance now? Close the account.

Written Exercise

50. 1. Close the following account and reopen it for October:

DR. RUFUS DAY CR.

190–					190–				
July	10	To Plough . . .	7	80	July	31	By cash . .	20	00
"	15	" Grass seed . .	3	30	Aug.	31	" cash . .	12	50
"	17	" Flower seed .	5	00	Sept.	28	" Provisions .	9	38
"	20	" Fertilizers . .	5	00					
"	30	" Lawn mower .	9	00					
Aug.	7	" Trellis . . .	1	75					
"	12	" Garden hose .	3	88					
"	18	" Sprinkler . .		67					
"	20	" Wheelbarrow .	7	50					
Sept.	28	" Paints . . .	13	76					

2. The following is an account for the month of March, 1908. Close it and make the first entry for April.

DR. FAMILY EXPENSE CASH ACCOUNT CR.

1908					1908				
March	1	To balance . .	5	80	March	3	By Fuel . . .	14	50
"	1	" allowance .	125	00	"	4	" Rug . . .	7	00
					"	6	" Plumbing .	3	80
					"	7	" Chairs . .	5	00
					"	8	" Couch . .	15	00
					"	31	" Groceries .	29	46
					"	"	" Provisions .	15	18
					"	"	" Milk . .	4	96
					"	"	" Domestic .	20	00
					"	"	" Stoker . .	10	50
					"	"	" Balance .		

Written Exercise

51. Make the rulings for the following accounts and make up and close the accounts:

1. Briggs Thomas's debits are for: Goods bought Jan. 1, $762.87; Jan. 8, $922.87; Jan. 15, $46.13.

His credits are for: Cash paid Jan. 10, $1208.23; Jan. 15, $163.55.

2. Edward Homer & Co.'s debits are for: March 31, railroad fares, $48.75; coach fares, $2.25; livery charges, $6.88; hotel charges, $57.60; laundry, $3.63; expressage, $.45; porter, $1.90.

Their credits are for: Cash, Mar. 1, $50; Mar. 15, $30; Mar. 25, $25.

3. Freeman Holbrook's debits are for purchases: May 1, 1 table, $4.50; 1 chair, $1.38; May 16, 1 hat tree, $8.34; 1 lounge, $18.75; 1 towel rack, $0.88; 1 ice chest, $12.50; crating, $5.60.

His credits are for: Cash, May 25, $15; May 30, $15.

4. Blair, Blake & Co.'s debits for freight charges are: Jan. 5, flour, $96.75; Jan. 24, flour, $168.75; Jan. 31, oats, $36.32; Feb. 8, corn, $7.04; Feb. 20, potatoes, $16.30; Feb. 24, carriage, $3.20; horse, $17.

Their credits are for: Balance, Jan. 1, $15.80; cash, Feb. 3, $200; Feb. 24, $150.

5. A household expense account has "expenses," *i.e.* money paid out, called credits: Jan. 31, $109.39; Feb. 28, $82.83; Mar. 31, $123.42; Apr. 30, $121.67; May 31, $127.44; June 30, $101.51.

It has cash received called debits: Jan. 1, $125; Feb. 1, $122; April 1, $118; April 15, $50; May 1, $116; May 20, $95; June 1, $110.

Written Exercise

52. 1. Find the total number of acres of the free lands of the United States open to settlement in 1908, which were as follows:

Alabama	129,713	Louisiana	116,249	No. Dakota	2,322,150
Alaska	368,021,509	Michigan	235,551	Oklahoma	86,339
Arizona	42,769,202	Minnesota	1,788,705	Oregon	16,957,913
Arkansas	1,061,185	Mississippi	42,791	So. Dakota	6,561,295
California	29,872,493	Missouri	27,480	Utah	36,578,998
Colorado	23,696,697	Montana	46,532,440	Washington	4,635,001
Florida	414,942	Nebraska	3,074,658	Wisconsin	13,280
Idaho	26,785,002	Nevada	61,177,050	Wyoming	37,145,302
Kansas	171,446	New Mexico	44,777,905	Total	

2. Figure up the total of the scores in this bowling tournament, held at S.:

Games Reds	First Scores	Second Scores	Third Scores	Games Blues	First Scores	Second Scores	Third Scores
Moore	164	167	183	Hilton	140	178	149
Richards	151	145	202	Bruce	149	164	107
Patz	158	202	157	Burnham	157	211	189
Babcock	168	205	169	Naylor	169	177	196
Williams	176	153	146	Peard	216	136	182

Which team won, and by how much?

3. The average monthly fire loss for 10 months of a certain year in the United States and Canada was $41,446,000; for the single month of October it was $13,872,000. How much greater was the average?

4. The value of the imports into the United States one year was $1,194,341,792 and of the exports $1,860,773,346. By how much did the latter exceed the former?

Written Exercise

53. **1.** In 1845 the amount of copper produced in the United States was 200,000 lb. In 1907 the amount produced was 879,241,766 lb. What was the increase in the copper output during the interval?

2. In 1906 the copper output of the United States exceeded that of 1907 by 38,378,234 lb. What was the amount for 1906?

3. The value of the United States' output of precious stones for one year was as follows: sapphire, $125,000; turquoise, $65,000; tourmaline, $50,000; peridot, $10,000; crystal quartz, $10,000; miscellaneous stones, $10,100; aquamarine, $6000; Kunzite, $5000; gold quartz, $5000; chrysoprase, $5000; silicified wood, $5000; garnet, $5000; smoky quartz, $3000; chlorastrolite, $3000; amethyst, $2000; agate, $2000; pyrite, $2000; malachite, $2000; anthracite, $2000; catlinite, $2000; moss agate, $1500; beryl, $1000; rose quartz, $1000; arrow points, $1000; Amazon stone, $1000; topaz, $500; utahlite, $500; mesolite, $500; fossil coral, $250. Find the total value.

4. An ordinary passenger car weighs 90,000 lb. A Pullman sleeper weighs approximately 160,000 lb. Find the sum and difference of their weights.

5. The cost of living for the average American wage-earning family was recently estimated as follows: food, $326.90; clothing, $107.91; rent, $99.53; fuel and light, $40.34; furniture and utensils, $26.28; tobacco and liquors, $40.36; sickness and death, $20.52; amusements and vacations, $12.30; charity and religion, $9.99; books and newspapers, $8.38. Find the average total.

6. If the surplus earnings averaged $58.65, what was the average income?

Written Exercise

54. In the city of F. the vote for mayor by wards was:

Names	Ward 1	Ward 2	Ward 3	Ward 4	Ward 5	Ward 6	Total
Curtis B. Stickney . .	84	102	142	108	117	168	
Orrin J. Noyes	102	61	74	111	102	82	
Paul D. George . . .	30	23	66	116	12	25	
Harry Plummer . . .	18	2	4	0	2	3	
Albert C. Wood . . .	185	105	177	142	100	183	
Totals							

1. Which candidate was elected? How much did his vote exceed that of the next highest candidate? How much did it exceed the vote of the lowest candidate?

2. The National Fire Insurance Company of Hartford, Conn., issued the following statement of assets and liabilities, Jan. 1907:

Assets, $ 7,076,852.54. Liabilities, $ 7,077,152.45.

Find the difference between the assets and liabilities.

3. In the St. John river basin in Maine it was estimated that there were standing, one year, 6,942,000,000 ft. of spruce timber, 427,000,000 ft. of pine, and 1,830,000,000 ft. of cedar. What was the total estimate? How much did the estimated cedar exceed the estimated pine?

4. The receipts of the U. S. post office at A. for one quarter were $ 26,623.82 against $ 20,030.56 for the corresponding quarter of the year preceding. What was the increase?

5. The registry business of the A. post office for six months amounted to 12,157 pieces, against 9880 pieces for the corresponding six months of the year preceding. What was the difference?

Written Exercise

55. **1.** Find the distance covered in a single day by an automobile that took Mr. B. to and from his office 3 times, going 24 mi. in all, and on a business trip 3.5 mi.; that went once for a forgotten article 8 mi.; that took the children to school 2 mi.; that delivered a package 2.5 mi.; that took Mrs. B. shopping 10.5 mi., on an errand 1 mi., and on a calling trip 16 mi.; that made a trip to meet friends at the depot 8.5 mi., and twice to the theater 16 mi.; that made a trip for the doctor 1.5 mi., and back 1.5 mi.; and that went to the drug store and back 8 mi.

POLICE FORCE OF NEW YORK CITY IN 1908

[Exclusive of Inspectors and other special officers]

BOROUGHS	CAPTAINS	LIEUTENANTS	SERGEANTS	PATROLMEN	DOORMEN	POLICE MATRONS
Manhattan	37	325	276	4779	80	41
The Bronx	9	38	45	662	18	4
Brooklyn	31	178	171	2272	65	21
Queens	10	48	50	528	20	2
Richmond	4	18	21	157	6	2

2. Find the totals by boroughs, by offices, and the grand total.

Provide items and balance the following accounts:

3.		4.		5.	
DR.	CR.	DR.	CR.	DR.	CR.
$ 3.185	$ 59.00	$ 9.64	$ 8.28	$41.875	$50.50
41.07	18.05	17.18	13.63	98.21	5.84
63.98	121.48	26.73	5.41	3.67	17.63
0.44	62.18	19.48	7.86	58.42	2.075
37.50	49.56	0.29	28.45	8.625	27.68
25.00	25.00	30.48	4.18	7.34	18.28

56. There are 7 days in 1 week. In 9 weeks there are how many days? To find the answer we may (1) add thus: $7+7+7+7+7+7+7+7+7=63$, or (2) in a shorter way we may unite nine 7's, saying 9 times 7 are 63.

The process of uniting a given number of equal numbers to find their amount is called **multiplication.**

One of the equal numbers (or the number to be multiplied) is called the **multiplicand.**

The number by which we multiply (*i.e.* the number of equal numbers) is called the **multiplier.**

The number found by uniting a number of equal numbers is called the **product.**

The multiplier and the multiplicand are called **factors** (makers) of the product. The product is called a **multiple** of each of its factors.

57. Illustrative Example. If 1 faucet discharges 231 cubic inches of water a minute, how much will 12 faucets of the same capacity discharge in the same time?

Work	Proof
231	12
12	231
2772	2772

Solution and Explanation. Since 1 faucet discharges 231 cubic inches, 12 will discharge 12 times 231 cubic inches. 12 times 1 = 12, = 1(0) and 2(units). We write 2 units below the line and reserve the 1 ten. 12 times 3(0) = 36(0), with the 1(0) reserved = 37(0). We write 7 tens below the line and reserve the 3(00). 12 times 2(00) = 24(00), with the 3(00) reserved = 27(00). We write 27 hundreds below the line and find the 12 faucets will discharge 2772 cubic inches in 1 minute. *Ans.* 2772 cu. in.

The sign $\times$ is used to indicate multiplication.

The multiplication of 231 cu. in. by 12 may be expressed either 231 cu. in. $\times$ 12, read "231 cubic inches multiplied by 12," or 12 $\times$ 231 cu. in., read "12 times 231 cubic inches." When both are considered as abstract numbers, 231 $\times$ 12 is read "231 times 12," or "231 multiplied by 12."

To prove the work in multiplication, we multiply the multiplier by the multiplicand.

The multiplicand may be either an abstract or a concrete number.

The multiplier is always used as an abstract number.

The product and multiplicand are always like numbers.

The product is numerically the same whichever of the two factors is regarded as the multiplier.

58. The table given below is for reference purposes. To find the product of any two factors, find the multiplicand in the figures arranged horizontally at the top and the multiplier in the column at the left; the product is the number under the multiplicand and opposite the multiplier. The products underlined are *squares*, *e.g.* $4 = 2 \times 2$, $9 = 3 \times 3$.

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625

Oral Exercise

59. **1.** Multiply each of the numbers from 2 to 12 by each of the following numbers, giving only products:

2, 10, 5, 1, 4, 8, 3, 6, 9, 7, 11, 12.

Thus, the products by 2 are 4, 20, 10, 2, 8, 16, 6, 12, 18, 14, 22, 24.

2. Multiply each of the following numbers by each of the numbers from 2 to 12:

2, 10, 5, 1, 4, 8, 3, 6, 9, 7, 11, 12

3. How does the product of 12×2 compare with the product of 2×12? $231 \times 12 = 12 \times$?

4. Multiply: 3×4, 30×4, 300×4, 3×40, 30×40, 300×40.

Multiply:

5.	6.	7.	8.	9.
2×5	3×4	4×7	4×8	6×5
20×5	3×40	4×70	4×80	60×5
200×5	3×400	40×7	40×8	6×50
22×5	30×4	400×7	400×8	60×50
2×50	300×4	4000×7	4×800	600×50
2×500	33×4	44×7	4×88	600×500
2×55	3×44	4×77	44×8	66×5

Written Exercise

	a	*b*	*c*	*d*	*e*
60. **1.** Multiply	2736	4859	2543	8679	3952
by	2	2	3	3	4
2. Multiply	8764	5723	6079	6234	7589
by	4	5	5	6	6

Written Exercise

61. Multiply:

1. 7435×7	9. 6549×8	17. 2806×11
2. 8693×7	10. 1257×9	18. 3798×12
3. 9018×7	11. 9834×9	19. 7504×11
4. 2368×8	12. 8067×9	20. 2634×11
5. 9999×7	13. 2806×9	21. $23{,}798 \times 12$
6. 9765×8	14. 7809×8	22. $14{,}506 \times 12$
7. 4085×9	15. 3456×7	23. $62{,}436 \times 10$
8. 6274×7	16. 4098×9	24. $71{,}478 \times 8$

62. Illustrative Examples. Multiply (1) 625 by 1110; (2) \$6.25 by 110.

WORK

(1) 625
1110
6250, prod. by 10
62500, prod. by 100
625000, prod. by 1000
693750, prod. by 1110

EXPLANATION. (1) The multiplication of 625 by 10 is expressed by annexing a zero to the multiplicand, thus, 6250. The multiplication by 100 is expressed by annexing 2 zeros to the multiplicand, thus, 62,500. The multiplication by 1000 is expressed by annexing 3 zeros to the multiplicand, thus, 625,000. The sum of the partial products is 693,750. *Ans.* 693,750.

(2) \$6.25
110
62.50, prod. by 10
625.00, prod. by 100
\$687.50, prod. by 110

(2) The multiplication of \$6.25 by 10 is expressed by moving the decimal point one place to the right, thus, \$62.5. The multiplication by 100 is expressed by moving the point two places to the right, thus, \$625.00. The sum of the partial products is \$687.50. *Ans.* \$687.50.

In multiplying a number by 10, 100, 1000, *etc., the product may be expressed by annexing to the multiplicand as many zeros as the multiplier has zeros.*

If there is a decimal point in the multiplicand, we move it as many places to the right as the multiplier has zeros.

Written Exercise

63. Find the products:

1. 45×110	**3.** 4692×1110	**5.** $\$48.71 \times 110$
2. 689×110	**4.** 4692×10000	**6.** $\$6.375 \times 1010$

Written Exercise

64. Illustrative Examples. Multiply (1) 643 by 202; (2) \$6.43 by 200.

WORK

(1) 643
202
1286, prod. by 2
1286, prod. by 200
129886, prod. by 202

EXPLANATION. (1) The product of 643 by 2 is 1286. There are no tens in the multiplier, hence there is no product by tens. The product by 200 is expressed by annexing 2 zeros to the multiplicand, and multiplying by 2, which gives 1286(00). By writing this partial product with 6 in the hundreds' place, the zeros in the tens' and units' places can be omitted.

Ans. 129,886.

(2) \$6.43
200
\$1286.00

(2) The product of \$6.43 by 100 is expressed by moving the decimal point 2 places to the right. On multiplying the result by 2, the product is \$1286. *Ans.* \$1286.00.

In practice, we multiply a decimal by the number of tens, hundreds, etc., as if the decimal were a whole number, and insert the decimal point afterward.

Multiply:

1. 475 by 220	**8.** \$6.25 by 110	**15.** 48,076 by 505
2. 286 by 440	**9.** \$970 by 707	**16.** 29,135 by 909
3. \$8.60 by 550	**10.** \$5872 by 303	**17.** \$87.50 by 990
4. \$3.75 by 660	**11.** \$30.19 by 880	**18.** \$39.40 by 7700
5. \$1.28 by 990	**12.** \$7.75 by 404	**19.** \$25.05 by 880
6. \$9.40 by 770	**13.** \$35.08 by 220	**20.** \$98.40 by 6600
7. \$8.96 by 880	**14.** \$17.86 by 909	**21.** \$37.60 by 5500

Written Exercise

65. Illustrative Examples. Multiply (1) 3605 by 137; (2) 3.605 by 137.

WORK

(1) 3605		(2) 3.605	
137		137	
25235,	prod. by 7	25 235,	prod. by 7
10815 ,	prod. by 3(0)	108 15 ,	prod. by 3(0)
3605 ,	prod. by 1(00)	360 5 ,	prod. by 1(00)
493885,	prod. by 137	493.885,	prod. by 137

We multiply decimals by integers as if we were multiplying integers, and point off as many places in the product as the multiplicand has decimal places.

Multiply and prove:

1. 237 × 321	**7.** 243 × 576	**13.** $56.40 × 761
2. 432 × 342	**8.** 542 × 392	**14.** 63.49 × 645
3. 806 × 528	**9.** 1065 × 965	**15.** 405.9 × 9007
4. 871 × 604	**10.** $56.70 × 908	**16.** 6.874 × 650
5. 596 × 846	**11.** $7.88 × 837	**17.** .2138 × 407
6. 679 × 978	**12.** $9.08 × 842	**18.** .07919 × 590

19. 847 steps each 2.25 feet long, lack how much of being a mile, which equals 5280 feet?

20. How much can a clerk save in a year (52 weeks), if his income is $1500, his board $6.75 a week, and his other expenses $505 a year?

21. If my automobile makes 3 trips daily to my office and back, the distance each way being 3.5 miles, how many miles does it go in 313 days?

Written Exercise

66. 1. A gardener expended $42.625 per acre for fertilizers. What was the cost for 17 A.?

2. How much can a cabman save a year if he earns $1200 a year and has $20.80 of expenses per week? (Reckon 52 weeks as a year.)

3. When .768 is one factor and 231 the other, what is the product?

4. What is the tax on $6780 if the rate is 3 mills on a dollar?

NOTE. The tax is 6780 times $.003. It is easier, however, to use the smaller number as the multiplier, thus $6780 multiplied by .003.

5. An iceberg extends 375 ft. below the surface of the water; it extends .89 as many feet above the surface. What is its entire height?

6. To supply a boiler consuming 74 T. of coal a year, how much less was the cost in 1908 when coal was $6.75 a ton, than in 1864 when it was $12.50 a ton?

67. Illustrative Examples. Multiply $62.50 (1) by .1; (2) by .01; (3) by .001.

WORK

(1) $62.50 × .1 = $6.25
(2) $62.50 × .01 = $.625
(3) $62.50 × .001 = $.0625

EXPLANATION. To multiply $62.50 by .1 is to take one tenth of $62.50. To multiply it by .01 is to take 1 hundredth of it, and so on. 1 tenth of $62.50 is expressed by moving the decimal point 1 place to the left; thus, $6.25. 1 hundredth is expressed by moving the decimal point 2 places to the left; thus, $.625. 1 thousandth is expressed by moving the decimal point 3 places to the left; thus, $.0625.

To multiply an integral number by .1, *we put the decimal point one place to the left of the units; to multiply by* .01, *we put the point two places to the left; and so on. We prefix zeros to the multiplicand when necessary.*

Written Exercise

68. Illustrative Examples. Multiply 320 (1) by .11 ; (2) by .011; (3) by .0101.

Work

(1) 320	(2) 320	(3) 320
.11	.011	.0101
3.20, prod. by .01	.320, prod. by .001	.0320, prod. by .0001
32.0 , prod. by .1	3.20 , prod. by .01	3.20 , prod. by .01
35.20, prod. by .11	3.52 , prod. by .011	3.232Ø, prod. by .0101

1.	2.	3.	4.
Multiply 547 by .1101	4687 ×.111	$842.16 ×.111	$40000.56 ×.1111

Written Exercise

69. Illustrative Example. Multiply 773.56 by 2.14.

Work

773.56
2.14
30.9424, prod. by .04
77.356 , prod. by .1
1547.12 , prod. by 2
1655.4184, prod. by 2.14.

Explanation. We first multiply by .04. To multiply 773.56 by .04 is to take .04 of it; .01 is expressed by moving the decimal point 2 places to the left; thus, 7.7356; .04 is 4 times 7.7356 = 30.9424. .1 of 773.56 = 77.356. 2 times 773.56 = 1547.12. The sum of the partial products is 1655.4184.

Ans. 1655.4184.

In practice in multiplication of decimals, we multiply as we multiply integers and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier together.

Multiply and prove:

1. .14 × 9	4. 1.12 × .08	7. 8.25 × 2.03
2. .014 × 9	5. 17.7 × 3.07	8. 30.09 × .005
3. 1.69 × 2.4	6. 60.37 × 2.6	9. 42.05 × .007

Written Exercise

70. Multiply and prove:

1.	$43.7 \times .045$	**9.**	1.463×7.07
2.	6.002×2.08	**10.**	$.0098 \times 6.2$
3.	77.034×10.02	**11.**	$758.9 \times .0851$
4.	58.009×6.709	**12.**	$29{,}000 \times .00805$
5.	$.0075 \times 25$	**13.**	$75{,}800 \times .00078$
6.	$.046 \times .095$	**14.**	$91{,}000 \times 3.0059$
7.	$.0892 \times 1.0031$	**15.**	$90{,}000 \times .7076$
8.	9.806×4.603	**16.**	$80{,}000 \times .0009$

17. There are 16.5 ft. in one rod. How many feet are there in 26.5 rd.?

18. An express train runs at an average speed of 52.65 mi. per hour for 8.5 hr. How far does it go?

19. Find the cost of 17 lb. of chicken at \$.22 a pound, and 15 lb. of turkey at \$.28 a pound.

20. What will be the cost of 5 gal. of sirup at \$.65 a gallon, 30 bbl. of apples at \$3.75 a barrel, and 16 bu. of potatoes at \$.80 a bushel?

21. Multiply 48.564 by 10, by 100, by 1000, and add the products.

22. Multiply 3.709 by 100, by 10, by 1000, and add the products.

23. What is the cost of 16 yd. of silk at \$1.125 a yard, and 25 yd. of cambric at \$.07 a yard?

24. Multiply 30.007 by 10, by 100, by 1000, and find (*a*) the difference between the products by 10 and by 100; (*b*) the difference between the products by 100 and 1000.

Written Exercise

71. 1. At 30 ¢ an hour, how much can a person earn in 15 da. of 9 hr. each?

2. At \$.17 apiece for tumblers, what is the cost of 5 lots of 100 each?

3. What is the product if 18 thousandths is multiplied by 14 ten-thousandths?

4. One factor is 18,459, the other is 6.05. Find the product.

5. Two thousand four hundred and nine tenths multiplied by seven hundred six thousandths gives what product?

6. What is the cost of tuition for two pupils for a winter term of 13 wk. at \$1.25 each per week?

7. What amount was paid for a substitute teacher for a spring term of 11 wk. (5 da. to a week) at \$2.75 per day?

8. A town has 9500 school children and appropriates \$26.50 per child for school purposes. What is the total amount appropriated?

9. At a rate of \$15.40 per \$1000, what tax will be paid on property worth \$90,000?

10. At a cost of \$1.65 per foot, how much will be paid to bore a well that is 175 ft. deep?

11. 465 gross of pens at \$.23 a gross are ordered to supply the schools of the city of B. What is the cost?

12. Find the total weight of 15 cases of currants weighing 96 lb. each, 24 half cases weighing 56 lb. each, and 34 quarter cases weighing 30 lb. each. How much will they all cost at \$.125 a pound?

13. One factor is 9.08, the other is 908. What is the product?

Written Exercise

72. **1.** If it takes 21.8 lb. of milk to make 1 lb. of butter, how many pounds of milk will make 75 lb. of butter?

2. Milk is .9 water. In 565 gal. of milk there are how many gallons of water?

3. Butter is .14 water. How many pounds of water are there in 350 lb. of butter? [How many gallons of water are there if a pint weighs a pound?]

4. If meat loses .35 of its weight in roasting, what is the loss in weight of an 8.5-lb. roast?

5. A heavy steel rail weighs 29 lb. per foot. How much does it weigh per mile (5280 ft.)?

6. If it takes 4.0625 lb. of feed to make a hog gain 1 lb. in weight, how many pounds would be needed to effect a gain of 165 lb. in weight?

7. The profit of a flock of 125 hens and chickens averaged $\$1.12\frac{1}{2}$ each. What was the total profit from the flock?

8. A cubic foot of asphalt weighs about 127.7 lb. In paving a street 5703.75 cu. ft. were used. How much did it weigh?

9. A wood paving averages .75 of an inch in wear per year. In 9.33 years what is the wear?

10. A piece of land yielded 116.37 bu. of potatoes at $\$.65$ a bushel. What was the total value?

11. A circular water tank measures 24.9 ft. across. The distance around it is 3.1416 times as great. What is the distance around it?

12. At .7 ¢ a pound for shelled corn, find the value of the contents of 4 bins containing severally 63 bu., 54 bu., 37 bu., and 29 bu., reckoning 56 lb. to the bushel.

Written Exercise

73. Illustrative Example. A gallon measure contains 4 qt. How many gallons are there in 492 qt.?

WORK

divisor → 123, quotient
4)492, dividend

SOLUTION AND EXPLANATION. Since there are 4 qt. in 1 gal., in 492 qt. there are as many gallons as 4 is contained times in 492. 4 is contained in 4(00) 1 hundred times. We write 1 in the hundreds' place above the line. 4 is contained in 9(0) 2 tens times, and 1 ten over. We write 2 in the tens' place above the line. The 1 ten left over with the 2 = 12. 4 is contained in 12, 3 times; we write 3 in the units' place above the line, and find that the number of gallons in 492 qt. is 123. *Ans.* 123 gal.

The process of finding how many times one number is contained in another is called **division.**

The process of separating a number into equal parts is also called division. This kind of division is sometimes called **partition.**

The number to be divided is called the **dividend.**

The number we divide by is called the **divisor.**

The result of the division is called the **quotient.**

Division is expressed as above, also as follows:

$492 \div 4 = 123$, read 492 divided by 4 = 123, or

$\frac{492}{4} = 123$, read as above, or 1 fourth of 492 = 123.

The sign $\div$ means *divided by.*

In practice, in dividing 492 by 4, we say or think simply, 4 in 4, 1, and write 1; 4 in 9, 2, and 1 over, and write 2; 4 in 12, 3, and write 3.

Divide:

1. $2\overline{)254}$	**4.** $3\overline{)681}$	**7.** $4\overline{)872}$	**10.** $5\overline{)590}$	**13.** $5\overline{)950}$
2. $2\overline{)396}$	**5.** $3\overline{)951}$	**8.** $4\overline{)488}$	**11.** $5\overline{)615}$	**14.** $4\overline{)648}$
3. $5\overline{)840}$	**6.** $6\overline{)906}$	**9.** $7\overline{)840}$	**12.** $8\overline{)960}$	**15.** $9\overline{)981}$

Oral Exercise

74. There are how many :

1.	2's in	4	14	40	400	4000	44	6	16	600
2.	3's in	3	6	60	600	66	9	90	18	2400
3.	3's in	21	24	210	27	270	2700	30	33	3000
4.	3's in	33	330	333	9	90	900	99	990	9090
5.	4's in	4	40	44	400	440	444	8	80	880
6.	4's in	48	420	808	488	888	12	120	124	1224
7.	4's in	16	160	1600	1680	1620	20	2000	220	2400
8.	5's in	5	50	55	500	550	555	10	100	1100
9.	5's in	100	20	200	220	2000	2200	25	250	6600
10.	6's in	6	60	600	66	660	666	12	120	210
11.	6's in	18	180	1800	24	240	2400	30	300	3300
12.	7's in	70	700	77	770	14	140	1400	21	1050
13.	7's in	714	28	280	350	49	490	4900	420	6300
14.	8's in	8	80	16	160	24	240	56	560	136
15.	8's in	40	400	4000	72	720	7200	84	840	1840
16.	8's in	800	808	880	48	480	4800	176	1760	520
17.	9's in	9	90	900	18	180	27	2700	720	378
18.	10's in	10	100	500	800	1000	120	1200	110	2220
19.	11's in	11	110	22	220	55	550	77	770	8800
20.	12's in	12	120	1200	24	240	48	480	60	960

Written Exercise

75. Divide :

1.	$4\overline{)856}$	3.	$3\overline{)1506}$	5.	$5\overline{)1105}$	7.	$6\overline{)1812}$	9.	$7\overline{)2198}$
2.	$4\overline{)1288}$	4.	$2\overline{)1524}$	6.	$5\overline{)3115}$	8.	$6\overline{)2580}$	10.	$7\overline{)7021}$

Oral Exercise

76. 1. At 5¢ each how many bags of peanuts can be bought for 35¢? for 60¢? for $1?

2. How many cords of wood at $8 a cord can be bought for $48? for $64? for $96? for $56? for $72?

3. Going at the rate of 9 mi. an hour, how many hours will it take to ride 45 mi.? 72 mi.?

4. At 20¢ an hour, how many hours will it take to earn 40¢? 80¢? 60¢? $1.10?

5. How many 5-lb. pails of lard will weigh 45 lb.? 55 lb.? 100 lb.? 200 lb.?

6. At 6¢ apiece, how many gimlets can be bought for 40¢ and how many cents remain?

In the above example, name the dividend; name the divisor; name the quotient.

In dividing, the number left over is called the **remainder**.

7. What is the remainder in Example 6?

8. At $7 each, how many plows can be bought for $20?

9. A man who has $100 can buy how many hats worth $8 each? worth $9 each? worth $11 each? worth $12 each?

10. From 32 oz. of silver there can be made how many spoons weighing 3 oz. each? 4 oz. each? 5 oz. each? 6 oz. each?

11. How many 10¢ pieces should be given for 50¢? for 500¢? for 80¢? for 800¢? for $2?

12. How many 10¢ pieces with how many cents will equal 63¢? 87¢? $1.15? $2.27?

13. How many dozen eggs are there in 30 eggs? in 90 eggs? in 120 eggs?

Written Exercise

77. Illustrative Examples. Divide (1) \$967 by \$4: (2) \$967 among 4 men.

WORK

(1) 241, quotient
\$4)\$967 —\$3, remainder

(2) \$241, quotient
4)\$967 —\$3, remainder
or, \$241¾, quotient

EXPLANATION. (1) To divide \$967 by \$4 is to find how many times \$4 there are in \$967. The quotient is an *abstract* number.

(2) If we divide \$967 among 4 men, each man will have one fourth of \$967. One fourth is found by dividing by 4. 1 fourth of 9(00) is 2(00) with 1(00) undivided. We write 2 in the hundreds' place in the quotient. 1(00) united with 6(0) = 16(0). 1 fourth of 16(0) = 4(0). We write 4 in the tens' place in the quotient. 1 fourth of 7 = 1 with 3 undivided. We write 1 in the quotient and express the division of 3 by 4, thus, ¾, in the quotient. *Ans.* \$241¾.

Here the quotient is a *concrete* number, as it is of the same denomination as the dividend.

PROOF (1)

\$4 × 241 = \$964
\$964 + \$3 = \$967

PROOF (2)

4 × \$241 = \$964
\$964 + \$3 = \$967

To prove the work in division, we multiply the divisor by the quotient and add to the product the remainder when there is one. The result should equal the dividend.

Divide and prove, using either of the preceding explanations:

1. 2)3524	6. 5)1135	11. 6)5007	16. 8)19291
2. 3)1070	7. 4)1185	12. 6)9991	17. 8)72981
3. 5)3040	8. 5)9433	13. 7)2208	18. 8)4988
4. 4)3224	9. 6)2725	14. 7)3822	19. 9)3427
5. 5)2505	10. 6)1860	15. 7)2400	20. 9)4483

Written Exercise

78. Illustrative Example. Divide 5480 by 10; by 100; by 1000.

WORK

5480, dividend
548.0, quotient by 10
54.80, quotient by 100
5.480, quotient by 1000

EXPLANATION. To divide 5480 by 10 is to take .1 of it; to divide by 100 is to take .01 of it, and so on. 1 tenth of 5480 is expressed by putting a decimal point 1 place to the left of the units; thus, 548.0; 1 hundredth is expressed by putting a decimal point 2 places to the left of the units; thus, 54.80; 1 thousandth by putting a decimal point 3 places to the left; thus, 5.480.

Express the quotient:

1. Of 7840 divided by 10; by 100; by 1000; by 10,000.
2. Of 64,061 divided by 100; by 10; by 1000; by 10,000.
3. Of 8709 divided by 1000; by 100; by 10; by 10,000.

Written Exercise

79. Illustrative Example. Divide 7620 by 500.

WORK

$$5|00\overline{)}76|20 \quad 15.24$$

or, $15\frac{24}{100}$

EXPLANATION. 500 = 5 times 100; so we may first divide by 100, and then by 5. $7620 \div 100 = 76.20$; $76.20 \div 5 = 15.24$. *Ans.* 15.24.

When we divide 6(.00) by 5, the 1(.00) which remains, with $20 = 1.20$ remainder.

When the divisor is a number of 10's, 100's, 1000's, *etc., we put a decimal point as many places to the left of units as the divisor has zeros at its right and then divide by the number of* 10's, 100's, *etc.*

1. Divide 5740 by 20; by 200.
2. Divide 6900 by 50; by 800.
3. $86{,}950 \div 40{,}000$.
4. $6830 \div 7000$.
5. In 36,000 minutes there are how many hours?

Oral Exercise

80. **1.** In 6 hr. a man travels 36 mi. How far does he go per hour?

SOLUTION. If he travels 36 mi. in 6 hr., in 1 hr. he will go 1 sixth of 36 mi. 1 sixth of 36 is 6. *Ans.* 6 mi.

2. In 10 rows there are 160 trees. How many is this to a row?

3. 11 cans contain 88 qt. of milk. How much is this per can?

4. What is the price of 1 orange when 9 cost 36¢? 54¢?

5. A circle is divided into 360 equal parts called degrees. How many degrees are there in a half circle? in a quarter circle?

6. A yard contains 36 in. How many inches are there in 1 third of a yard? in 1 half of a yard? in 1 fourth of a yard?

7. There are 365 days in one year. How many days are there in 1 fifth of a year? in 1 fourth of a year?

8. Ten telegrams of ten words each from New York to Alaska cost $27. How much did 1 cost?

Written Exercise

81. **1.** There are 480 sheets in a ream of paper and 20 quires in a ream. How many sheets are there in a quire?

2. A man walks 162 mi. in 6 da., walking 8 hr. a day. What is the distance per day? per hour?

3. A 25-trip railroad ticket sells for $2.40. What is the cost per trip?

4. A legal bushel of coarse salt in Illinois weighs 50 lb. If 40 bu. in Pennsylvania equal 64 bu. in Illinois, what is the legal weight per bushel in Pennsylvania?

Written Exercise

82. Illustrative Examples. (1) Divide 174.6 by 4; (2) 170 by 7.

WORK

(1) $4\overline{)174.6}$ — $43.6\frac{2}{4}$

(2) $7\overline{)170.000}$ — 24.285^{+}

EXPLANATION. (1) 174.6 = 1746 tenths; 1746 tenths ÷ 4 = $436\frac{2}{4}$ tenths = $43.6\frac{2}{4}$. By annexing a zero to 174.6 thus, 174.60, and dividing, the quotient is 43.65. *Ans.* 43.65.

(2) Here 170 ÷ 7 = 24, with 2 remainder. Annexing zeros to the dividend, we extend the division to thousandths. *Ans.* 24.285+.

In dividing a decimal by a whole number, we point off as many places in the quotient as the dividend has decimal places.

1. Divide 26.3 by 5; by 6; by 4; by 7.

Divide:

2. 86.436 by 3	6. 244.034 by 7	10. 734.8 by 11
3. 223.28 by 4	7. 8.9075 by 5	11. $1.68 by 6
4. 240.08 by 8	8. 294.7 by 7	12. 288.9 by 9
5. 369.009 by 9	9. 246.036 by 6	13. 240.12 by 12

Compare the quotient of 6 ÷ 2 with that of 10 times 6 ÷ 10 times 2. The above comparison shows that the quotient is not affected by multiplying both dividend and divisor by 10. The principle applies to all numbers.

Multiplying or dividing both dividend and divisor by the same number does not alter the quotient. ◆

83. Illustrative Examples. (1) Divide 172.68 by .4; (2) 17 by .6.

WORK

(1) $4\overline{)172.6.8}$ — 431.7

(2) $6\overline{)17.0.000}$ — 28.333^{+}

EXPLANATION. (1) In dividing 172.68 by .4 we multiply 172.68 and .4 both by 10, which we express by moving the decimal point one place to the right. 1726.8 ÷ 4 = 431.7, *Ans.*

(2) In (2) we annex one zero to multiply by 10, the others to extend the division to 10ths, 100ths, and 1000ths. *Ans.* 28.333+.

Written Exercise

84. *In dividing a decimal by a decimal, we multiply both divisor and dividend by a number that will change the divisor to a whole number, and then divide and point off as in dividing a decimal by an integer.*

Divide, extending the quotient to thousandths if not exact.

1. 75.96 by .2
2. 87.09 by .07
3. 9.072 by .6
4. 436.2 by .5
5. 1.739 by .8
6. 5.782 by .005
7. 208.87 by .8
8. 774.92 by .7
9. 5930.83 by .04
10. 3062.2 by .7
11. 1728.9 by .03
12. 8076.085 by .004
13. $.002\overline{)4273.284}$
14. $.009\overline{).5778}$
15. $.04\overline{)6834.8}$
16. $.003\overline{)438.51}$
17. $.06\overline{)72147.6}$
18. $.9\overline{)6207.39}$

19. If 6.25 times a number is 87.5, what is the number?

20. How many rods (16.5 ft.) are there in a mile (5280 ft.)?

21. A French coin called a franc equals \$0.193; a German coin called a mark equals \$0.238. \$3 = how many francs? how many marks?

22. The price per ride being 9.84¢, how many rides will \$2.46 pay for?

23. How many yards of silk .75 of a yard wide will line a piece of goods 9 yd. long and 1.5 yd. wide?

24. The circumference of a wheel is about 3.1416 times its diameter. How long is the diameter of a wheel whose tire is 47.124 in.? Since there are 12 in. in 1 ft., what is the length around this tire in feet?

25. There are 5280 ft. in a mile. How many times will the wheel described in Ex. 24 revolve in going 1 mile?

26. One factor of 6.75 is .375. What is the other factor?

Written Exercise

85. Illustrative Examples. (1) Divide 29,515 by 12; (2) 29,515 by 72.

WORK

(1) SHORT DIVISION.

2459.583+ *Ans.*
12)29515.000

(2) LONG DIVISION.

409$\frac{7}{12}$, quotient
72)29515, dividend
288, product by 4(00)
715, undivided part of dividend
648, product by 9
67, remainder.

EXPLANATION. (2) 72 is not contained in 2(0000) any ten thousand times, nor in 29(000) any thousand times. It is contained in 295(00) about 4(00) times. We write 4 in the hundreds' place in the quotient. 72 multiplied by 4(00) = 288(00); 288(00) from 295(00) leaves 7(00). Uniting 1(0) of the dividend with 7(00) gives 71(0). 72 is contained in 71(0) no tens times. We write 0 in the tens' place in the quotient. Uniting 5 of the dividend with 71(0) gives 715. 72 is contained in 715 9 times with a remainder of 67. We write 9 in the units' place of the quotient. $1 \div 72 = \frac{1}{72}$ and $67 \div 72 = \frac{67}{72}$, which we write in the quotient, and for the entire quotient we have $409\frac{7}{12}$. *Ans.* $409\frac{7}{12}$.

If the division is extended to hundredths, the quotient is 409.93+.

When, as in (1) above, the products and remainders are not written down, the process is called **short division.**

When, as in (2), they are written down, the process is called **long division.**

Divide and prove, carrying the division to 100ths if not exact:

1. 3864 by 21
2. 6438 by 31
3. 9607 by 42
4. 4076 by 62
5. 5081 by 59
6. 52)1357
7. 63)3579
8. 74)2543
9. 81)3857
10. 89)68750
11. 30,742 ÷ 78
12. 48,685 ÷ 227
13. 743,054 ÷ 56
14. 938,601 ÷ 29
15. 87,256 ÷ 94

Written Exercise

86. Illustrative Examples. (1) Divide $371.84 by 83; (2) $166.42 by .83.

WORK

(1)		(2)
$4.48		$200.506
83)$371.84		.83)$166.42.000
332,	prod. by 4.	166
39 8,	undivided part of dividend	42 0
33 2,	prod. by 4	41 5
6 64,	undivided part of dividend	500
6 64,	prod. by 8	498
		2

EXPLANATION. (2) In dividing $166.42 by .83 we multiply 166.42 and .83 both by 100, which we express by moving the decimal point 2 places to the right. 16,642 ÷ 83 = 200 with 42 remainder. Annexing zeros to the dividend and continuing the division to mills, gives the quotient $200.506+, *Ans.*

In dividing a decimal by a decimal always change the divisor to an integral number.

Divide and prove, carrying the division to 1000ths if not exact:

1. 471.2 by 31
2. 96.32 by 43
3. 58.77 by .54
4. 67.48 by 6.2
5. 121.57 by 23
6. 24.898 by .72
7. 36.872 by 8.4
8. 360.72 by 19.1
9. 6.842 ÷ 2.2
10. 13.872 ÷ .15
11. 16.8259 ÷ .028
12. 4917.6 ÷ .016

WORK OF EXAMPLE 12

307 350

.016)4917.600.

117, undivided part of dividend

5 6, undivided part of dividend

800, undivided part of dividend

down only the remainders, as shown in the model.

EXPLANATION. In Example 12 to multiply the dividend by 1000 we have to annex 2 zeros. The work in long division can be abbreviated by writing

Ans. 307,350.

Written Exercise

87. Divide, extending the quotients to 1000ths if not exact:

1. $.2\overline{)439.75}$	12. $80\overline{)3427.8}$	23. $15\overline{)6207.09}$
2. $.02\overline{)695.37}$	13. $800\overline{)4.6735}$	24. $.17\overline{)3062.2}$
3. $.002\overline{)500}$	14. $.008\overline{)438.96}$	25. $.13\overline{)3122.6}$
4. $30\overline{)1505.4}$	15. $.9\overline{)191.42}$	26. $.014\overline{)185090}$
5. $300\overline{)3111.9}$	16. $90\overline{)38.106}$	27. $3.2\overline{)78.9386}$
6. $.003\overline{)1235.28}$	17. $.004\overline{)547.25}$	28. $1600\overline{)5.4691}$
7. $.06\overline{)2184}$	18. $900\overline{)90895}$	29. $170\overline{)226.51}$
8. $60\overline{)21865}$	19. $.04\overline{)4200.8}$	30. $.84\overline{)98834}$
9. $7\overline{)82564}$	20. $.0005\overline{)6207.09}$	31. $.018\overline{)2385.61}$
10. $700\overline{)6688}$	21. $50\overline{).37398}$	32. $1900\overline{)4700}$
11. $7000\overline{)58800}$	22. $.012\overline{)3062.2}$	33. $12800\overline{)6.4944}$

Work of Example 33

$$
\begin{array}{r}
.00\ 0507\frac{8}{8} \\
128.00\overline{).06\,4944} \\
\underline{944}\text{, part of undivided dividend} \\
\underline{48}\text{, part of undivided dividend}
\end{array}
$$

Explanation. In Example 33 we first divide by 100 and then by 128. To express the division by 100 we move the decimal point in 6.4944 two places to the left, thus, .064944. .064944 ÷ 128 gives no 10ths, no 100ths, and no 1000ths. We write zeros in these places in the quotient. .064944 ÷ 128 = .0005, with .0009 undivided. We write (.000)5 in the quotient. Uniting .00004 to .0009 gives .00094. .00094 ÷ 128 gives no hundred-thousandths. We write zero in hundred-thousandths' place in the quotient. Uniting .000004 to .00094 gives .000944. .000944 ÷ 128 = .000007. We write (.00000)7 in the quotient. The remainder, $\frac{48}{128} = \frac{8}{8}$.

Ans. $.000507\frac{8}{8}$.

Written Exercise

88. Divide, extending quotients 4 decimals if not exact:

1. 13,398 by 462	9. $3.847 \div .462$	17. $.659\overline{)2641.54}$
2. 63,840 by 760	10. $1.246 \div .0573$	18. $.0492\overline{)609}$
3. 47,613 by 807	11. $427.18 \div 2610$	19. $6890\overline{)1.586}$
4. 12,753 by 351	12. $40.875 \div 1.83$	20. $.704\overline{)346.22}$
5. 28,032 by 573	13. $3.847 \div 61.02$	21. $9.047\overline{)480}$
6. 84,262.5 by 963	14. $147{,}500 \div 6190$	22. $8790\overline{)10{,}000}$
7. 766 by 2.14	15. $87.363 \div 548$	23. $9.04\overline{)362.052}$
8. 2.54 by 35.1	16. $15.767 \div 78.5$	24. $.0914\overline{).188{,}284}$

25. How many pairs of socks at \$.375 a pair can be bought for \$33.75?

26. 7.5 gal. of water weigh 62.5 lb. What is the weight of 1 gal.?

27. A balloon moving forward at the rate of 35.2 mi. an hour will require how much time to go 255.2 mi.?

28. Cab fares for a mile being \$.50, and \$.25 for any added distance within a half mile, how many fares for 1.5 mi. will \$9 pay for?

29. How many railroad fares at \$1.56 will \$73.32 pay for?

30. During recent years the average annual price of corn per bushel in the United States was 38.9¢, and the average annual value \$869,575,309. What was the number of bushels raised?

31. Estimating a cubic foot of pine wood to weigh .65 as much as the same volume of water, and a cubic foot of oak to weigh .845 as much as a cubic foot of water, what part of the weight of oak does the weight of pine equal? (Decimal to 5 places.)

Oral Exercise

89. **1.** If \$3.50 per day is paid for 7 days' work, at the same rate how much will be paid for 10 days' work?

2. How many hours' work @ \$.20 an hour will pay for 3 loaves of bread @ \$.10 and 4 lb. of beef at \$.125 a pound?

3. If 9 rows of corn yield 36 bu., how many bushels will 20 similar rows yield?

4. If an express train runs 120 mi. in 3 hr., how far can it go in 4 hr.?

5. If \$2 will buy 6 pk. of peas, how many pecks will \$12 buy?

6. If 6 boys can put the schoolroom in order in .1 of an hour, how many boys can do the work in .2 of an hour?

7. A field of rye was reaped by 12 men in .5 of a day. What length of time would be required for 15 men to reap the same amount?

8. If a cistern can be emptied in .2 of an hour by 3 pipes of equal size, in what time can it be emptied when only 2 of the pipes are open?

9. An easterly sea breeze was felt on the coast at half-past four. If its velocity was 5 mi. an hour, at what time was it felt, at a point directly west, 9 mi. inland?

10. What is the profit in buying 100 shares of stock at \$50 per share and selling them at \$65 a share?

11. If it takes 1 doz. lemons to make lemonade for 25 persons, how many lemons must be allowed for 300 persons?

12. At 3 for 5¢, how many oranges can be bought for \$1?

13. If 12 yd. of ribbon cost \$1, how much will 3 yd. of ribbon cost at the same rate?

Written Exercise

90. **1.** Find the sum of $272.91, $1423.96, $22, $908, $6034.89, and write the answer in words.

2. The sum of two numbers is 2,049,871; the greater is 1,029,546. What is the smaller number?

3. What number must be added to the sum of 3421, 1964, 134, and 269,635, to make 480,000?

4. Find the sum of 29.784, 2.68403, 5.9001, 8.405, 121.36, .529, and write the answer in words.

5. Subtract 4.26897 from 17.541.

6. How many dollars does each person receive if $1,321,515 are divided equally among 105 persons?

7. How many times is 4.63 contained in 364.1985?

8. Multiply 56,241 by 124.

9. Multiply 49.27 by 532.

10. A high school has a membership of 590 pupils. .5 of the entire number are in the 1st-year class, 120 are in the 2d-year class, 102 in the 3d-year class, and the rest are in the 4th-year class. How many are there in each class?

11. To the difference between 43.511 and 3.8127, add 9057.075.

12. Multiply 408,903 by 3508.

13. Multiply .5182 by .8215.

14. A man bought 115 shoats at $7.50 each and 42 at $5.25 each. If he sold the lot at $7.90 each, how many dollars did he gain or lose?

15. One factor of the number 913.68 is 1296. What is the other factor?

16. If 14 is added to a number and the sum multiplied by 6, the product is 90. What is the number?

Written Exercise (*Continued*)

17. Multiply 42.8635 by 5.269.

18. Divide 359,206 by 820.

19. Divide 60831.7 by 421.

20. Multiply .5778 by 6.741.

21. Divide .34833 by 17.

22. Divide 500 by .0025.

23. Divide 95.72040 by .00268.

24. Divide 270 marbles among 3 boys so that the first gets .2 of them, the second .3 of them, and the third .5 of them.

25. A man sold a number of suits of clothes for $885 which cost him $825. If he gained $2 on each, how many suits did he sell? How much did each cost him? For how much did he sell each suit?

26. If 1 cu. ft. of granite weighs 170 lb., what is the weight of a block containing 64 cu. ft.?

27. What is the total weight of 26 bales of cotton weighing an average of 450 lb. each?

28. What is the cost of 94 thousand bricks at $9.50 per thousand?

29. How many square yards of concrete at 62.5¢ per yard can be laid for $1055?

30. A silver dollar weighs 412.5 grains. How many grains are there in 105 silver dollars?

31. An ounce of pure gold is worth $20.67. How many ounces can be bought for $1736.28?

32. The birth rate in a certain city was 32 per thousand people. There were 45,000 people in the city. What was the total number of births?

33. If standard silver is .9 pure silver and .1 copper, how many ounces of pure silver are there in a quantity of standard silver weighing 167.5 oz.?

Oral Exercise

91. 1. What two integers multiplied together make 6? 7?

Integers which multiplied together make a number are called **factors** of the number.

A number like 7 that has no factors but itself and 1 is called a **prime number.**

A number like 6 that has other factors besides itself and 1 is called a **composite number.**

Factors that are prime numbers are called **prime factors.**

2. Tell which of the following are prime numbers:

1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 15, 17, 19, 20.

3. Name the composite numbers from 4 to 24.

As every number is the product of its factors, it must be exactly divisible by each of them; also by the product of two or more of them.

An integer that divides a number and leaves no remainder is called an **exact divisor** of the number.

A number that is exactly divisible by 2 is called an **even number**; and a number that is not exactly divisible by 2 is called an **odd number.**

4. Name the even numbers from 2 to 30; the odd numbers.

The following are tests by which we can tell at sight some of the factors, or exact divisors, of a number.

Any number is exactly divisible

(1) by 2 if its units' figure is 0, 2, 4, 6, or 8.

(2) by 4 or 25 if its units and tens are thus divisible.

(3) by 3 or 9 if the sum of its digits are thus divisible.

(4) by 6 if it is divisible by 3 and its units' figure is even.

(5) by 5 if its units' figure is 0 or 5.

(6) by 10 if its units' figure is 0.

(7) by 11 if the sums of the alternate digits are equal, or if their difference is thus divisible.

Oral and Sight Exercise

92. Applying the tests given on the preceding page, name at sight the numbers below that are exactly divisible by 2, 4, 3, 5, 10, 25 :

1. 24	**5.** 18	**9.** 507	**13.** 114	**17.** 484	**21.** 8064
2. 12	**6.** 490	**10.** 225	**14.** 893	**18.** 540	**22.** 7876
3. 45	**7.** 850	**11.** 459	**15.** 275	**19.** 922	**23.** 8569
4. 27	**8.** 738	**12.** 738	**16.** 672	**20.** 946	**24.** 9100

When the exact divisor of a given number is a composite number, its *factors* are exact divisors of the given number.

An exact divisor of a number is an exact divisor of any *multiple* of the number.

An exact divisor of two numbers is an exact divisor of the *sum* and of the *difference* of the numbers.

93. Illustrative Example. Find the prime factors of 360.

WORK

1st FORM

2	360
5	180
3	36
2	12
2	6
	3

2d FORM

10	360
9	36
4	4
	1

$360 = 2^3 \times 3^2 \times 5$

EXPLANATION. (1st Form.) The units' figure in 360 being 0, the numbers 2 and 5 are prime factors of 360. The sum of the digits being 9, 3 is a prime factor of 360. After dividing by 2, 5, and 3 as in the model, the quotient is 12, of which the prime factors are 2, 2, and 3.

Ans. The prime factors of 360 are 2, 5, 3, 2, 2, 3, or 2^3, 3^2, 5.

The small figures ^{3} and ^{2} written to the right of and above the factors 2 and 3 show how many times each factor occurs, and are called **exponents.**

The 2d Form above is left for the pupil to explain.

Written Exercise

94. *To find the prime factors of a number, we first divide the given number by one of its factors. The quotient thus obtained we then divide by one of its factors, and so continue dividing by factors till a prime number is obtained as quotient. Divisors that are composite numbers we resolve into their prime factors. These prime factors with the last quotient are the prime factors sought.*

Find the prime factors of the following numbers:

1. 116	**5.** 204	**9.** 364	**13.** 567	**17.** 4200
2. 164	**6.** 252	**10.** 600	**14.** 644	**18.** 1980
3. 280	**7.** 270	**11.** 880	**15.** 814	**19.** 6225
4. 175	**8.** 342	**12.** 720	**16.** 625	**20.** 5400

21. Name a factor or divisor of 12 that is also a factor of 30.

22. Name the largest factor common to 12 and 30.

Because 6 is the largest factor or divisor common to 12 and 30 it is called their **greatest common factor** or **divisor.**

We write g.c.f. for greatest common factor, and g.c.d. for greatest common divisor.

Written Exercise

95. Illustrative Example. Find the g.c.f. or g.c.d. of 70, 42, and 182.

WORK

$70 = 2 \times 5 \times 7$

$42 = 2 \times 3 \times 7$

$182 = 2 \times 7 \times 13$

$2 \times 7 = 14$, the g. c. d.

EXPLANATION. Factoring 70, 42, and 182, we find that 2 and 7 are the only prime factors common to 70, 42, and 182; their product, 14, must be the g.c.d. *Ans.* 14.

Find the g.c.f. or g.c.d. of:

1. 18 and 24	**3.** 20 and 30	**5.** 36, 48, and 72
2. 22 and 33	**4.** 63 and 81	**6.** 45, 60, and 150

Written Exercise

96. Illustrative Example. Divide 5 times 56 by 5 times 21.

WORK

$$\frac{8}{\cancel{56}\times\cancel{5}}{\cancel{21}\times\cancel{5}}=2\tfrac{2}{3}$$

$$3$$

EXPLANATION. We express the division of 5 times 56 by 5 times 21 as in the margin. 5 times 21 is contained in 5 times 56 as many times as once 21 is contained in once 56. Hence, rejecting the factor 5 in both divisor and dividend does not affect the quotient. This leaves 56 to be divided by 21. But 56 is the product of 8 by 7, and 21 is the product of 3 by 7, so we may reject 7 without affecting the quotient. Dividing 8 by 3 gives the quotient $2\frac{2}{3}$. *Ans.* $2\frac{2}{3}$.

Rejecting equal factors in dividend and divisor is called **canceling.**

Divide by canceling:

1. $6\times12\times9$ by 3×8
2. $8\times14\times5$ by 4×42
3. $30\times5\times8$ by 15×24
4. $4\times44\times2$ by 33×8
5. $14\times27\times84$ by 56×21
6. $50\times16\times3$ by 10×24
7. $132\times16\times54$ by 84×48
8. $64\times35\times18\times6$ by 63×8
9. $8\times8\times22\times46$ by $44\times8\times2$
10. $48\times32\times5$ by $96\times40\times10$
11. $52\times34\times12$ by $17\times26\times9$
12. $38\times56\times30$ by $14\times18\times100$
13. $169\times51\times33$ by $17\times26\times66$
14. $125\times42\times38$ by $75\times63\times70$

15. An iron bar 8 ft. long weighs 36 lb. Find the weight of a bar of the same thickness 100 ft. long. (Cancel when possible.)

16. A tree 60 ft. high casts a shadow 90 ft. long. What length of shadow will a tree 120 ft. high cast?

17. The work of 9 boys equals the work of 6 men. How many men's work equals that of 21 boys?

18. How many towels at 25¢ each can I get for a credit check received on the return of 7 yd. of silk at $1.75 a yard?

19. A boy worked 5 hr. a day for 18 da. at 18¢ an hour. How many music lessons at 90¢ each could he pay for with his earnings?

Written Exercise

97. Illustrative Example. Divide (1.75×1.3) by .65.

WORK

$$\frac{1.75 \times \cancel{1}\cancel{3}\cancel{0}^{2}}{\cancel{6}\cancel{5}} = 3.5$$

EXPLANATION. In order to make the divisor .65 a whole number we multiply it by 100, and at the same time multiply *one factor* (1.3) of the dividend by 100. We then have $\frac{1.75 \times 130}{65}$. On canceling 65 and 130, we have left in the dividend 2×1.75 which equals 3.5. *Ans.* 3.5.

1. Divide 7.5×3.33 by .045.

2. Divide $\$4.80 \times .66$ by .12.

3. Divide $\$8.75 \times 5.4$ by $6.3 \times .25$.

4. Divide $\$96.12 \times .17$ by $1.53 \times .6$.

5. A yacht has provisions for 12 persons 60 da. How long will the provisions serve 20 persons?

6. 126 lb. of sugar at 6¢ a pound will pay for how many eggs at 28¢ a dozen?

7. How many yards of cloth 24 in. wide will be required to line 6 yd., 27 in. wide?

8. The freight on 9.5 T. of merchandise was \$7.60. What was the freight on 2.240 T.?

9. How long will it take to pay a debt of \$390 at the rate of \$6.50 a month?

10. A plot of ground contains 6.3×5.76 square yards. How many pieces of turf each containing 6×1.5 sq. ft. will cover it, 9 sq. ft. making a square yard?

11. 20 bbl. of apples at \$2.375 a barrel were given in exchange for wheat at \$0.80 a bushel. How many bushels were received?

98. The product of two or more factors is called a **multiple** of each of the factors.

Name some products made by using 2 as a factor; by using 3; by using 5; by using 7.

Name the smallest product of both 2 and 3; of 2, 3, and 5; of 2, 3, 5, and 7.

The smallest product common to two or more numbers is called their **least common multiple** (l. c. m.).

We write l. c. m. for least common multiple.

The l. c. m. of 2 and 3 is 6; of 2, 3, and 5 is 30; of 2, 3, 5, and 7 is 210.

A multiple is exactly divisible by its factors; a common multiple of two or more numbers is exactly divisible by each of the numbers; and the least common multiple of two or more numbers is the least number that is exactly divisible by each of the given numbers.

99. Illustrative Example. Find the l. c. m. of 30, 63, and 70.

WORK

1st Form

$30 = 2 \times 3 \times 5$

$63 = 3 \times 3 \times 7$

$70 = 2 \times 5 \times 7$

Thus, $2 \times 3 \times 3 \times 5 \times 7 = 630$.

EXPLANATION. (1st Form.) We separate the numbers into their prime factors. The l. c. m. must have the factors of 30, viz. 2, 3, and 5. These we write. The l. c. m. must have the factors of 63, viz. 3, 3, and 7. We have one 3 in the factors of 30, so we need write only 3 and 7. The l. c. m. must have the factors of 70, viz. 2, 5, and 7. These we have in 30 and 63, so we need not write them. The product of 2, 3^2, 5, and 7 = 630, which is the l. c. m. *Ans.* 630.

WORK

2d Form

$70 \times 9 = 630$

(2d Form.) If we take the largest number, 70, for one factor, it is evident that in 63, or 7×9, the 7 is contained in 70. Hence, we take only the factor 9, thus, 70×9; $30 = 10 \times 3$; but 10 is already contained in 70 and 3 in 9. Hence, the l. c. m. is 70×9, or 630. *Ans.* 630.

Written Exercise

100. *To find the least common multiple of two or more numbers:* (1) *We separate each of the numbers into its prime factors. The product of all the different prime factors, each taken the largest number of times it occurs as a factor in any one of the numbers, is the least common multiple sought.*

Or, (2) *We may multiply the largest number by such factors of the other numbers as are not contained in the largest number.*

Find the l. c. m. :

1. Of 15, 21, 35, 45.
2. Of 12, 24, 16, 80.
3. Of 24, 56, 28, 32.
4. Of 14, 26, 32, 84.
5. Of 13, 15, 18, 52.
6. Of 52, 80, 30, 26.
7. Of 66, 42, 21, 120.
8. Of 90, 110, 165.

WORK OF EXAMPLE 8

5	90	110	165
2	18	22	33
3	9	11	33
11	3	11	11
	3	1	1

$5 \times 2 \times 3^2 \times 11 = 990.$

EXPLANATION. Here, by successive divisions, we find all the factors that are common to the several numbers. The product of the divisors with the remaining factors is the least common multiple. *Ans.* 990.

The above method for finding the l. c. m. is convenient when the numbers are many or large.

9. Find the l. c. m. of 36, 76, and 56, using either of the methods that have been explained.

10. What is the width of the narrowest street across which stepping stones either 5 ft., 4 ft., or 8 ft. long will exactly reach?

11. What is the width of the narrowest box that will exactly pack ribbons 3 in., 4 in., or 5 in. wide?

101. When a single thing or a unit of any kind is divided into equal parts, one or more of the parts is called a **fraction.**

If a foot measure is divided into 12 equal parts, 1 part is 1 twelfth of a foot; three parts are 3 twelfths of a foot; the whole is 12 twelfths of the foot.

1 twelfth, 5 twelfths, 1 third, 2 thirds, 1 fourth, 3 fourths, are called **common fractions.**

We write the common fraction 5 twelfths thus, $\frac{5}{12}$.

The figure written below the line shows the number of parts into which the unit is divided; it is called the **denominator.**

Denominator means *namer.* It gives the name to the parts.

The figure written above the line shows the number of parts in the fraction; it is called the **numerator.**

Numerator means *numberer.*

The numerator and denominator are called the **terms** of the fraction.

Oral and Written Exercise

102. 1. Read each of the following fractions:

$\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{6}{9}$, $\frac{11}{12}$, $\frac{11}{16}$, $\frac{15}{36}$, $\frac{37}{45}$, $\frac{81}{92}$.

2. Name the numerator of each of the above fractions.

3. Name the denominator of each of the above; tell what each shows.

Express in figures:

4. Ten thirteenths; twenty-nine thirtieths; nine eighths.

5. Six hundredths; ninety-nine one hundred twentieths.

6. One hundred seventy-eight one hundred seventy-ninths.

7. Five hundred thirty-three one thousandths.

8. One hundred sixty-five one hundred sixty-fifths.

9. Ten thousand one ten-millionths.

10. Three thousand five ten-thousandths.

Oral and Sight Exercise

103. 1. Tell which of these fractions is less than an integer, or whole number. $\$\frac{5}{8}$, $\$\frac{5}{2}$, $\$\frac{10}{10}$, $\frac{23}{15}$, $\frac{56}{24}$, $\frac{125}{125}$, $\frac{120}{200}$, $\frac{365}{30}$.

A fraction that is less than an integer or whole number is called a **proper fraction.**

A fraction that equals or exceeds an integer or whole number is called an **improper fraction.**

2. Name the proper fractions above. Name the improper fractions.

3. The improper fraction $\$\frac{5}{2}$ equals how many dollars and what besides?

An integer, or whole number, with a fraction, as $\$2\frac{1}{2}$, is called a **mixed number.**

$\$2\frac{1}{2}$ is read two and one half dollars.

Read the following:

4. $5\frac{1}{8}$ T. 6. $300\frac{4}{7}$ wk. 8. $6001\frac{3}{8}$ 10. $87\frac{1}{2}$

5. $65\frac{3}{8}$ pk. 7. $120\frac{9}{12}$ yr. 9. $\$300\frac{4}{10}$ 11. $\$9.83\frac{1}{3}$

Oral Exercise

104. Illustrative Examples. (1) Change \$5 to half dollars; to dimes; to quarters. (2) Change the mixed number $\$2\frac{3}{5}$ to a fraction.

SOLUTION. (2) $\$1 = \$\frac{5}{5}$; $\$2 = 2$ times $\$\frac{5}{5} = \$\frac{10}{5}$; $\$\frac{10}{5}$ with $\$\frac{3}{5} = \$\frac{13}{5}$.
Ans. $\$\frac{13}{5}$.

The process of changing the form of a number without changing its value is called **conversion** (or **reduction**).

Convert, or change, to improper fractions at sight:

	a	*b*	*c*	*d*	*e*	*f*
1.	$4 = \frac{}{5}$	$8 = \frac{}{2}$	$4 = \frac{}{7}$	$9 = \frac{}{6}$	$8 = \frac{}{12}$	$7 = \frac{}{8}$
2.	$3 = \frac{}{3}$	$12 = \frac{}{7}$	$9 = \frac{}{11}$	$1 = \frac{}{8}$	$50 = \frac{}{2}$	$100 = \frac{}{8}$
3.	$6 = \frac{}{4}$	$10 = \frac{}{10}$	$8 = \frac{}{6}$	$12 = \frac{}{4}$	$20 = \frac{}{5}$	$10 = \frac{}{10}$

Oral and Sight Exercise

105. Change to improper fractions at sight.

	a	*b*	*c*	*d*	*e*	*f*	*g*
1.	$5\frac{1}{2}$	$1\frac{9}{10}$	$8\frac{7}{8}$	$7\frac{5}{8}$	$10\frac{6}{7}$	$5\frac{6}{7}$	$8\frac{5}{6}$
2.	$6\frac{2}{3}$	$6\frac{3}{4}$	$7\frac{8}{9}$	$9\frac{2}{5}$	$5\frac{7}{8}$	$9\frac{6}{7}$	$8\frac{7}{12}$
3.	$3\frac{5}{9}$	$11\frac{4}{5}$	$1\frac{4}{5}$	$6\frac{5}{9}$	$16\frac{1}{2}$	$5\frac{8}{11}$	$4\frac{11}{20}$
4.	$6\frac{3}{7}$	$8\frac{6}{7}$	$12\frac{10}{11}$	$8\frac{4}{9}$	$9\frac{9}{10}$	$7\frac{11}{12}$	$9\frac{5}{6}$
5.	$3\frac{3}{4}$	$4\frac{5}{6}$	$5\frac{5}{9}$	$9\frac{4}{15}$	$4\frac{3}{11}$	$3\frac{7}{25}$	$40\frac{1}{8}$
6.	$4\frac{2}{5}$	$7\frac{3}{8}$	$7\frac{5}{11}$	$3\frac{3}{17}$	$7\frac{5}{12}$	$2\frac{3}{16}$	$16\frac{1}{3}$

Written Exercise

106. Illustrative Example. Change $48\frac{3}{5}$ to fifths.

WORK

$$48\frac{3}{5}$$
$$5$$
$$\frac{240}{5}+\frac{3}{5}=\frac{243}{5}$$

SOLUTION. $1=\frac{5}{5}$, $48=48$ times $\frac{5}{5}=\frac{240}{5}$, with $\frac{3}{5}=\frac{243}{5}$. *Ans.* $\frac{243}{5}$.

In practice, to change an integer or a mixed number to an equivalent fraction, we multiply the integer by the denominator of the fraction, and in case of the mixed number, add the numerator of the fraction to the product.

Change to improper fractions:

1.	$39\frac{2}{5}$	**7.**	$47\frac{3}{4}$	**13.**	$45\frac{5}{6}$	**19.**	$257\frac{1}{2}$
2.	$47\frac{5}{7}$	**8.**	$94\frac{11}{12}$	**14.**	$89\frac{7}{10}$	**20.**	$453\frac{4}{5}$
3.	$64\frac{7}{12}$	**9.**	$87\frac{7}{8}$	**15.**	$10\frac{10}{11}$	**21.**	$166\frac{2}{3}$
4.	$76\frac{8}{9}$	**10.**	$97\frac{4}{9}$	**16.**	$83\frac{1}{3}$	**22.**	$897\frac{56}{100}$
5.	$26\frac{5}{6}$	**11.**	$58\frac{3}{11}$	**17.**	$264\frac{4}{25}$	**23.**	$567\frac{7}{16}$
6.	$39\frac{5}{8}$	**12.**	$47\frac{5}{12}$	**18.**	$387\frac{18}{20}$	**24.**	$189\frac{16}{29}$

Oral and Sight Exercise

107. 1. How many dollars should be given in exchange for 10 half dollars? for 11 half dollars? for $\$11\frac{1}{4}$?

Change to whole numbers, at sight:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
2.	$\frac{6}{2}$	$\frac{12}{3}$	$\frac{16}{4}$	$\frac{24}{4}$	$\frac{30}{10}$	$\frac{56}{8}$	$\frac{96}{16}$	$\frac{60}{20}$
3.	$\frac{12}{2}$	$\frac{18}{6}$	$\frac{16}{8}$	$\frac{24}{12}$	$\frac{30}{6}$	$\frac{560}{8}$	$\frac{960}{16}$	$\frac{60}{15}$

Change to integers or mixed numbers:

	i	*j*	*k*	*l*	*m*	*n*	*o*	*p*
4.	$\frac{5}{2}$	$\frac{13}{3}$	$\frac{19}{4}$	$\frac{25}{5}$	$\frac{10}{10}$	$\frac{56}{7}$	$\frac{96}{10}$	$\frac{60}{20}$
5.	$\frac{10}{2}$	$\frac{20}{7}$	$\frac{33}{8}$	$\frac{46}{9}$	$\frac{51}{7}$	$\frac{36}{9}$	$\frac{48}{6}$	$\frac{100}{11}$
6.	$\frac{56}{5}$	$\frac{25}{4}$	$\frac{40}{5}$	$\frac{75}{6}$	$\frac{75}{8}$	$\frac{75}{9}$	$\frac{70}{7}$	$\frac{70}{14}$

Written Exercise

108. Illustrative Example. Change $\frac{564}{7}$ to a mixed number.

WORK

$$7\overline{)564}$$
$$80\frac{4}{7}$$

SOLUTION. $\frac{7}{7} = 1$, then in $\frac{564}{7}$ or 564 sevenths there are as many 1's as there are 7's in 564, which is $80\frac{4}{7}$. *Ans.* $80\frac{4}{7}$.

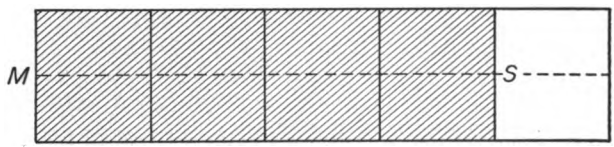
To change an improper fraction to an integer or a mixed number, we divide the numerator of the improper fraction by its denominator. The quotient is the integer or the mixed number required.

Change to whole or mixed numbers:

1. $\frac{145}{9}$	6. $\frac{175}{5}$	11. $\frac{96}{13}$	16. $\frac{191}{6}$	21. $\frac{269}{42}$
2. $\frac{316}{5}$	7. $\frac{576}{12}$	12. $\frac{584}{11}$	17. $\frac{781}{8}$	22. $\frac{1004}{10}$
3. $\frac{254}{6}$	8. $\frac{156}{7}$	13. $\frac{93}{7}$	18. $\frac{121}{9}$	23. $\frac{401}{84}$
4. $\frac{987}{7}$	9. $\frac{144}{16}$	14. $\frac{671}{11}$	19. $\frac{218}{28}$	24. $\frac{466}{150}$
5. $\frac{639}{9}$	10. $\frac{225}{15}$	15. $\frac{140}{13}$	20. $\frac{964}{190}$	25. $\frac{915}{250}$

Oral and Sight Exercise

109. 1. Into how many equal parts is the diagram divided by the vertical lines? What part of the whole is the one part not shaded?



2. What fraction of the whole is the shaded part?

3. Into how many equal parts is each of the $\frac{1}{5}$ divided by the dotted line *MS*? How many such parts are there in the whole diagram? What fraction of the whole is 1 of the 10 parts? How many tenths are there in the $\frac{1}{5}$?

4. By what number are the terms of $\frac{1}{5}$ multiplied to change it to $\frac{8}{10}$?

When both terms of a fraction are multiplied by the same number, the fraction is said to be **converted, reduced,** or **changed,** to **higher terms.**

Change to higher terms at sight:

5. $\frac{3}{4}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{4}{9}$, $\frac{5}{8}$, $\frac{3}{7}$, $\frac{3}{5}$

6. $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{5}{6}$, to 12ths

7. $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{2}$, $\frac{3}{4}$, to 20ths

8. $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{1}{4}$, to 16ths

9. $\frac{11}{12}$, $\frac{3}{8}$, $\frac{1}{3}$, $\frac{3}{4}$, to 24ths

10. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{5}{9}$, to 27ths

11. $\frac{1}{12}$, $\frac{2}{9}$, $\frac{2}{3}$, $\frac{3}{4}$, to 36ths

Tell how to change a fraction to higher terms.

Supply the numerator for each denominator below:

12. $\frac{2}{5} = \frac{\quad}{15} = \frac{\quad}{20} = \frac{\quad}{25} = \frac{\quad}{30} = \frac{\quad}{40} = \frac{\quad}{45} = \frac{\quad}{55} = \frac{\quad}{60} = \frac{\quad}{70}$.

13. $\frac{3}{4} = \frac{\quad}{8} = \frac{\quad}{12} = \frac{\quad}{16} = \frac{\quad}{20} = \frac{\quad}{24} = \frac{\quad}{36} = \frac{\quad}{40} = \frac{\quad}{48} = \frac{\quad}{80}$.

14. $\frac{4}{9} = \frac{\quad}{18} = \frac{\quad}{27} = \frac{\quad}{45} = \frac{\quad}{54} = \frac{\quad}{63} = \frac{\quad}{90} = \frac{\quad}{72} = \frac{\quad}{81} = \frac{\quad}{108}$.

15. $\frac{5}{6} = \frac{\quad}{12} = \frac{\quad}{18} = \frac{\quad}{24} = \frac{\quad}{30} = \frac{\quad}{36} = \frac{\quad}{42} = \frac{\quad}{54} = \frac{\quad}{72} = \frac{\quad}{66}$.

Written Exercise

110. Illustrative Example. Change $\frac{23}{25}$ to 100ths.

WORK

$$\frac{23}{25} \times \frac{4}{4} = \frac{92}{100}$$

SOLUTION AND EXPLANATION. Since $1 = \frac{100}{100}$, $\frac{1}{25}$ of $1 = \frac{1}{25}$ of $\frac{100}{100} = \frac{4}{100}$, which is found by dividing 100 by 25.

$\frac{23}{25} = \frac{23 \times 4}{25 \times 4} = \frac{92}{100}$. *Ans.* $\frac{92}{100}$.

To change a given fraction to higher terms, we multiply both terms of the given fraction by the number found by dividing the denominator of the required fraction by the denominator of the given fraction.

Change to equivalent fractions with denominators as indicated:

1. $\frac{11}{15}$ to $\frac{}{195}$
2. $\frac{52}{63}$ to $\frac{}{126}$
3. $\frac{99}{100}$ to $\frac{}{300}$
4. $\frac{5}{12}$ to $\frac{}{192}$
5. $\frac{6}{11}$ to $\frac{}{121}$
6. $\frac{17}{18}$ to $\frac{}{72}$
7. $\frac{28}{36}$ to $\frac{}{144}$
8. $\frac{25}{28}$ to $\frac{}{112}$
9. $\frac{18}{30}$ to $\frac{}{180}$
10. $\frac{9}{13}$ to $\frac{}{156}$
11. $\frac{7}{20}$ to $\frac{}{500}$
12. $\frac{57}{75}$ to $\frac{}{150}$
13. $\frac{12}{30}$ to $\frac{}{180}$
14. $\frac{36}{45}$ to $\frac{}{180}$
15. $\frac{6}{25}$ to $\frac{}{500}$

111. We have learned, Art. 109, that $\frac{4}{5} = \frac{8}{10}$, and generally that multiplying both terms of a fraction by the same number does not affect its value.

Dividing both terms of a fraction by the same number does not affect its value, thus, $\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$.

When both terms of a fraction are divided by a common factor, the fraction is said to be reduced, changed, or converted, to **lower terms**.

When both terms of the fraction are divided by the greatest common factor, the fraction is said to be changed to its **lowest terms**.

A fraction may be changed to its lowest terms by canceling all the factors common to both terms.

Written Exercise

112. Illustrative Example. Change $\frac{21}{36}$ to its lowest terms by canceling.

WORK

$$\frac{\cancel{21}^{7}}{\cancel{36}_{12}}$$

EXPLANATION. 3 is the only prime factor common to 21 and 36. On canceling 3 in $\frac{21}{36}$, the resulting fraction, $\frac{7}{12}$, expresses the fraction $\frac{21}{36}$ in its lowest terms.

Change to their lowest terms by canceling:

1. $\frac{9}{21}$ 2. $\frac{10}{60}$ 3. $\frac{18}{36}$ 4. $\frac{12}{84}$ 5. $\frac{20}{45}$ 6. $\frac{9}{54}$ 7. $\frac{8}{16}$ 8. $\frac{36}{72}$ 9. $\frac{10}{30}$ 10. $\frac{16}{20}$ 11. $\frac{20}{35}$ 12. $\frac{28}{48}$ 13. $\frac{70}{90}$ 14. $\frac{35}{56}$

Written Exercise

113. Illustrative Example. Change $\frac{52}{182}$ to its lowest terms.

WORK

$$\frac{\cancel{52}^{2}}{\cancel{182}_{7}}$$

$52 = 2 \times 2 \times 13$

$182 = 2 \times 7 \times 13$

EXPLANATION. We find on factoring the terms of the fraction $\frac{52}{182}$, that the g. c. d. is $2 \times 13 = 26$. Dividing both terms by 26 gives the result $\frac{2}{7}$. *Ans.* $\frac{2}{7}$.

$26)\overline{52} = \frac{2}{7}$

$26)\overline{182}$

$52)182(3$

156

$26)52(2$

52

When the g. c. d. is not easily seen, it can be found as here shown, by dividing the greater number by the less, the less by the remainder, and continuing so to divide till the remainder is 0. The last divisor is the g. c. d.

To change a given fraction to an equivalent fraction in its lowest terms, we cancel all the factors that are common to the terms of the fraction, or we divide both terms by their g. c. d.

Change to equivalent fractions in their lowest terms:

1. $\frac{89}{104}$ 2. $\frac{72}{88}$ 3. $\frac{66}{99}$ 4. $\frac{52}{91}$ 5. $\frac{108}{324}$ 6. $\frac{210}{240}$ 7. $\frac{148}{361}$ 8. $\frac{272}{546}$ 9. $\frac{2000}{2240}$ 10. $\frac{1.44}{7.2}$

114. 1. Read the fractions .2, .06, .25, .004, .125, .000008.

2. Write the above in the form of common fractions. Written as common fractions, each has for its denominator 10 or a number that can be made by using 10's only as factors.

Thus, the denominator $100 = 10 \times 10$; $1000 = 10 \times 10 \times 10$.

A fraction whose denominator is 10 or a number that can be made by using 10's only as factors, is called a **decimal fraction**, or a **decimal**.

Oral and Written Exercise

115. Illustrative Example. Change .26 to thousandths; .260 to millionths.

WORK

$.26 = .260$

$.260 = .260000$

EXPLANATION. To change a decimal to lower denominations we annex zeros to the figures till the place of the required denomination is filled.

1. Change 4.5 to hundredths; to thousandths; to ten-thousandths.

2. Change 4 to hundredths; to hundred-thousandths; to millionths.

Written Exercise

116. Illustrative Example. Change to common fractions .008; .26; .064.

WORK

$.008 = \frac{8}{1000} = \frac{1}{125}$

$.26 = \frac{26}{100} = \frac{13}{50}$

$.064 = \frac{64}{1000} = \frac{8}{125}$

EXPLANATION. We omit the decimal point and write the decimal with its denominator and then reduce the resulting fraction to its lowest terms.

Change to common fractions expressed in their lowest terms:

1. .4
2. .62½
3. .045
4. .06¼
5. .555
6. .56¼
7. .068
8. .087½
9. .0848
10. .016⅔

NOTE. In Ex. 2, $.62\frac{1}{2} = .625 = \frac{625}{1000}$, etc.

Written and Oral Exercise

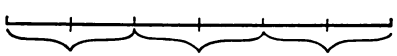
117. Change the following mixed decimals to mixed numbers with common fractions in their lowest terms:

1. 8.15	**4.** 12.625	**7.** 654.57	**10.** 915.66
2. 26.492	**5.** 84.05	**8.** 52.035	**11.** 57.435
3. 60.408	**6.** 126.16	**9.** 9.74	**12.** 18.036

Written Exercise

118. To change a common fraction to a decimal.

Illustrative Examples. (1) $\frac{2}{3}$ of 1 inch equals what part of 2 inches?



$\frac{2}{3}$ of 1 inch $= \frac{1}{3}$ of 2 inches

EXPLANATION. By the diagram we see that $\frac{2}{3}$ of 1 in. equals $\frac{1}{3}$ of 2 in. In the same way it may be shown that $\frac{3}{5}$ of 1 in. equals $\frac{1}{5}$ of 5 in.; that $\frac{3}{4}$ of 1 ft. equals $\frac{1}{4}$ of 3 ft.; that $\frac{4}{5}$ of 1 dime equals $\frac{1}{5}$ of 4 dimes, and generally, that any fraction equals the quotient of its numerator divided by its denominator.

(2) Change $\frac{3}{8}$ to a decimal fraction.

WORK

$$8\overline{)3.000}\quad .375$$

EXPLANATION. Since $\frac{3}{8}$ equals $\frac{1}{8}$ of 3, to change $\frac{3}{8}$ to a decimal, we divide 3 by 8, first changing 3 to 10ths, to 100ths, and 1000ths. 3 = 3.0 (thirty 10ths); $\frac{1}{8}$ of 3.0 = .3 with .6 undivided; .6 = .60 (sixty 100ths); $\frac{1}{8}$ of .60 = .07 with .04 undivided; .04 = .040 (forty 1000ths); $\frac{1}{8}$ of .040 = .005; which gives the decimal .375. *Ans.* 375.

Change to equivalent decimals:

1. $\frac{2}{5}$	**5.** $\frac{3}{4}$	**9.** $\frac{5}{8}$	**13.** $\frac{9}{8}$	**17.** $\frac{1}{16}$	**21.** $\frac{27}{20}$
2. $\frac{1}{4}$	**6.** $\frac{1}{8}$	**10.** $\frac{6}{8}$	**14.** $\frac{5}{16}$	**18.** $\frac{15}{16}$	**22.** $\frac{7}{40}$
3. $\frac{3}{5}$	**7.** $\frac{7}{8}$	**11.** $\frac{7}{16}$	**15.** $\frac{4}{25}$	**19.** $\frac{7}{32}$	**23.** $3.01\frac{1}{5}$
4. $\frac{4}{5}$	**8.** $\frac{3}{16}$	**12.** $\frac{3}{20}$	**16.** $\frac{19}{25}$	**20.** $\frac{5}{64}$	**24.** $871\frac{1}{4}$

Written Exercise

119. Illustrative Example. Change $\frac{5}{6}$ to a decimal.

WORK

$$\begin{array}{r} .83\frac{1}{3} \\ \hline 6)5.00 \end{array}$$

EXPLANATION. We cannot express as a decimal the exact equivalent of any common fraction whose denominator contains other prime factors than 2 and 5. However far the division is carried there will be a remainder. In such a case, when the figures begin to repeat, the rest of the quotient may be expressed as a common fraction, as in the illustrative example. *Ans.* $.83\frac{1}{3}$.

Or, the division may be carried to any desired number of decimal places. Thus, $\frac{5}{6}$ carried to 4 places equals .8333+; $3\frac{1}{7}$ expressed in decimals to 6 places equals 3.142857+.

The part of the decimal fraction that repeats is called a **repeating decimal.**

To change a common fraction to a decimal, we annex zeros to the numerator of the fraction and divide by the denominator.

Change to equivalent decimals carrying none to more than 6 places:

1. $\frac{5}{6}$	4. $\frac{8}{11}$	7. $\frac{13}{15}$	10. $\frac{19}{30}$	13. $.48\frac{2}{11}$	16. $\frac{56}{420}$
2. $\frac{7}{12}$	5. $\frac{17}{21}$	8. $\frac{17}{32}$	11. $.16\frac{4}{15}$	14. $.00\frac{7}{32}$	17. $402\frac{5}{18}$
3. $\frac{6}{7}$	6. $\frac{9}{14}$	9. $\frac{7}{25}$	12. $.05\frac{1}{14}$	15. $.9\frac{1}{11}$	18. $3.14\frac{6}{7}$

LEAST COMMON DENOMINATOR

120. Fractions that have the same denominator are said to have a **common denominator.**

To what fractions having a common denominator can we change $\frac{1}{3}$ and $\frac{1}{4}$?

We can change $\frac{1}{3}$ to $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, $\frac{5}{15}$, $\frac{6}{18}$, $\frac{7}{21}$, $\frac{8}{24}$, $\frac{9}{27}$, $\frac{10}{30}$, $\frac{11}{33}$, $\frac{12}{36}$, etc.

We can change $\frac{1}{4}$ to $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, $\frac{5}{20}$, $\frac{6}{24}$, $\frac{7}{28}$, $\frac{8}{32}$, $\frac{9}{36}$, etc.

Thus, we see that we can change $\frac{1}{3}$ and $\frac{1}{4}$ to fractions having the common denominators 12, 24, etc. The *smallest common denominator* to which both can be changed is 12.

The smallest common denominator of two or more fractions is called their **least common denominator.**

We write l. c. d. for least common denominator.

Oral and Written Exercise

121. Illustrative Example. Change $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to fractions having the least common denominator.

WORK

l. c. d. $6 \times 2 = 12$.

$$\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$$

$$\frac{5}{6} = \frac{5\times2}{6\times2} \times \frac{10}{12}$$

EXPLANATION. To get the l. c. d. of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ we take the l. c. m. of the denominators. This we find by multiplying the largest denominator, 6, by 2, which is the only factor in the denominators 3 and 4 not contained in 6. $6 \times 2 = 12$, the l. c. d. We change $\frac{2}{3}$ to 12ths by multiplying its terms by 4, $\frac{3}{4}$ to 12ths by multiplying its terms by 3, and $\frac{5}{6}$ to 12ths by multiplying its terms by 2.

Ans. $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{5}{6} = \frac{10}{12}$.

To change fractions to equivalent fractions having the least common denominator, we take for the least common denominator the least common multiple of the denominators of the fractions.

To obtain the new numerators, we multiply the numerator of each fraction by the number which its denominator must be multiplied by to produce the common denominator.

Change to fractions having the l. c. d.:

1. $\frac{2}{3}, \frac{1}{6}, \frac{3}{4}, \frac{1}{3}$
2. $\frac{1}{2}, \frac{3}{4}, \frac{7}{24}, \frac{2}{3}$
3. $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{4}{15}$
4. $\frac{1}{2}, \frac{1}{4}, \frac{3}{8}, \frac{3}{16}$
5. $\frac{1}{8}, \frac{7}{8}, \frac{9}{10}, \frac{3}{5}$
6. $\frac{5}{24}, \frac{11}{15}, \frac{1}{6}, \frac{3}{10}$
7. $\frac{4}{9}, \frac{5}{6}, \frac{3}{4}, \frac{7}{8}$
8. $\frac{9}{30}, \frac{9}{10}, \frac{3}{25}, \frac{7}{8}$
9. $\frac{7}{6}, \frac{2}{5}, \frac{11}{12}, \frac{2}{15}$
10. $\frac{2}{3}, \frac{2}{5}, \frac{3}{10}, \frac{5}{6}$
11. $\frac{1}{6}, \frac{5}{9}, \frac{1}{4}, \frac{7}{12}$
12. $\frac{1}{50}, \frac{1}{4}, \frac{1}{5}, \frac{3}{10}$
13. $\frac{1}{4}, \frac{7}{16}, \frac{3}{8}, \frac{5}{8}$
14. $\frac{4}{9}, \frac{5}{12}, \frac{5}{6}, \frac{7}{10}$
15. $\frac{5}{6}, \frac{4}{15}, \frac{7}{12}, \frac{3}{20}$
16. $\frac{1}{12}, \frac{7}{18}, \frac{4}{5}, \frac{7}{18}$
17. $\frac{4}{7}, \frac{2}{9}, \frac{3}{14}, \frac{5}{24}$
18. $\frac{2}{3}, \frac{7}{8}, \frac{17}{16}, \frac{5}{24}$
19. $\frac{3}{4}, \frac{4}{9}, \frac{5}{12}, \frac{2}{3}$
20. $\frac{3}{5}, \frac{5}{8}, \frac{1}{4}, \frac{3}{10}$
21. $\frac{7}{8}, \frac{1}{16}, \frac{2}{3}, \frac{5}{12}$
22. $\frac{5}{12}, \frac{8}{15}, \frac{2}{3}, \frac{3}{20}$
23. $\frac{11}{12}, \frac{2}{7}, \frac{4}{3}, \frac{5}{42}$
24. $\frac{5}{16}, \frac{7}{20}, \frac{5}{4}, \frac{4}{5}$

Written Exercise

122. Illustrative Example. Change $\frac{8}{57}$, $\frac{6}{114}$, $\frac{7}{95}$ to equivalent fractions having the least common denominator.

WORK

$$\begin{array}{r|lll} 3) & 57 & 114 & 95 \\ 19) & 19 & 38 & 95 \\ \hline & 1 & 2 & 5 \end{array}$$

EXPLANATION. By Art. 100, we find that the l. c. m. of the denominators 57, 114, and 95 is 570. We then proceed as in Art. 121. *Ans.* $\frac{80}{570}$, $\frac{25}{570}$, $\frac{42}{570}$.

l. c. m. $3 \times 19 \times 2 \times 5 = 570$

$$\frac{8 \times 10}{57 \times 10} = \frac{80}{570}; \quad \frac{5 \times 5}{114 \times 5} = \frac{25}{570}; \quad \frac{7 \times 6}{95 \times 6} = \frac{42}{570}.$$

Change to equivalent fractions having the least common denominator:

1. $\frac{5}{52}$, $\frac{19}{104}$, $\frac{28}{260}$
2. $\frac{7}{54}$, $\frac{8}{63}$, $\frac{19}{81}$
3. $\frac{17}{308}$, $\frac{8}{154}$, $\frac{21}{176}$
4. $\frac{7}{225}$, $\frac{12}{325}$, $\frac{17}{250}$
5. $\frac{31}{144}$, $\frac{13}{176}$, $\frac{7}{396}$
6. $\frac{8}{255}$, $\frac{16}{85}$, $\frac{19}{340}$
7. $\frac{17}{432}$, $\frac{41}{252}$, $\frac{7}{720}$
8. $\frac{14}{20}$, $\frac{17}{19}$, $\frac{13}{50}$
9. $\frac{57}{84}$, $\frac{18}{25}$, $\frac{21}{60}$

123. Comparing the fractions $\frac{3}{4}$ pk. and $\frac{5}{4}$ pk., we see that they are like parts of the same unit, 1 pk.

Fractions that are like parts of the same unit are called **similar fractions.**

Only similar fractions can be added or subtracted.

Oral and Sight Exercise

124. Illustrative Example. Find the sum and difference of $\frac{9}{10}$ and $\frac{7}{10}$.

WORK

$\frac{9}{10} + \frac{7}{10} = \frac{16}{10} = \frac{8}{5} = 1\frac{3}{5}$ The sum of $\frac{9}{10}$ and $\frac{7}{10}$ is $1\frac{3}{5}$. *Ans.* $1\frac{3}{5}$.

$\frac{9}{10} - \frac{7}{10} = \frac{2}{10} = \frac{1}{5}$ The difference of $\frac{9}{10}$ and $\frac{7}{10}$ is $\frac{1}{5}$. *Ans.* $\frac{1}{5}$.

Find the sum and difference of:

1. $\frac{7}{8}$ and $\frac{5}{8}$
2. $\frac{11}{12}$ and $\frac{5}{12}$
3. $\frac{9}{7}$ and $\frac{6}{7}$
4. $\frac{11}{15}$ and $\frac{9}{15}$
5. $\frac{17}{20}$ and $\frac{5}{20}$
6. $\frac{16}{5}$ and $\frac{11}{5}$
7. $\frac{25}{18}$ and $\frac{5}{18}$
8. $\frac{25}{7}$ and $\frac{15}{7}$

Written Exercise

125. Illustrative Example. Add $\frac{4}{9}$ yd., $\frac{5}{12}$ yd., and $\frac{5}{6}$ yd.

WORK

$12 \times 3 = 36$ l. c. d.

$\frac{4}{9} = \frac{16}{36}$

$\frac{5}{12} = \frac{15}{36}$

$\frac{5}{6} = \frac{30}{36}$

$16 + 15 + 30 = 61$, sum

$\frac{61}{36} = 1\frac{25}{36}$

EXPLANATION. To add $\frac{4}{9}$, $\frac{5}{12}$, and $\frac{5}{6}$, we change them to equivalent fractions having the l. c. d. and then add the new numerators. The l. c. d. is the l. c. m. of the denominators 9, 12, 6, which is 36. $\frac{4}{9} = \frac{16}{36}$; $\frac{5}{12} = \frac{15}{36}$; $\frac{5}{6} = \frac{30}{36}$; $\frac{16 + 15 + 30}{36}$

$= \frac{61}{36} = 1\frac{25}{36}$, the sum.

Ans. $1\frac{25}{36}$.

1. $2\frac{23}{24} + \frac{1}{36} + \frac{2}{3} + \frac{1}{2}$
2. $\frac{1}{6} + \frac{2}{9} + \frac{13}{30} + \frac{4}{5}$
3. $1\frac{17}{48} + \frac{7}{8} + \frac{3}{4} + \frac{5}{6}$
4. $\frac{5}{36} + \frac{9}{16} + \frac{2}{9} + \frac{13}{24}$
5. $\frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{8}{9} + 1\frac{1}{12}$
6. $\frac{2}{3} + \frac{7}{15} + \frac{7}{10} + \frac{7}{45} + \frac{5}{18}$
7. $\frac{7}{18} + \frac{7}{9} + 1\frac{1}{16} + \frac{5}{6} + \frac{5}{24}$
8. $\frac{75}{144} + \frac{53}{72} + \frac{14}{48} + \frac{25}{36}$

Written Exercise

126. Illustrative Example. From $\frac{7}{12}$ subtract $\frac{9}{20}$.

WORK

$20 \times 3 = 60$, l. c. d.

$\frac{7}{12} = \frac{35}{60}$

$\frac{9}{20} = \frac{27}{60}$

$35 - 27 = 8$

$\frac{8}{60} = \frac{2}{15}$

EXPLANATION. To subtract $\frac{9}{20}$ from $\frac{7}{12}$ we change the fractions to equivalent fractions having the l. c. d., which is 60.

$\frac{7}{12} = \frac{35}{60}$; $\frac{9}{20} = \frac{27}{60}$; $\frac{35}{60} - \frac{27}{60} = \frac{8}{60} = \frac{2}{15}$

Ans. $\frac{2}{15}$.

Subtract the following:

	a	*b*	*c*	*d*	*e*	*f*
1.	$\frac{3}{4} - \frac{1}{3}$	$\frac{2}{3} - \frac{8}{25}$	$\frac{5}{11} - \frac{3}{10}$	$\frac{3}{5} - \frac{4}{7}$	$\frac{4}{5} - \frac{2}{3}$	$\frac{8}{11} - \frac{10}{27}$
2.	$\frac{3}{4} - \frac{2}{3}$	$\frac{3}{4} - \frac{4}{15}$	$\frac{7}{10} - \frac{1}{2}$	$\frac{3}{8} - \frac{2}{7}$	$1\frac{3}{10} - 1\frac{21}{25}$	$\frac{15}{7} - \frac{7}{8}$
3.	$\frac{3}{4} - \frac{3}{5}$	$\frac{4}{5} - \frac{7}{20}$	$\frac{7}{4} - \frac{8}{9}$	$\frac{6}{7} - \frac{5}{8}$	$\frac{7}{12} - \frac{9}{20}$	$\frac{7}{9} - \frac{5}{8}$
4.	$\frac{19}{20} - \frac{8}{25}$	$\frac{9}{10} - \frac{3}{20}$	$\frac{2}{9} - \frac{4}{27}$	$\frac{15}{16} - \frac{3}{64}$	$\frac{7}{8} - \frac{5}{9}$	$\frac{15}{24} - \frac{4}{15}$
5.	$\frac{7}{8} - \frac{5}{16}$	$\frac{6}{5} - \frac{2}{15}$	$\frac{9}{13} - \frac{2}{39}$	$\frac{9}{20} - \frac{2}{15}$	$\frac{5}{12} - \frac{7}{24}$	$\frac{20}{21} - \frac{9}{14}$

Written Exercise

127. Illustrative Example. Of $18\frac{5}{6}$ and $8\frac{4}{15}$ find the sum and the difference.

WORK

$15 \times 2 = 30$, l. c. d.

$18\frac{5}{6} = 18\frac{25}{30}$

$8\frac{4}{15} = 8\frac{8}{30}$

$27\frac{33}{30} = 27\frac{1}{10}$, sum

$10\frac{17}{30}$, difference

EXPLANATION. The fractions $\frac{5}{6}$ and $\frac{4}{15}$ are changed to fractions having the l. c. d., 30, $\frac{5}{6} = \frac{25}{30}$ and $\frac{4}{15} = \frac{8}{30}$;

$\frac{25}{30} + \frac{8}{30} = \frac{33}{30} = 1\frac{3}{30} = 1\frac{1}{10}$.

Adding the 1 with the integers, gives the sum $27\frac{1}{10}$. *Ans.* $27\frac{1}{10}$.

$\frac{25}{30} - \frac{8}{30} = \frac{17}{30}$; $18 - 8 = 10$;

the difference is $10\frac{17}{30}$. *Ans.* $10\frac{17}{30}$.

Find the sum and the difference of the following:

1. $18\frac{3}{4}$ and $4\frac{1}{2}$
2. $17\frac{5}{14}$ and $9\frac{5}{21}$
3. $50\frac{8}{25}$ and $7\frac{4}{15}$
4. $62\frac{6}{7}$ and $19\frac{22}{35}$
5. $17\frac{37}{72}$ and $5\frac{4}{9}$
6. $18\frac{25}{33}$ and $2\frac{8}{11}$
7. $60\frac{29}{30}$ and $18\frac{7}{54}$
8. $71\frac{9}{10}$ and $35\frac{11}{20}$
9. $78\frac{2}{3}$ and $72\frac{5}{8}$
10. $18\frac{7}{10}$ and $14\frac{8}{15}$
11. $52\frac{1}{4}$ and $36\frac{5}{64}$
12. $13\frac{39}{80}$ and $2\frac{7}{16}$

To add similar fractions, we find the sum of their numerators; to subtract them, we find the difference of their numerators.

To add or subtract fractions not having a common denominator, we change them to equivalent fractions having the least common denominator, and then add or subtract their numerators.

To add or subtract mixed numbers, we first add or subtract the fractions, then add or subtract the integers and unite the results.

Written Exercise

128. 1. Cork is $\frac{6}{25}$, and white pine $\frac{13}{20}$, as heavy as water. How much heavier is pine than cork?

2. Copper is $8\frac{23}{25}$, and anthracite coal is $1\frac{3}{10}$, as heavy as water. How much lighter is coal than copper?

3. A lives $17\frac{3}{4}$ mi. from the city by road, B lives $8\frac{1}{3}$ mi. on the same road, and C $5\frac{2}{5}$ in the opposite direction. How far apart do A and B live? B and C?

4. A dress pattern contained $15\frac{1}{2}$ yd., the lining contained $7\frac{1}{8}$ yd. less. The pattern cost $\$21\frac{7}{8}$, the lining $\$5\frac{3}{8}$. How many yards did the lining contain? How much did the dress pattern and lining cost?

5. Flour of 2 grades is sold, one at $\$5\frac{3}{10}$, the other at $\$6\frac{3}{4}$, a barrel. What is the difference in the price?

6. A man who owns $25\frac{3}{4}$ A. of land, sows $15\frac{2}{3}$ A., plants $4\frac{4}{5}$ A., and makes an orchard of the rest. The orchard is how many acres?

7. What is the total weight of 3 bbl. of sugar weighing severally $247\frac{1}{2}$ lb., 229 lb., and $260\frac{7}{8}$ lb?

Add the following in lines and in columns:

11.	12.	13.	14.	18.	19.	20.	21.	25.	26.
8. $\frac{21}{24}$	$+\frac{11}{36}$	$+\frac{2}{3}$	$+\frac{1}{2}$	15. $\frac{5}{6}$	$+\frac{7}{8}$	$+\frac{8}{9}$	$+\frac{11}{12}$	22. $45\frac{2}{3}$	$+23\frac{3}{4}$
9. $\frac{1}{6}$	$+\frac{5}{9}$	$+\frac{13}{36}$	$+\frac{4}{5}$	16. $\frac{9}{15}$	$+\frac{7}{10}$	$+\frac{17}{45}$	$+\frac{15}{18}$	23. $67\frac{3}{5}$	$+26\frac{6}{9}$
10. $\frac{17}{48}$	$+\frac{7}{8}$	$+\frac{3}{4}$	$+\frac{5}{6}$	17. $\frac{17}{18}$	$+\frac{7}{9}$	$+\frac{11}{12}$	$+\frac{15}{24}$	24. $24\frac{14}{25}$	$+97\frac{3}{4}$

Written Exercise

129. Illustrative Example. From $71\frac{4}{45}$ subtract $62\frac{11}{20}$.

WORK

$45 \times 4 = 180$ l. c. d.

$71\frac{4}{45} = 71\frac{16}{180}$ | 196

$62\frac{11}{20} = 62\frac{99}{180}$ | 99

$8\frac{97}{180}$ | $\frac{97}{180}$

EXPLANATION. The fractions $\frac{4}{45}$ and $\frac{11}{20}$ changed to fractions having the l. c. d., 180, are $\frac{16}{180}$ and $\frac{99}{180}$. We cannot take $\frac{99}{180}$ from $\frac{16}{180}$. Changing 1 of the 71 to $\frac{180}{180}$, and adding it to $\frac{16}{180}$, we have $\frac{196}{180}$. $\frac{196}{180} - \frac{99}{180} = \frac{97}{180}$. *Ans.* $8\frac{97}{180}$.

1.	3.	5.	7.	9.
$13\frac{5}{7}$	$37\frac{4}{15}$	$75\frac{3}{20}$	$49\frac{7}{16}$	$351\frac{2}{9}$
$-9\frac{13}{14}$	$-18\frac{3}{4}$	$-19\frac{7}{8}$	$-29\frac{7}{9}$	$-242\frac{94}{126}$
2.	**4.**	**6.**	**8.**	**10.**
$24\frac{5}{10}$	$35\frac{5}{16}$	$66\frac{8}{21}$	$88\frac{4}{25}$	$401\frac{5}{42}$
$-16\frac{17}{30}$	$-22\frac{3}{8}$	$-55\frac{5}{9}$	$-33\frac{1}{8}$	$-361\frac{41}{54}$

Written Exercise

130. 1. Of a lot of grass seed containing $24\frac{7}{8}$ lb., $15\frac{9}{10}$ lb. was redtop, the rest clover. How much was clover?

2. The cost of the seed, Ex. 1, was $\$4\frac{1}{4}$. Lawn seed costing \$8 to cover the same ground would cost how much more?

3. If by selling wheat at $\$\frac{4}{5}$ a bushel you lose $\$\frac{1}{8}$, how much did it cost you? For how much should you have sold it if you had lost $\$\frac{3}{8}$?

4. In $5\frac{1}{6}$ lb. of powder there is $\frac{2}{3}\frac{3}{3}$ lb. each of charcoal and sulphur. The rest is saltpeter. How much is saltpeter?

5. I had $\$19\frac{3}{4}$ and spent $\$7\frac{1}{5}$. What sum was left?

6. From the sum of $\frac{2}{4}\frac{3}{0}$ and $\frac{1}{2}\frac{3}{0}$ subtract their difference.

7. Out of a barrel of vinegar containing $31\frac{1}{2}$ gal., were drawn $1\frac{1}{3}$ gal., $3\frac{5}{8}$ gal., $2\frac{1}{4}$ gal., and $12\frac{1}{2}$ gal. How many gallons did the barrel then contain?

8. How much silk is there in 7 remnants measuring as follows: $12\frac{5}{8}$ yd., $25\frac{7}{16}$ yd., $10\frac{2}{3}$ yd., $8\frac{3}{4}$ yd., $6\frac{1}{2}$ yd., $7\frac{5}{8}$ yd., and $16\frac{3}{4}$ yd.?

9. How much more railing do I need to fence a court whose sides are $4\frac{1}{11}$ rd., $3\frac{5}{2}\frac{5}{2}$ rd., $2\frac{1}{2}$ rd., and $7\frac{3}{4}$ rd., if I have enough to fence $10\frac{3}{4}$ rd.?

10. John weighs $93\frac{1}{4}$ lb., James $75\frac{5}{16}$ lb., Charles $65\frac{7}{8}$ lb., and Richard $88\frac{1}{4}$ lb. What is the sum of their weights? What is the difference between the weights of John and James? Charles and James? Charles and Richard?

11. From a piece of ribbon measuring $10\frac{2}{3}$ yd., $3\frac{5}{8}$ yd., $2\frac{1}{2}$ yd., and $3\frac{3}{8}$ yd. were cut. How much remained?

12. A new silver dollar weighs $412\frac{1}{2}$ gr. What has been the loss in weight for a silver dollar that weighs $405\frac{5}{9}$ gr.?

Written Exercise

131. **1.** To heat a school building during the month of February, the following amounts of coal were consumed per week: $7\frac{1}{4}$ T., $6\frac{5}{8}$ T., $7\frac{3}{5}$ T., $8\frac{11}{20}$ T. How much was consumed in the month?

2. Six carloads of coal weigh as follows: $24\frac{7}{8}$ T., $31\frac{1}{5}$ T., $31\frac{6}{10}$ T., $22\frac{1}{4}$ T., $28\frac{5}{8}$ T., and $25\frac{2}{5}$ T. Find the total weight of the six loads.

3. I have on hand $2\frac{3}{4}$ cd. of wood. I complete my winter's supply by two lots of $5\frac{1}{2}$ cd. and $3\frac{5}{8}$ cd. How much have I after completing these purchases?

4. On three wood lots are stacked $263\frac{5}{6}$ cd., $121\frac{4}{5}$ cd., $36\frac{1}{8}$ cd. of wood. What is the total in these 3 lots?

5. Out of $28\frac{3}{5}$ T., in a carload of coal, $2\frac{1}{2}$ T. were lost in transit from the mine to the coal yard. How much of this coal remained?

6. On a 100-mi. run, a locomotive started with $2\frac{1}{2}$ T. of coal. At the end of the first half of the run $1\frac{1}{8}$ T. had been burned. How much remained for the last half of the run? During the last half of the run only $\frac{3}{4}$ of a ton was actually burned. How much remained at the end of the entire run?

7. If I order $12\frac{3}{4}$ T. of coal and only $3\frac{3}{10}$ T. are delivered, how many tons remain undelivered?

8. What is the total cost of 1150 tons of coal delivered, if it costs \$3.25 per ton at the mines, \$2.90 per ton for freight, and 50¢ per ton for cartage?

9. What is the total cost of 63 cords of rock maple wood if it costs \$3.60 per cord on the wood lot, \$1.75 per cord for hauling, and \$1.50 per cord for sawing, splitting, and piling it up?

Oral Exercise

132. Illustrative Examples. (1) Multiply $\frac{4}{5}$ by 12; (2) Multiply $\$3\frac{3}{4}$ by 12.

WORK

(1) $\frac{4 \times 12}{5} = \frac{48}{5} = 9\frac{3}{5}$. *Ans.* $9\frac{3}{5}$.

(2) $\frac{\$3 \times 12^{3}}{\cancel{4}_{1}} = \$9$. *Ans.* \$9.

To multiply a fraction by an integer, we multiply the numerator of the fraction by the integer, and cancel when possible.

Multiply at sight:

1. $\frac{2}{5}$ yd. by 4
2. $\frac{1}{8}$ lb. by 6
3. $\frac{1}{3}$ ft. by 2
4. $\frac{2}{11}$ rd. by 10
5. $\frac{3}{8}$ yd. by 7
6. $\frac{2}{7}$ wk. by 4
7. $\frac{2}{15}$ mo. by 5
8. $\frac{9}{10}$ T. by 7
9. $\frac{7}{8}$ da. by 8
10. \$.9 by 3
11. \$.25 by 2
12. \$.008 by 9

Oral Exercise

133. Illustrative Example. Find the cost of 12 chairs at $\$10\frac{4}{5}$ each.

WORK

$\$10\frac{4}{5}$
12
$9\frac{3}{5}$
120
$\$129\frac{3}{5}$

SOLUTION. 1 chair costs $\$10\frac{4}{5}$; 12 chairs will cost 12 times $\$10\frac{4}{5}$; 12 times $\frac{4}{5} = \frac{48}{5} = 9\frac{3}{5}$; 12 times 10 = 120; 120 and $9\frac{3}{5} = 129\frac{3}{5}$. *Ans.* $\$129\frac{3}{5}$.

To multiply a mixed number by an integer, we multiply the fractional and the integral parts separately, and then add the products.

1. What is the cost of 2 planes at $\$1\frac{1}{2}$ each? at $\$\frac{4}{5}$ each?
2. How much ink will fill 7 bottles holding $\frac{7}{8}$ pt. each?
3. How many bushels are contained in 6 bbl. of $3\frac{4}{5}$ bu. each?

134. Written Exercise

1. $\frac{3}{4} \times 18$
2. $\frac{3}{8} \times 14$
3. $22\frac{5}{7} \times 16$
4. $.25\frac{3}{5} \times 15$
5. $15\frac{5}{6} \times 40$
6. $.37\frac{9}{10} \times 14$
7. $2.31\frac{1}{12} \times 64$
8. $33\frac{4}{7} \times 11$
9. $98\frac{2}{3} \times 20$
10. $80\frac{5}{8} \times 15$
11. $38\frac{7}{15} \times 12$
12. $96\frac{5}{9} \times 28$
13. $10\frac{3}{11} \times 42$
14. $16\frac{2}{3} \times 21$
15. $13\frac{8}{9} \times 16$
16. $17\frac{5}{7} \times 14$
17. $28\frac{9}{10} \times 27$
18. $48\frac{3}{4} \times 20$
19. $56\frac{10}{11} \times 35$
20. $89\frac{1}{5} \times 40$
21. $22\frac{6}{7} \times 17$
22. $1.21\frac{3}{5} \times 46$
23. $3.74\frac{1}{4} \times 50$
24. $19\frac{3}{5} \times 33$
25. $81\frac{1}{9} \times 48$
26. $73\frac{4}{7} \times 69$
27. $6.34\frac{2}{3} \times 35$

28. How many yards wide is a floor that takes exactly 6 strips of carpeting $\frac{3}{4}$ of a yard wide?

29. At the rate of $30\frac{4}{5}$ mi. an hour, how far does an automobile travel in 28 hr.?

30. At $16\frac{2}{3}¢$ each, what is the cost of 2 doz. handkerchiefs?

31. How many yards of cloth are there in 96 webs of cotton sheeting containing $44\frac{3}{4}$ yd. to the web?

32. How long must a line be drawn to represent a length of 26 ft. on a scale of $\frac{3}{4}$ in. to 1 ft.?

33. Corn is quoted at $50\frac{7}{8}¢$ a bushel. How much must be paid for 500 bu.?

34. What is the value of a 450-lb. bale of cotton at $18\frac{1}{6}¢$ per pound?

35. What is the weight in grains of 1728 cu. in. of water, 1 cu. in. weighing $252\frac{24}{25}$ gr.?

36. Find the cost of 300 lambs at $\$3.37\frac{1}{2}$ each.

Oral and Written Exercise

135. Illustrative Examples. (1) What is $\frac{2}{3}$ of 10 feet? (2) $\frac{2}{3}$ of 9 feet?

WORK

(1) $\frac{10 \times 2}{3} = \frac{20}{3} = 6\frac{2}{3}$. *Ans.* $6\frac{2}{3}$ feet.

(2) $\frac{\overset{3}{\cancel{9}} \times 2}{\underset{1}{\cancel{3}}} = 6$. *Ans.* 6 feet.

Finding the fractional part of a number is called multiplying by a fraction.

To multiply an integer by a fraction, we multiply the integer by the numerator of the fraction, and divide by the denominator, canceling when possible.

Give answers at sight:

	a	*b*	*c*	*d*	*e*
1.	$\frac{2}{3}$ of 9	$\frac{5}{6}$ of 18	\$20 $\times \frac{4}{5}$	60¢ $\times \frac{2}{3}$	\$.48 $\times \frac{11}{12}$
2.	$\frac{3}{4}$ of 12	$\frac{5}{9}$ of 36	\$45 $\times \frac{4}{9}$	75¢ $\times \frac{24}{25}$	\$.88 $\times \frac{5}{8}$

Multiply:

3.	8 by $\frac{2}{3}$	30 by $\frac{3}{4}$	25 by $\frac{5}{6}$	33 by $\frac{3}{8}$	\$84 by $\frac{5}{8}$
4.	20 by $\frac{2}{3}$	40 by $\frac{4}{9}$	15 by $\frac{4}{7}$	60 by $\frac{2}{11}$	\$16 by $\frac{8}{5}$

Written Exercise

136. Illustrative Examples. (1) At \$12 a barrel, what is the cost of $\frac{3}{4}$ bbl. of beef? (2) of $\frac{3}{5}$ bbl.?

SOLUTION. (1) 1 barrel costs \$12: $\frac{1}{4}$ barrel will cost $\frac{1}{4}$ of \$12, and $\frac{3}{4}$ barrel will cost 3 times $\frac{1}{4}$ of \$12. $\frac{1}{4}$ of \$12 is \$3; $\frac{3}{4}$ of \$12 is $3 \times \$3 = \$9$. *Ans.* \$9.

(2) $\frac{1}{5}$ barrel will cost $\frac{1}{5}$ of \$12, and $\frac{3}{5}$ bbl. will cost 3 times $\frac{1}{5}$ of \$12. $\frac{1}{5}$ of \$12 is $\$2\frac{2}{5}$; $\frac{3}{5}$ of \$12 is 3 times $\$2\frac{2}{5} = \$7\frac{1}{5}$. *Ans.* $\$7\frac{1}{5}$.

What is the cost of:

1. $\frac{3}{4}$ doz. ties at \$2 a dozen?
2. $\frac{2}{5}$ bbl. of flour at \$7?
3. $\frac{7}{12}$ doz. napkins at \$9?

Find the number of:

4. Hours in $\frac{3}{8}$ of a day.
5. Cents in $\$\frac{7}{8}$; in $\$\frac{2}{3}$.
6. Months in $\frac{5}{6}$ yr.; $\frac{10}{9}$ yr.

Written Exercise

137. Illustrative Example. Multiply 658 by $17\frac{5}{8}$.

WORK

658
$17\frac{5}{8}$
$411\frac{1}{4}$, prod. by $\frac{5}{8}$
4606
658
$11597\frac{1}{4}$, *Ans.*

$\frac{329}{\cancel{658} \times 5}{\cancel{8}_{4}} = \frac{1645}{4} = 411\frac{1}{4}$.

To multiply an integer by a mixed number, we multiply by the fractional and the integral parts separately and then add the products.

Multiply:

1. $56 \times \frac{7}{8}$
2. $84 \times \frac{11}{12}$
3. $60 \times \frac{4}{5}$
4. $90 \times \frac{7}{15}$
5. $\frac{7}{10} \times 130$
6. $\frac{7}{8}$ of 150
7. $\frac{12}{13}$ of 104
8. $\frac{2}{3}$ of 315
9. 26 by $\frac{3}{13}$
10. 64 by $\frac{5}{16}$
11. 68 by $\frac{5}{17}$
12. 96 by $\frac{7}{8}$
13. $467 \times 27\frac{7}{18}$
14. $296 \times 28\frac{5}{9}$
15. $637 \times 17\frac{2}{9}$

Multiply:

16. 516 by $95\frac{1}{10}$
17. 49.3 by $83\frac{8}{9}$
18. 8.31 by $37\frac{2}{9}$
19. 282 by $.62\frac{7}{8}$
20. 504 by $9.4\frac{9}{10}$
21. 6.11 by $15\frac{2}{3}$
22. 6.27 by $1.60\frac{2}{9}$
23. 16.1 by $50.4\frac{6}{7}$
24. 4.04 by $8.44\frac{5}{6}$
25. 3.84 by $6.01\frac{4}{5}$
26. 85.9 by $30.9\frac{4}{7}$
27. .393 by $.73\frac{3}{8}$

Written Exercise

138. 1. Find the cost of $13\frac{2}{3}$ T. of coal at \$7.50 per ton.

2. At \$14 per week what are the wages of a man for $48\frac{3}{5}$ wk.?

3. What is the cost for lighting a factory for one week of 5 days' work, if it takes 560 burners for $2\frac{1}{2}$ hr. each day, at a cost of $\frac{3}{4}$¢ per hour?

4. Find the value of 400 bu. of wheat at $90\frac{2}{3}$¢ per bushel.

5. It takes $4\frac{1}{2}$ bu. of wheat to make a barrel of flour. How many bushels will make 167 bbl. of flour?

6. How far will a train run in $17\frac{1}{2}$ hr. at a speed of 56 mi. an hour?

7. The total length of the St. Lawrence River is 1720 mi. How long would a river $\frac{2}{5}$ that length be?

8. What is the wholesale value of 960 sacks of coffee containing 132 lb. each, at $15\frac{1}{4}$¢ a pound?

9. Find the value of $12\frac{3}{5}$ A. of land at \$860 per acre.

10. At $45\frac{2}{3}$¢ an hour what is a man's daily wage for 9 hours' work? What is his weekly wage for 52 hours' work?

11. The driving wheel of a locomotive has a circumference of $19\frac{5}{6}$ ft. In making 361 revolutions how many feet are covered? How many miles and feet? (5280 ft. = 1 mi.)

12. What is the total weight of a steel rail that is 80 ft. long and weighs $21\frac{3}{4}$ lb. per foot?

13. What is the weight of 50 new silver dollars weighing $412\frac{1}{2}$ gr. each?

14. What is the rent for $35\frac{7}{8}$ A. of pasture land at \$3 per acre for the season?

Oral and Written Exercise

139. Illustrative Example. Multiply $\frac{7}{8}$ by $\frac{4}{5}$, or find $\frac{4}{5}$ of $\frac{7}{8}$.

WORK

(1st Form)

$$\frac{7 \times 4}{8 \times 5} = \frac{28}{40} = \frac{7}{10}$$

(2d Form)

$$\frac{7 \times \cancel{4}^{1}}{\cancel{8}_{2} \times 5} = \frac{7}{10}$$

EXPLANATION. (1st Form) To multiply $\frac{7}{8}$ by $\frac{4}{5}$ is to take $\frac{4}{5}$ of $\frac{7}{8}$. To get $\frac{1}{5}$ of $\frac{7}{8}$, we must divide $\frac{7}{8}$ into 5 equal parts. This is expressed by multiplying the denominator 8 by 5 thus, $\frac{1}{8 \times 5} = \frac{1}{40}$. Since $\frac{1}{5}$ of $\frac{1}{8}$ $= \frac{1}{40}$, $\frac{1}{5}$ of $\frac{7}{8} = \frac{7}{40}$, and $\frac{4}{5}$ of $\frac{7}{8} = 4$ times $\frac{7}{40} = \frac{28}{40} = \frac{7}{10}$. *Ans.* $\frac{7}{10}$.

Or, (2d Form) we may cancel the common factor 4 and obtain the same answer, $\frac{7}{10}$.

To multiply a fraction by a fraction, we multiply the numerators together for a new numerator, and the denominators together for a new denominator, canceling when possible.

Give the answers at sight, then give the solution:

1. $\frac{1}{2}$ of $\frac{2}{3}$	**5.** $\frac{4}{5}$ of $\frac{1}{2}$	**9.** $\frac{2}{7} \times \frac{2}{7}$	**13.** $\frac{7}{20}$ of $\frac{4}{5}$
2. $\frac{1}{2}$ of $\frac{3}{4}$	**6.** $\frac{3}{5}$ of $\frac{3}{4}$	**10.** $\frac{4}{7} \times \frac{3}{5}$	**14.** $\frac{3}{4} \times \frac{4}{15}$
3. $\frac{1}{3}$ of $\frac{3}{4}$	**7.** $\frac{5}{6}$ of $\frac{4}{5}$	**11.** $\frac{2}{9} \times \frac{3}{4}$	**15.** $\frac{3}{25} \times \frac{2}{3}$
4. $\frac{2}{3}$ of $\frac{3}{4}$	**8.** $\frac{2}{3}$ of $\frac{6}{7}$	**12.** $\frac{2}{5} \times \frac{10}{9}$	**16.** $1\frac{8}{9} \times \frac{3}{4}$

WORK OF EXAMPLE 16

(1) $1\frac{8}{9} = \frac{17}{9}$

$$\frac{17 \times \cancel{3}^{1}}{\cancel{9}_{3} \times 4} = \frac{17}{12} = 1\frac{5}{12}$$

(2) $1\frac{8}{9} \times \frac{3}{4}$

$$\frac{\cancel{8}^{2}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{4}_{2}} = \frac{2}{3}$$

$1 \times \frac{3}{4} = \frac{3}{4}$

$\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$

EXPLANATION. To multiply $1\frac{8}{9}$ by $\frac{3}{4}$, we first change $1\frac{8}{9}$ to an improper fraction, and then proceed as in multiplying a fraction.

Or, (2) we may multiply the fractional and integral parts separately and add the products, canceling when possible. *Ans.* $1\frac{5}{12}$.

Multiply, canceling when possible:

17. $\frac{2}{3}$ of $2\frac{1}{2}$
18. $\frac{2}{5}$ of $11\frac{1}{4}$
19. $\frac{5}{7}$ of $4\frac{1}{5}$
20. $\frac{3}{8}$ of $62\frac{1}{2}$
21. $\frac{5}{13}$ of $8\frac{2}{3}$
22. $\frac{2}{4}$ of $7\frac{1}{5}$
23. $\frac{4}{9}$ of $7\frac{5}{7}$
24. $\frac{5}{9}$ of $14\frac{2}{5}$
25. $\frac{12}{7}$ of $12\frac{5}{6}$
26. $15\frac{3}{8} \times \frac{2}{7}$
27. $18\frac{3}{4} \times \frac{14}{75}$
28. $\frac{3}{5}$ of $1\frac{3}{2}$
29. $87\frac{1}{2} \times 9$
30. $34\frac{2}{3} \times 15$
31. $18\frac{3}{4} \times \frac{2}{33}$

Written Exercise

140. Illustrative Example. What part of an estate is $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of it?

WORK

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \text{ of } \frac{\cancel{5}}{\cancel{6}_{2}} = \frac{1}{2}$$

EXPLANATION. $\frac{3}{4}$ of $\frac{4}{5} = \frac{3}{5}$; $\frac{3}{5}$ of $\frac{5}{6} = \frac{3}{6} = \frac{1}{2}$ But the work may be abridged by canceling as shown in the model. *Ans.* $\frac{1}{2}$.

A fraction of a fraction is called a **compound fraction.**

To change a compound fraction to a simple fraction, we multiply the numerators together for a new numerator, and the denominators for a new denominator, canceling if possible.

Change to simple fractions:

1. $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{4}{5}$
2. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{7}$
3. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$
4. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$
5. $\frac{1}{2}$ of $\frac{1}{8}$ of $2\frac{2}{5}$
6. $\frac{2}{5}$ of $\frac{2}{3}$ of $3\frac{1}{2}$
7. $\frac{1}{9}$ of $\frac{2}{3}$ of $1\frac{5}{7}$
8. $\frac{7}{12}$ of $\frac{4}{9}$ of $2\frac{2}{5}$
9. $\frac{5}{13} \times \frac{6}{9} \times 18\frac{2}{5}$
10. $5\frac{1}{2} \times \frac{3}{11} \times \frac{5}{12}$
11. $\frac{3}{14} \times 3\frac{1}{2} \times 8$
12. $4\frac{3}{8} \times \frac{6}{7} \times \frac{3}{10}$

13. A car fare was paid with $\frac{1}{5}$ of $\frac{1}{2}$ of \$$\frac{1}{2}$. What was the fare expressed as the fraction of a dollar? in cents?

14. What is the cost for insurance on $\frac{3}{4}$ of the value of a building worth \$9000 at the rate of $\frac{2}{5}$ ¢ on a dollar?

15. A suit of clothes costs a tailor \$37.50. He marks the suit at $\frac{5}{4}$ of the cost. In a mark down sale he gets only $\frac{2}{3}$ of the marked price. How much does he get?

16. 1 rd. is $5\frac{1}{2}$ yd.; 1 ft. is $\frac{1}{3}$ yd.; 1 in. is $\frac{1}{12}$ ft. What part of a rod is 1 in.?

17. 2 boys worked $3\frac{1}{2}$ da., earning together $\$1\frac{1}{2}$ a day. How much should each boy receive?

18. On foot a man can go 10 mi. in $3\frac{1}{2}$ hr.; on horseback he can go the same distance in $\frac{2}{5}$ the time, and by automobile in $\frac{1}{2}$ the time taken on horse. Find the time by automobile.

19. When 40 wk. make a school year, $\frac{3}{4}$ of it is completed by April 30. How many weeks are then completed? The rest of the school year equals $\frac{1}{3}$ of the $\frac{3}{4}$ that is completed. This is what fraction of the whole school year? It is how many weeks?

20. If it requires $1\frac{1}{2}$ yd. of wire costing $3\frac{1}{2}$¢ a yard to hang one picture, what will be the cost for wire to hang 6 pictures?

21. I bought 40 yd. of cloth at $\$2\frac{1}{2}$ a yard; I sold $\frac{7}{8}$ of it at $\$3\frac{1}{4}$ a yard, and the remainder at $\$2\frac{3}{4}$ a yard. Find the gain.

22. What is the value of $5\frac{1}{3}$ cd. of wood at $\$7\frac{1}{8}$ a cord?

23. $19\frac{7}{8} \times 2\frac{2}{5} = ?$ $\quad 33\frac{1}{3} \times 9\frac{1}{4} = ?$

24. In a sheet of tin $12\frac{1}{2}$ in. $\times$ $17\frac{1}{2}$ in. there are how many square inches?

25. Multiply each of the following by 14 and add the products: $\frac{11}{14}$, $\frac{6}{7}$, $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{21}$; multiply the sum of the products by 14.

26. At $\$1\frac{1}{8}$ a rod, what will be the cost of $16\frac{2}{3}$ rd. of fencing?

27. Find the value of 9700 lb. of wheat at $\$\frac{5}{8}$ per bushel (60 lb. in a bushel).

28. What is the cost of 9 suits, each requiring $7\frac{1}{2}$ yd., at $\$2\frac{2}{3}$ a yard?

Oral and Written Exercise

141. Illustrative Examples. (1) Divide equally among 3 boys $\frac{3}{4}$ of a bushel of nuts; (2) $\frac{5}{4}$ of a bushel.

WORK

(1) $\frac{\overset{1}{\cancel{3}}}{4 \times \cancel{3}} = \frac{1}{4}$

(2) $\frac{5}{4 \times 3} = \frac{5}{12}$

EXPLANATION. (1) Since 3 boys have $\frac{3}{4}$ bu., 1 boy will have $\frac{1}{3}$ of $\frac{3}{4}$ bu. $\frac{1}{3}$ of $\frac{3}{4}$ is $\frac{1}{4}$. Here we divide the numerator of the fraction by the integer, which divides the fraction by taking $\frac{1}{3}$ the number of parts, the size remaining the same. *Ans.* $\frac{1}{4}$ bu.

In example (2) we multiply the denominator, which divides the fraction by making the parts $\frac{1}{3}$ as large, the number remaining the same. *Ans.* $\frac{5}{12}$ bu.

To divide a fraction by an integer, we divide the numerator or multiply the denominator of the fraction by the integer.

In written work we express the division by writing the integer with the denominator; we cancel when possible.

Divide:

1. $\frac{5}{6}$ by 5	5. $\frac{21}{22}$ by 7	9. $\frac{3}{5}$ by 6	13. $\frac{16}{19}$ by 8
2. $\frac{9}{10}$ by 3	6. $\frac{3}{8}$ by 3	10. $\frac{6}{7}$ by 3	14. $\frac{2}{5}$ by 8
3. $\frac{1}{10}$ by 3	7. $\frac{14}{23}$ by 2	11. $\frac{25}{26}$ by 5	15. $\frac{7}{9}$ by 9
4. $\frac{3}{5}$ by 5	8. $\frac{4}{5}$ by 6	12. $\frac{5}{6}$ by 10	16. $3\frac{4}{5}$ by 4

WORK OF EXAMPLE 16: $3\frac{4}{5} = \frac{19}{5}$; $\frac{19}{5 \times 4} = \frac{19}{20}$

17. $2\frac{6}{7} \div 5$	24. $4\frac{3}{11} \div 2$	31. $9\frac{6}{7} \div 5$	38. $16\frac{2}{3} \div 12$
18. $9\frac{3}{8} \div 3$	25. $6\frac{2}{9} \div 6$	32. $18\frac{1}{2} \div 4$	39. $37\frac{1}{2} \div 3$
19. $5\frac{2}{3} \div 9$	26. $12\frac{1}{4} \div 7$	33. $22\frac{3}{5} \div 7$	40. $83\frac{1}{3} \div 20$
20. $8\frac{3}{4} \div 3$	27. $12\frac{3}{7} \div 3$	34. $14\frac{7}{8} \div 10$	41. $87\frac{1}{2} \div 14$
21. $9\frac{5}{6} \div 12$	28. $10\frac{4}{5} \div 9$	35. $9\frac{3}{11} \div 6$	42. $10\frac{10}{11} \div 8$
22. $7\frac{4}{7} \div 14$	29. $14\frac{2}{3} \div 11$	36. $4\frac{2}{7} \div 12$	43. $9\frac{9}{10} \div 11$
23. $5\frac{5}{8} \div 9$	30. $5\frac{5}{9} \div 10$	37. $87\frac{1}{2} \div 7$	44. $13\frac{1}{8} \div 10$

Written Exercise

142. 1. For 2 yd. of silk I pay $\$\frac{4}{5}$. How much do I pay per yard?

2. What is the cost of 1 doz. eggs if 2 doz. cost $62\frac{1}{2}¢$?

3. A school day of $5\frac{1}{4}$ hr. is divided into 6 equal periods. How long is each period?

4. A lot containing $\frac{7}{8}$ A. is divided into 5 equal sections. What part of an acre does each section contain?

5. If it takes 1 man $1\frac{1}{6}$ wk. to spade a garden, what part of a week should it take 2 men? 3 men?

6. A dealer bought 25 crates of strawberries for which he paid $80. He sold them for $\$87\frac{1}{2}$. How much did he gain on a crate? on a dollar?

7. If $9 will buy $3\frac{3}{4}$ bu. of cranberries, what quantity will $1 buy?

Written Exercise

143. Illustrative Example. Divide $306\frac{2}{3}$ by 15.

WORK

$$15\overline{)306\tfrac{2}{3}}\quad 20\tfrac{4}{9}$$

$$15)6\tfrac{2}{3} = \frac{\overset{4}{\cancel{20}}}{3 \times \cancel{15}_{3}} = \frac{4}{9}$$

EXPLANATION. If the mixed number is large when compared with the divisor, we divide the integral part as in the model, without changing it to the fractional form.

Ans. $20\frac{4}{9}$.

1. Find the number of days' work of 8 hours each required to do $45\frac{2}{3}$ hours' work.

2. If 12 oz. of gold cost $\$213\frac{1}{3}$, find the value per ounce.

3. If a miner mines $110\frac{5}{8}$ T. of coal in 24 da., how much does he average per day?

Written Exercise

144. Illustrative Example. Divide $365\frac{1}{4}$ by 52.

WORK

(1)
$$52\overline{)365\tfrac{1}{4}}\quad 7\tfrac{5}{208}$$
$$364$$
$$1\tfrac{1}{4} = \tfrac{5}{4}$$
$$\frac{5}{4 \times 52} = \frac{5}{208}$$

WORK

(2)
$$52\overline{)365\tfrac{1}{4}}\quad (7\tfrac{5}{208}$$
$$4 \quad\quad 4$$
$$208\overline{)1461}$$
$$1456$$
$$5$$

EXPLANATION. When it is necessary to use long division, the work can be done by either of the two ways illustrated.

(2) consists in changing both the dividend and divisor to fourths, and then dividing as we divide integral numbers.

Divide: *Ans.* $7\frac{5}{208}$.

1. $260\frac{1}{2}$ by 32
2. $272\frac{1}{4}$ by 21
3. $333\frac{3}{5}$ by 15
4. $175\frac{3}{4} \div 13$
5. $256\frac{2}{3} \div 26$
6. $521\frac{1}{6} \div 24$
7. $264\frac{3}{8} \div 31$
8. $198\frac{3}{7} \div 16$
9. $625\frac{3}{10} \div 47$
10. $1432\frac{7}{9} \div 84$
11. $1728\frac{1}{12} \div 17$
12. $4760\frac{1}{7} \div 14$

13. The expenses of a trip costing $\$2324\frac{5}{8}$ were shared by 15 persons. What was the share of each?

14. 31 acres of land were bought for \$271. The expense attending the purchase was $\$24\frac{7}{10}$. Find the total cost per acre.

15. Land that extends along the street $103\frac{1}{2}$ rd. is made into 18 house lots of equal frontage. The frontage of each lot is how many rods?

16. 10 trees, each equally distant from the next, are set in a row $8.62\frac{1}{2}$ rd. long. One rod being $16\frac{1}{2}$ ft., how many feet apart are the trees? (There are 9 spaces.)

17. If a boy earns \$45 a month, how long will it take him to earn $\$240\frac{3}{4}$?

18. If 18 fish weigh $41\frac{5}{8}$ lb., what is the average weight of each?

Oral and Written Exercise

145. Illustrative Example. When peaches are $\$\frac{7}{8}$ a bushel, how many bushels can be bought for \$12?

WORK

$\$12 = \$\frac{96}{8}$

$\$\frac{96}{8} \div \$\frac{7}{8} = 13\frac{5}{7}$

SOLUTION. For \$12 we can buy as many bushels as $\$\frac{7}{8}$ is contained times in \$12. $\$12 = \$\frac{96}{8}$; $\$\frac{7}{8}$ is contained in $\$\frac{96}{8}$ as many times as 7 is contained times in 96, which is $13\frac{5}{7}$ times. *Ans.* $13\frac{5}{7}$ bushels.

In this example we make the dividend, 12, of the same denomination as the divisor and then divide the numerator of the dividend by the numerator of the divisor.

Divide at sight:

1. 6 by $\frac{3}{5}$	**5.** $5 \div \frac{3}{5}$	**9.** $4 \div \frac{8}{11}$	**13.** 15 by $\frac{3}{4}$
2. 3 by $\frac{3}{8}$	**6.** $10 \div \frac{7}{8}$	**10.** $6 \div \frac{2}{9}$	**14.** 13 by $\frac{1}{3}$
3. 1 by $\frac{6}{7}$	**7.** $5 \div \frac{2}{3}$	**11.** $2 \div \frac{5}{12}$	**15.** 18 by $\frac{9}{10}$
4. $6 \div \frac{3}{4}$	**8.** $12 \div \frac{4}{5}$	**12.** $14 \div \frac{1}{2}$	**16.** 16 by $1\frac{1}{4}$

WORK OF EXAMPLE 16: $16 = \frac{64}{4}$; $1\frac{1}{4} = \frac{5}{4}$; $\frac{64}{4} \div \frac{5}{4} = 12\frac{4}{5}$

Divide:

17. $26 \div \frac{7}{10}$	**21.** $42 \div 1\frac{7}{8}$	**25.** $18 \div 5\frac{1}{3}$	**29.** $25 \div 6\frac{1}{4}$
18. $39 \div \frac{11}{12}$	**22.** $12 \div \frac{14}{15}$	**26.** $10 \div 4\frac{7}{8}$	**30.** $80 \div 33\frac{1}{3}$
19. $42 \div \frac{7}{24}$	**23.** $31 \div 3\frac{9}{11}$	**27.** $17 \div 8\frac{2}{3}$	**31.** $100 \div 16\frac{2}{3}$
20. $36 \div \frac{8}{9}$	**24.** $6 \div 2\frac{6}{17}$	**28.** $9 \div 7\frac{6}{7}$	**32.** $100 \div 11\frac{1}{9}$

33. How many pounds of butter at $\$\frac{2}{5}$ a pound can be bought for \$19?

34. How many bottles each holding $\frac{7}{8}$ gal. will contain 14 gal. of vinegar?

35. How many pillow slips each $1\frac{1}{4}$ yards will 35 yards of cotton make?

36. How many months' storage at $\$1\frac{5}{6}$ a month will \$26 pay for?

Written Exercise

146. 1. If 3 yd. of cloth cost $\$4\frac{1}{2}$, how much does 1 yd. cost? $\frac{2}{3}$ of a yard?

2. A grain dealer paid $\$37\frac{1}{2}$ for 45 bu. of wheat. Find the cost of 1 bu. at the same rate; of $3\frac{1}{3}$ bu.

3. A cask of oil containing $40\frac{1}{3}$ gallons cost \$8. How many gallons will \$1 buy?

4. Find the cost of $6\frac{1}{2}$ gal. of oil at $\$\frac{1}{5}$ a gallon.

5. If 7 heirs share equally $\frac{8}{11}$ of a piece of property, what part does each one receive?

6. My horse eats $424\frac{1}{5}$ lb. of hay in 4 weeks. This is an average of how many pounds daily?

7. Divide $\$96\frac{3}{4}$ equally among 5 men. What amount should three of the men receive?

8. If $\$13\frac{3}{5}$ is paid for 1600 lb. of hay, how much is paid for a ton (2000 lb.)?

9. A dividend is $95\frac{3}{4}$, the divisor is 12, what is the quotient?

10. What number multiplied by 9 will give a product of $66\frac{2}{3}$?

11. If 13 suits are cut from $125\frac{3}{3}$ yd. of cloth, how many yards will be required for each suit?

12. The steel rails for $3\frac{2}{7}$ mi. of railroad track cost \$4260. What was the cost per mile?

13. If a gallon of kerosene weighs $6\frac{2}{5}$ lb., how many pounds does a tank contain that holds 5400 gal.?

14. If it takes a match factory $9\frac{1}{3}$ da. to manufacture 160 million matches, how many are manufactured per day?

15. What number multiplied by 35 will give a product of $196\frac{7}{8}$?

Oral and Written Exercise

147. Illustrative Example. At $\$\frac{4}{5}$ a yard how many yards of cloth will $\$\frac{5}{6}$ buy?

Work

$$\$\frac{4}{5} = \$\frac{24}{30} \text{ or } \frac{\$4 \times 6}{30}$$

$$\$\frac{5}{6} = \$\frac{25}{30} \text{ or } \frac{\$5 \times 5}{30}$$

$$\$\frac{25}{30} \div \$\frac{24}{30} = \frac{25}{24} = 1\frac{1}{24}$$

Explanation. $\$\frac{1}{30}$ will buy as many yards as $\$\frac{1}{5}$ is contained times in $\$\frac{1}{6}$. After changing $\frac{1}{5}$ and $\frac{1}{6}$ to fractions having a common denominator, we have $\$\frac{1}{30}$ to divide by $\$\frac{24}{30}$, which gives for a quotient $1\frac{1}{24}$. *Ans.* $1\frac{1}{24}$ yd.

Or, if we express the work in full, thus,

$$\frac{5 \times 5}{30} \div \frac{4 \times 6}{30} = \frac{5 \times 5}{4 \times 6} = \frac{25}{24} = 1\frac{1}{24},$$

the quotient is obtained by multiplying the dividend by the divisor inverted. *Ans.* $1\frac{1}{24}$.

To divide a fraction by a fraction, we change both fractions to fractions having a common denominator, and then divide the numerator of the dividend by the numerator of the divisor.

Or, *we multiply the dividend by the divisor inverted.*

Divide:

1. $\frac{6}{7}$ by $\frac{1}{2}$
2. $\frac{3}{5}$ by $\frac{1}{2}$
3. $\frac{5}{12}$ by $\frac{2}{3}$
4. $\frac{5}{8}$ by $\frac{1}{5}$
5. $\frac{1}{2}$ by $\frac{1}{3}$
6. $\frac{1}{2}$ by $\frac{3}{4}$
7. $\frac{2}{3}$ by $\frac{5}{6}$
8. 2 by $\frac{1}{3}$
9. $1\frac{3}{8}$ by $\frac{4}{5}$
10. $\frac{10}{14} \div 1\frac{8}{7}$
11. $\frac{5}{4} \div \frac{3}{2}$
12. $\frac{7}{5} \div \frac{2}{5}$
13. $\frac{7}{16}$ by $\frac{3}{8}$
14. $\frac{7}{75}$ by $2\frac{2}{25}$
15. $9\frac{1}{3}$ by $3\frac{1}{2}$

Work of Example 15: $9\frac{1}{3} = \frac{28}{3}$; $3\frac{1}{2} = \frac{7}{2}$; $\frac{28}{3} \div \frac{7}{2} = \frac{\overset{4}{\cancel{28}} \times 2}{3 \times \cancel{7}} = \frac{8}{3} = 2\frac{2}{3}$.

16. $11\frac{3}{5} \div \frac{2}{3}$
17. $12\frac{1}{2} \div \frac{3}{4}$
18. $\frac{5}{6} \div 3\frac{1}{2}$
19. $9\frac{3}{8} \div 2\frac{1}{4}$
20. $7\frac{1}{5} \div 3\frac{2}{3}$
21. $4\frac{7}{8} \div \frac{2}{5}$
22. $\frac{3}{5} \div 4\frac{1}{4}$
23. $\frac{7}{8} \div 3\frac{1}{16}$
24. $\frac{7}{8} \div 1\frac{2}{5}$
25. $12\frac{3}{4} \div 5\frac{1}{2}$
26. $\frac{3}{5} \div \frac{4}{5}$
27. $\frac{5}{8} \div 3\frac{1}{3}$

Written Exercise

148. Divide:

1.	$\frac{4}{5}$ by $\frac{5}{36}$	**8.**	$\frac{3}{5}$ by $3\frac{1}{2}$	**15.**	$40\frac{2}{9} \div 8\frac{1}{3}$
2.	$\frac{5}{18}$ by $\frac{3}{14}$	**9.**	$12\frac{4}{5}$ by $9\frac{7}{8}$	**16.**	$40\frac{1}{2} \div 8\frac{2}{9}$
3.	$15\frac{3}{4}$ by $\frac{7}{8}$	**10.**	$9\frac{5}{6}$ by $\frac{5}{6}$	**17.**	$17\frac{4}{13} \div \frac{4}{7}$
4.	$34\frac{1}{2}$ by $\frac{5}{6}$	**11.**	$4\frac{1}{2} \div \frac{5}{6}$	**18.**	$25\frac{1}{2} \div 3\frac{2}{5}$
5.	$\frac{3}{8}$ by $5\frac{1}{6}$	**12.**	$27\frac{2}{5} \div 3\frac{1}{8}$	**19.**	$15\frac{2}{3} \div 5\frac{1}{6}$
6.	$33\frac{5}{6}$ by $1\frac{1}{2}$	**13.**	$\frac{3}{20} \div \frac{11}{13}$	**20.**	$3\frac{5}{6} \div 12\frac{1}{2}$
7.	$15\frac{7}{8}$ by $\frac{3}{4}$	**14.**	$\frac{7}{12} \div \frac{3}{20}$	**21.**	$12\frac{1}{2} \div 3\frac{5}{6}$

22. What are the daily wages of a boy who earns $\$3\frac{3}{4}$ in $8\frac{1}{2}$ da.?

23. Find the rate per hour if a horse goes $17\frac{1}{2}$ mi. in $\frac{7}{8}$ hr.

24. How many $\frac{3}{4}$-lb. packages of starch can be filled from a box containing $6\frac{1}{2}$ lb.?

How many yards of cloth can be bought for $150, the value per yard being as follows?

25.	$\$.33\frac{1}{3}$	**27.**	$\$.12\frac{1}{2}$	**29.**	$\$.62\frac{1}{2}$	**31.**	$\$\frac{5}{8}$
26.	$\$.16\frac{2}{3}$	**28.**	$\$.37\frac{1}{2}$	**30.**	$\$\frac{3}{4}$	**32.**	$\$.87\frac{1}{2}$

33. $66\frac{2}{3}¢$ **34.** $6\frac{1}{4}¢$

35. Find the cost of 5 spools of cotton at $37\frac{1}{2}¢$ per dozen.

36. $20 will buy how many bushels of wheat at $56\frac{1}{4}¢$ each?

37. How long will a barrel of flour (196 lb.) last 12 persons, each using $\frac{7}{8}$ lb. a day?

38. The circumference of a wheel is $9\frac{11}{12}$ ft. How many times will it turn in going a mile (5280 ft.)?

39. How many francs worth $19\frac{1}{4}¢$ can be bought for $8.50, with how many cents remaining?

40. A steam shovel handled $46\frac{1}{8}$ cu. yd. of earth in $4\frac{2}{3}$ min. How much was this per minute?

149. Express as common fractions: $.6\frac{1}{4}$; $.37\frac{1}{2}$; $.333\frac{1}{3}$.

WORK

$$.6\tfrac{1}{4} = \frac{6\frac{1}{4}}{10}; \quad .37\tfrac{1}{2} = \frac{37\frac{1}{2}}{100}; \quad .333\tfrac{1}{3} = \frac{333\frac{1}{3}}{1000}$$

A fraction that has a fraction in one or both of its terms is called a **complex fraction.**

Complex fractions are treated as expressing division, the numerator being the dividend, the denominator the divisor.

Written Exercise

150. Illustrative Examples. Change to simple fractions:

(1) $\frac{37\frac{1}{2}}{100}$; (2) $\frac{10}{16\frac{2}{3}}$; (3) $\frac{\frac{3}{4}}{\frac{4}{5}}$.

WORK

(1) $\frac{37\frac{1}{2}}{100} = \frac{75}{2} \div \frac{200}{2} = \frac{75}{200} = \frac{3}{8}$

(2) $\frac{10}{16\frac{2}{3}} = \frac{30}{3} \div \frac{50}{3} = \frac{30}{50} = \frac{3}{5}$

(3) $\frac{\frac{3}{4}}{\frac{4}{5}} = \frac{15}{20} \div \frac{16}{20} = \frac{15}{16}$

EXPLANATION. To change these complex fractions to simple fractions we perform the division indicated.

Ans. $\frac{3}{8}$; $\frac{3}{5}$; $\frac{15}{16}$.

151. Performing the division expressed by a complex fraction is called **simplifying** the fraction.

Simplify the following complex fractions:

1. $\frac{2\frac{1}{2}}{10}$	**5.** $\frac{8}{12\frac{4}{5}}$	**9.** $\frac{4\frac{1}{6}}{8}$	**13.** $\frac{8\frac{1}{3}}{40}$
2. $\frac{4\frac{1}{2}}{6\frac{1}{4}}$	**6.** $\frac{3\frac{6}{7}}{2\frac{5}{8}}$	**10.** $\frac{.16\frac{2}{3}}{.62\frac{1}{2}}$	**14.** $\frac{66\frac{2}{3}}{90}$
3. $\frac{3}{5\frac{1}{4}}$	**7.** $\frac{2\frac{1}{2}}{20}$	**11.** $\frac{2.5\frac{1}{2}}{4\frac{2}{5}}$	**15.** $\frac{\frac{2}{3} \text{ of } 7\frac{1}{2}}{\frac{2}{5}}$
4. $\frac{\frac{1}{2}}{\frac{2}{3}}$	**8.** $\frac{\frac{6}{7}}{\frac{7}{8}}$	**12.** $\frac{18\frac{3}{4}}{80}$	**16.** $\frac{7.5}{66\frac{2}{3}}$

Written Exercise

152. Illustrative Examples. (1) Find the value of $8 - 3 \times 2$; (2) of $(8 - 3) \times 2$.

EXPLANATION. (1) $8 - 3 \times 2$ shows that 3×2 is to be taken from 8; the result, or value, is 2. (2) $(8 - 3) \times 2$ shows that 3 is to be taken from 8, and the remainder multiplied by 2; the result, or value, is 10.

A vinculum is sometimes used in place of a parenthesis, thus, $\overline{8 - 3} \times 2 = 10$.

In performing a series of operations indicated by signs:

(1) Operations indicated by $\times$ and $\div$ signs are to be done first, those indicated by $+$ and $-$ signs afterward.

(2) $\times$ and $\div$ signs, also $+$ and $-$ signs, are used in order as they occur.

(3) Numbers inclosed by parentheses or a vinculum or by brackets [ ] are treated each as a single number.

Find the value of the following:

1. $(12 + 8) \times 4$
2. $12 + 8 \times 4$
3. $(20 - 8) \div 2$
4. $20 - 8 \div 2$
5. $4 \times 6 \div 2 \times 3$
6. $4 \times 6 \div (2 \times 3)$
7. $24 \div (4 \times 2)$
8. $24 \div 4 \times 2$
9. $(3 + 8) \times 4 - [(8 - 3) \div 5]$
10. $(7 + 11) \div (54 \div 9)$
11. $28 + \dfrac{15 \times 4 - 6 \times 3}{7}$

12. How many yards of fence will surround a lot of land 200 ft. long, 40 ft. wide?

13. How much will it cost to fence the above lot at \$2 per rod? ($16\frac{1}{2}$ ft. = 1 rd.)

14. From the freezing to the boiling point in the Centigrade thermometer is 100°; in the Fahrenheit it is 180°. How many degrees of Fahrenheit equal 40° of Centigrade?

15. How many degrees Centigrade equal 36° Fahrenheit?

Oral Exercise

153. **1.** What product is made by using 2 as a factor twice? 3 times? 4 times? 5 times? 6 times?

2. What product is made by using 3 as a factor twice? 3 times? By using 5 as a factor twice? 3 times?

A product made by using equal factors is called a **power.** One of the equal factors of a power is called the **root** of the power.

The product made by using the factor twice is called the **second power.**

The product made by using the factor 3 times is called the **third power,** and so on.

3. Give the second power of each integer from 1 to 12.

4. Give the third power of each of the nine digits.

The second power of 5 is indicated thus, 5^2; the third power thus, 5^3; and so on.

The figure written above and at the right of a number to indicate the power is called the **exponent.**

The second power of a fraction is the second power of its numerator divided by the second power of its denominator. The third power of a fraction is the third power of its numerator divided by the third power of its denominator.

Find the powers of the following as indicated:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
5.	3^2	5^2	2^5	10^2	4^3	9^2	2^7	10^4
6.	4^2	3^4	12^2	10^3	5^3	1^4	12^3	$(\frac{1}{2})^4$

Written Exercise

154. Find the powers of the following as indicated:

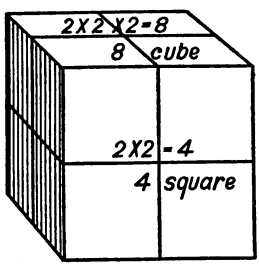
1. 25^2	**4.** 64^2	**7.** 3.6^2	**10.** $(\frac{4}{7})^3$	**13.** $(\frac{3.6}{8})^2$
2. 16^2	**5.** 125^2	**8.** $.27^2$	**11.** $.46^3$	**14.** $(5\frac{1}{2})^2$
3. 13^4	**6.** 81^3	**9.** 14.2^2	**12.** $(\frac{8}{9})^4$	**15.** $(3\frac{1}{8})^2$

16. $(2\frac{1}{2})^3$ 18. $(1.1)^3$ 20. $(.25)^3$ 22. $(.002)^2$ 24. $(4\frac{1}{2})^3$

17. $(4\frac{1}{4})^2$ 19. $(3\frac{2}{3})^3$ 21. $(.02)^4$ 23. $(.005)^3$ 25. $(5\frac{3}{5})^2$

Oral Exercise

155. The side of a square being 2 in. long, the square contains 2 × 2, or 4, sq. in. The side being 3 in. long, the square contains 3 × 3, or 9, sq. in., and so on.



Hence, finding the second power of a number is called **squaring** the number, and one of the two equal factors that make a number is called the **square root** of the number.

The edge of a cube being 2 in. long, the cube contains 2 × 2 × 2, or 8, cu. in. The edge being 3 in. long, the cube contains 3 × 3 × 3, or 27, cu. in. Hence, finding the third power of a number is called **cubing** the number, and one of the three equal factors that make a number is called the **cube root** of the number.

A root is indicated by the sign $\sqrt{\ }$, which is called the **radical sign.** Thus, $\sqrt{49} = 7$ indicates and is read, "The square root of $49 = 7$;" $\sqrt[3]{64} = 4$ indicates and is read, "The cube root of $64 = 4$."

The small figure in the opening of the radical sign is called the **index** of the root.

1. $\sqrt{16}$ 5. $\sqrt{64}$ 9. $\sqrt{144}$ 13. $\sqrt[3]{125}$ 17. $\sqrt[3]{64}$

2. $\sqrt{49}$ 6. $\sqrt{81}$ 10. $\sqrt[3]{27}$ 14. $\sqrt[4]{16}$ 18. $\sqrt{100}$

3. $\sqrt{25}$ 7. $\sqrt[3]{8}$ 11. $\sqrt[3]{216}$ 15. $\sqrt[3]{729}$ 19. $\sqrt{121}$

4. $\sqrt{36}$ 8. $\sqrt[3]{343}$ 12. $\sqrt[4]{81}$ 16. $\sqrt[4]{256}$ 20. $\sqrt[3]{512}$

Written Exercise

156. Illustrative Example. Find the square, or second power, of 23.

WORK

1st Form	2d Form
23	$20+3$
23	$20+3$
69	$60+9$
46	$400+60$
529	$400+2\times60+9=$
	$20^2+2\times20\times3+3^2$

EXPLANATION. (2d Form):

$23=20+3$. $23^2=(20+3)^2$.

Here, by keeping the partial products separate, we see that the square, or second power, of 23 contains the square of the tens + twice the product of the tens by the units + the square of the units.

Using the initial letters t for tens and u for units, the square, or second power, of any number may be represented by:

$$t^2+2\times t\times u+u^2, \text{ or } t^2+2\,tu+u^2.$$

Find the square of the following by the second form above:

1. 12^2	4. 16^2	7. 20^2	10. 34^2	13. 28^2	16. 87^2
2. 13^2	5. 17^2	8. 21^2	11. 35^2	14. 29^2	17. 96^2
3. 14^2	6. 18^2	9. 22^2	12. 36^2	15. 30^2	18. 243^2

WORK OF EXAMPLE 18

$$243^2=\big(24(t)+3(u)\big)^2$$
$$\big(24(t)\big)^2=57{,}600$$
$$2\times\big(24(t)\times3\big)=\quad1440$$
$$\big(3(u)\big)^2=\quad\quad9$$
$$\big(24(t)\big)^2+2\big(24(t)\times3(u)\big)+\big(3(u)\big)^2=59{,}049$$

EXPLANATION. Any integral number of two or more terms may be considered as made up of tens and units; thus, 243 = 24 tens and 3 units.

19. Read the following as tens and units: 126; 583; 1728.

Find the square, or second power, of the following:

20. 125^2	22. 143^2	24. 200^2	26. 306^2	28. $.5^2$	30. $.007^2$
21. 132^2	23. 154^2	25. 212^2	27. 412^2	29. 25^2	31. $(.039)^2$

32. Find the square of $2\frac{1}{2}$.

WORK OF EXAMPLE 32

$2\frac{1}{2} \times 2\frac{1}{2} = (2 + \frac{1}{2})^2$

$2(\frac{1}{2} \times \frac{1}{2})$

$\frac{3}{6\frac{1}{4}}$

EXPLANATION. To multiply $2\frac{1}{2}$ by $2\frac{1}{2}$, we multiply $\frac{1}{2}$ by $\frac{1}{2}$, which equals $\frac{1}{4}$. We multiply 2 by 2, and by $\frac{1}{2}$, and $\frac{1}{2}$ by 2. 2 by $\frac{1}{2}$, and $\frac{1}{2}$ by $2 = 2$ by 1; 2 by 1 with 2 by 2, gives 2 by 3. *Ans.* $6\frac{1}{4}$.

Hence, to square an integer with $\frac{1}{2}$, we multiply the integer by a number 1 greater than itself and annex to the product $\frac{1}{4}$.

33. Square $1\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{1}{2}$, $9\frac{1}{2}$, $10\frac{1}{2}$, $11\frac{1}{2}$, $12\frac{1}{2}$, $13\frac{1}{2}$.

SQUARE ROOT

157. Illustrative Example. Find the square of 1 and 9; 10 and 99; 100 and 999; 1000 and 9999.

$1^2 = 1$	$10^2 = 100$	$100^2 = 10000$	$1000^2 = 1000000$
$9^2 = 81$	$99^2 = 9801$	$999^2 = 998001$	$9999^2 = 99980001$

The results above show that the number of orders of units in the second power of a number is double, or one less than double, the number of orders in the root. Hence,

To find the number of figures in the square root of a second power, we point off the figures into periods of two figures each, beginning at units. The number of figures in the root will equal the number of periods, and the square of the highest order of units will be contained in the left-hand period.

Written Exercise

158. Illustrative Example. Find the square root of 576.

WORK

$5'76(24$

$2(0)^2 = 4 = t^2$

$1\ 76$

$44 \times 4 = 1\ 76 = 2t \times u + u^2$

EXPLANATION. By separating 576 into periods, we find that its square root has tens and units and that the square of its tens is contained in 5(00). The largest square in 5(00) is 4(00), the square root of which is 2(0). We write 2 for the tens of the root, and

subtracting (4)00 from 5(76) have 176 left. We know that 176 contains 2 × 2(0) multiplied by the units, + units^2. To find the units, we divide 176 by 2 × 2(0), = 4(0). Using 4(0) as a trial divisor, we think 4 is the units. Writing 4 with 4(0), thus, 44, and 4 for the units of the root and multiplying 44 by 4, we complete the square and have for the root 24. *Ans.* 24.

Find the square root of:

1. 625	**4.** 1681	**7.** 3136	**10.** 8281	**13.** 1156
2. 841	**5.** 1936	**8.** 3721	**11.** 9025	**14.** 2704
3. 961	**6.** 2209	**9.** 4356	**12.** 9409	**15.** 6724

Oral and Sight Exercise

159. Find the square root of the following by finding the square root of both terms of each fraction. Prove the results.

1. $\frac{1}{4}$	**4.** $\frac{4}{9}$	**7.** $\frac{81}{144}$	**10.** $\frac{144}{169}$	**13.** $\frac{900}{1600}$	**16.** $\frac{1}{3600}$
2. $\frac{1}{9}$	**5.** $\frac{9}{16}$	**8.** $\frac{49}{100}$	**11.** $\frac{1}{36}$	**14.** $\frac{400}{900}$	**17.** .25
3. $\frac{1}{16}$	**6.** $\frac{25}{36}$	**9.** $\frac{16}{64}$	**12.** $\frac{1}{25}$	**15.** $\frac{2500}{4900}$	**18.** .0025

Written Exercise

160. Illustrative Example. Find the square root of $\frac{81}{1369}$.

WORK

$\sqrt{81} = 9$

$\sqrt{13'69} = 37$

$3(0)^2 = \underline{9} = t^2$

$4\ 69$

$67 \times 7 = \underline{4\ 69} = 2tu + u^2$

EXPLANATION. The second power of a fraction being the square of its separate terms, the square root of the separate terms of a fraction will be its square root. The square root of the numerator, 81, is 9. The square root of the denominator 1369, is 37. Hence the square root of $\frac{81}{1369}$ is $\frac{9}{37}$. *Ans.* $\frac{9}{37}$.

The proof consists in multiplying $\frac{9}{37}$ by $\frac{9}{37}$; the result is $\frac{81}{1369}$.

Extract the square root and prove:

1. $\frac{400}{529}$ **2.** $\frac{256}{900}$ **3.** $\frac{1600}{324}$ **4.** $\frac{2500}{1764}$ **5.** $\frac{3721}{7569}$ **6.** $\frac{529}{676}$.

Written Exercise

161. Illustrative Example. Find the square root of 1056.25.

WORK

$$\begin{array}{rl} & 10'56.25(32.5 \\ 3(0)^2 = & \underline{9} \qquad = t^2 \\ & 1\,56 \\ 62 \times 2 = & \underline{1\,24} \qquad = 2t \times u + u^2 \\ & 32.25 \\ 645 \times 5 = & 32.25 = 2t \times u + u^2 \end{array}$$

EXPLANATION. The square root of 1056.25 has three root figures. The first two are found as in Ill. Example, Art. 160, to be 32. 32 is now to be regarded as tens relatively to the next term of the root. 32^2 has been taken out of 1056.25 and 32.25, which remains, contains $2 \times 32\,t \times u + u^2$. Using $2 \times 32(0), = 64(0)$, as a trial divisor we think .5 is the next term of the root. Writing 5 with 64(0), thus, 645, and writing 5 also for the tenths of the root, we multiply 645 by .5 which completes the square. *Ans.* 32.5.

1. *Beginning at units, separate the number whose square root is to be found into periods of two figures each.*

2. *Find the greatest square in the left-hand period, and write its square root as the first term of the root.*

3. *Subtract this square from the left-hand period and to the remainder annex the next period for a dividend.*

4. *Double the part of the root already found considered as tens for a trial divisor, and by this divide the dividend, writing the quotient figure as the next term of the root, also adding it to the trial divisor to make the true divisor.*

5. *Multiply the true divisor by the term of the root last found and subtract the product from the dividend.*

6. *To the remainder last found annex the next period for the next dividend, take double the part of the root now found, regarded as tens, for a trial divisor, and proceed as before.*

NOTE 1. The last step is to be repeated until all the terms of the given number have been used, or until the root has been found to any desired number of decimal places. In the latter case two zeros are annexed to each successive remainder.

NOTE 2. When a dividend does not contain a trial divisor, place a zero as the next figure of the root. Place also a zero at the right of the trial

divisor; for the next dividend unite the next period to the previous dividend, and proceed as before.

The square root of a common fraction can be found by taking the square root of both numerator and denominator.

NOTE 3. If the denominator is not a perfect square, the fraction should be changed to an equivalent fraction whose denominator is a perfect square. Thus, $\frac{2}{3} = \frac{6}{9}$; $\frac{4}{7} = \frac{28}{49}$.

Written Exercise

162. Illustrative Example. Extract the square root of $6\frac{2}{3}$ to 3 places of decimals.

WORK

$6\frac{2}{3} = 6.66'66'66$

$6.66'66'66(2.581\cdots$

$2(0)^2 = 4 \quad = t^2$

266

$45 \times 5 = 225 \quad = 2t \times u + u^2$

4166

$508 \times 8 = 4064 = 2t \times u + u^2$

10266

$5161 \times 1 = 5161$

5105

EXPLANATION. To find the square root of $6\frac{2}{3}$ we first change the common fraction to a decimal carried to double the number of decimals required in the root, and proceed to extract the root as for integral numbers. The root is 2.581 ..., the dots indicating that there is a remainder.

Ans. 2.581 ...

The root could be found by changing $6\frac{2}{3}$ to the improper fraction $\frac{20}{3}$, changing $\frac{20}{3}$ to $\frac{60}{9}$, and extracting the root of both terms, which gives $\frac{7.745}{3}$, or 2.581.

Find the square roots to two decimal places when not exact:

1.	81,225	8.	5241.76	15.	30.25	22.	$\frac{3}{4}$
2.	119,025	9.	7396	16.	7.7284	23.	$3\frac{1}{3}$
3.	242,064	10.	1069.29	17.	1.423	24.	$12\frac{5}{9}$
4.	531,441	11.	217,156	18.	.023	25.	$16\frac{4}{49}$
5.	567,009	12.	25.8064	19.	53.29	26.	$2\frac{1}{2}$
6.	622,521	13.	318,096	20.	3.42	27.	$3\frac{1}{4}$
7.	76.7376	14.	45.0241	21.	.555025	28.	$6\frac{1}{4}$

Oral and Written Exercise

163. Illustrative Examples. (1) Find $\frac{3}{5}$ of 20¢; (2) $\frac{5}{8}$ of $\$\frac{16}{25}$; (3) $\frac{3}{5}$ of $\frac{8}{11}$ rd.

SOLUTION. (1) $\frac{1}{5}$ of 20¢ = 4¢; $\frac{3}{5}$ of 20¢ = 3 × 4¢ = 12¢, *Ans.*

(2) $\frac{1}{8}$ of $\$\frac{16}{25} = \$\frac{2}{25}$; $\frac{5}{8}$ of $\$\frac{16}{25} = 5 \times \$\frac{2}{25} = \$\frac{10}{25} = \$\frac{2}{5}$, *Ans.*

(3) $\frac{1}{5}$ of $\frac{8}{11}$ rd. = $\frac{8}{55}$ rd : $\frac{3}{5}$ of $\frac{8}{11}$ rd. = 3 × $\frac{8}{55}$ rd. = $\frac{24}{55}$ rd, *Ans.*

To find the fractional part of a number, we divide the given number by the denominator expressing the fractional part, and multiply the result by the numerator; or we multiply the given number by the fraction (Art. 140).

1. $\frac{2}{3}$ of 18	10. $\frac{2}{5}$ of 35	19. $\frac{3}{10}$ of 50	28. $\frac{2}{7}$ of $\frac{3}{5}$
2. $\frac{3}{4}$ of 16	11. $\frac{4}{9}$ of 36	20. $\frac{10}{7}$ of 56	29. $\frac{9}{8}$ of 72
3. $\frac{5}{7}$ of 21	12. $\frac{2}{3}$ of 39	21. $\frac{4}{11}$ of $5\frac{1}{2}$	30. $\frac{3}{5}$ of 75
4. $\frac{7}{8}$ of 24	13. $\frac{7}{8}$ of 40	22. $\frac{8}{14}$ of 56	31. $\frac{3}{4}$ of 80
5. $\frac{4}{5}$ of 25	14. $\frac{1}{8}$ of $\frac{5}{6}$	23. $\frac{3}{2}$ of 60	32. $\frac{3}{5}$ of 80
6. $\frac{6}{7}$ of 28	15. $\frac{6}{7}$ of 42	24. $\frac{2}{5}$ of $62\frac{1}{2}$	33. $\frac{2}{7}$ of $87\frac{1}{2}$
7. $\frac{7}{9}$ of $\frac{3}{2}$	16. $\frac{2}{3}$ of 45	25. $\frac{4}{9}$ of 63	34. $\frac{5}{12}$ of 96
8. $\frac{3}{4}$ of 32	17. $\frac{5}{12}$ of 48	26. $\frac{5}{11}$ of 66	35. $\frac{4}{5}$ of $3\frac{1}{3}$
9. $\frac{7}{11}$ of 33	18. $\frac{2}{7}$ of 49	27. $\frac{3}{4}$ of $66\frac{2}{3}$	36. $\frac{5}{8}$ of $4\frac{4}{9}$

37. In $\frac{2}{5}$ of a day there are how many hours? In $\frac{2}{5}$ of a year there are how many days?

38. Make the following entries in an account against John Boltwood, find the amount due, and close the account:

$30\frac{3}{8}$ yd. of crash at \$2.80 a yard.
$27\frac{3}{4}$ yd. of silk at \$1.66 a yard.
$30\frac{2}{3}$ yd. of cashmere at \$.75 a yard.
$12\frac{1}{3}$ yd. of velvet at \$2.25 a yard.
$31\frac{1}{4}$ yd. of worsted at \$5.00 a yard.

Oral Exercise

164. 1. What part of 12 is 1? 5? 7? 11?

2. Express the relation that 10 bears to 12; that 12 bears to 10.

The relation that one number bears to another is called the **ratio** of the numbers.

The ratio of 10 to 12 is expressed thus, 10 : 12; the ratio of 12 to 10, thus, 12 : 10. These ratios are expressed as fractions in their lowest terms, thus, $10 : 12 = \frac{5}{6}$; $12 : 10 = \frac{6}{5}$.

The numbers compared are called the **terms** of the ratio. The number whose value is sought is called the **antecedent**; the number with which it is compared is called the **consequent.**

Expressed as a fraction the antecedent is the numerator; expressed with the sign, the antecedent precedes the sign.

Written Exercise

165. Express as fractions in their lowest terms the ratio of:

1. 2 to 6	5. 5 ft. to 15 ft.	9. 9 to 36	13. 24 to 12
2. 3 to 6	6. 1 da. to 1 wk.	10. 8 to 16	14. 20 to 15
3. 4 to 6	7. \$5 to \$30	11. 16 to 48	15. 7 to 21
4. \$10 to \$5	8. \$20 to \$24	12. 36 to 48	16. 28 to 21

17. What part of his debts can a man pay who is able to pay only 30¢ on \$1?

18. A bicycle rider can ride a mile in 8 min. What part of a mile can he ride in 6 min.?

19. A 50¢ article sells for 75¢. What is the ratio of gain to value?

20. Eggs costing 30¢ a dozen sell for 24¢. What is the ratio of loss?

21. Oranges costing 35¢ a dozen sell for 50¢ a dozen. What is the ratio of gain?

Written Exercise

166. Illustrative Example. Find the ratio of $\frac{3}{4}$ to $\frac{4}{5}$.

WORK

$\frac{3}{4} = \frac{15}{20}$

$\frac{4}{5} = \frac{16}{20}$

EXPLANATION. $\frac{3}{4}$ and $\frac{4}{5}$ changed to fractions having the same common denominator, 20, become $\frac{15}{20}$ and $\frac{16}{20}$. *Ans.* The ratio of $\frac{15}{20}$ to $\frac{16}{20}$ is $\frac{15}{16}$.

The result could also be obtained by making $\frac{3}{4}$ the numerator of a fraction and $\frac{4}{5}$ the denominator, and simplifying the complex fraction. Thus, $\frac{\frac{3}{4}}{\frac{4}{5}} = \frac{15}{16}$.

Find the ratio of:

1. $5\frac{1}{2}$ to 11
2. $12\frac{1}{2}$ to 100
3. $\frac{3}{5}$ to $\frac{4}{7}$
4. $\frac{1}{12}$ to $\frac{5}{9}$
5. $\frac{8}{20}$ to $\frac{5}{6}$
6. $2\frac{2}{3}$ to $6\frac{1}{2}$
7. $\frac{2}{7}$ to $\frac{5}{8}$
8. $\frac{3}{4}$ to $\frac{5}{6}$
9. $\frac{3}{8}$ to $\frac{5}{6}$
10. $16\frac{2}{3}$ to 100
11. $31\frac{1}{4}$ to $62\frac{1}{2}$
12. $83\frac{1}{3}$ to 100

13. Charles and Fred buy a box of 150 oranges for $3.50. Charles takes 60 and Fred takes the remainder. What part of the cost should each bear?

14. A box of chocolates containing 3 layers of 1 doz. each is bought for 60¢ and sold for 2 cents each. What is the ratio of the gain to the cost?

15. Grape fruits bought at $4 a box, 66 fruits to the box, are sold 3 for 25¢. What is the gain? What is the ratio of the gain to the cost?

16. Pineapples bought at $3.75 a box of 36 are sold 2 for 25¢. What is the gain? the ratio of gain to cost?

Oral Exercise

167. Illustrative Example. At 24¢ for 12 eggs how much will 8 eggs cost?

SOLUTION. (1) Since 12 eggs cost 24¢, 1 egg will cost $\frac{1}{12}$ of 24¢ = 2¢, and 8 eggs will cost 8 times 2¢ = 16¢, *Ans.*

(2) Since 12 eggs cost 24¢, 8 will cost $\frac{8}{12} = \frac{2}{3}$, of 24¢ = 16¢, *Ans.*

Solve the following, using the briefest analysis:

1. 8 yd. of cloth cost 36¢. Find the cost of 2 yd.; of 6 yd.; of 16 yd.; of 12 yd.; of 10 yd. How many yards would 18¢ buy? 12¢? 4¢?

2. 10 rows of corn yield 35 bu. What is the yield of 2 rows? 6 rows? 12 rows? 14 rows? 20 rows? How many rows would yield 7 bu.? $10\frac{1}{2}$ bu.?

3. When 30 pk. of peas can be bought for $12, how many pecks can be bought for $3? for $6? for $9? for $4? for $8? for $2? for $2\frac{1}{2}$?

4. If a cistern can be emptied in 20 min. by 3 pipes of uniform size, in what time will it be emptied by 1 of the pipes? by 2 of the pipes?

5. How many pipes of the size in Ex. 4 would empty the above cistern in 10 min.? in 5 min.? in 4 min? in 60 min.?

6. If 4 dredgers can dredge a channel in 18 da., in how many days can 2 do it? In how many days can 8 do it?

7. How many dredgers would dredge the above channel in 9 da.? in 6 da.? in 3 da.?

Written Exercise

168. Illustrative Example. An express train goes 660 miles in 12 hours. How far will it go in 9 hours?

WORK

$$\frac{\overset{55}{\cancel{660}} \times 9}{\underset{1}{\cancel{12}}} = 495$$

EXPLANATION. We solve this problem as in Art. 167. We write the work as in the model and cancel.

Ans. 495 mi.

1. 60 yd. of plush cost $135. Find the cost of 130 yd.

2. If $11.55 is paid for 1540 bricks, what is the price per thousand?

3. If $18.75 is paid for 100 lb. of cheese, what is the cost of 1500 lb.?

4. The population of a city increased 85,638 from 1880 to 1890; at the same rate find the increase from 1890 to 1910.

5. At what rate was a wind storm traveling per hour that took 15 hr. to go from Chicago to New York, calling the distance 875 mi.?

6. A man's personal property is taxed $137.50, at a rate of $11 on $1000. How much would it be taxed if he moved to a town where the rate is $16.50 on $1000?

7. A roadway 45 ft. wide contains 30 A. of land. How much will it contain if it is made 80 ft. wide?

8. A 56 lb. cheese sells for $8.40. At the same price per pound for how much would a cheese weighing $66\frac{1}{2}$ lb. sell?

Written Exercise

169. 1. What part of $272\frac{1}{4}$ sq. ft. is 9 sq. ft.?

2. The material for a cloak cost $12\frac{1}{2}$, the making $11\frac{1}{2}$. What part of the entire cost was the cost of making?

3. If a man can do a piece of work in 25 days, what part of it can he do in $6\frac{1}{4}$ days?

4. A can lay a hall floor in 6 da., B in 8 da. What part of the work can each do in a day? What part can both do together in a day?

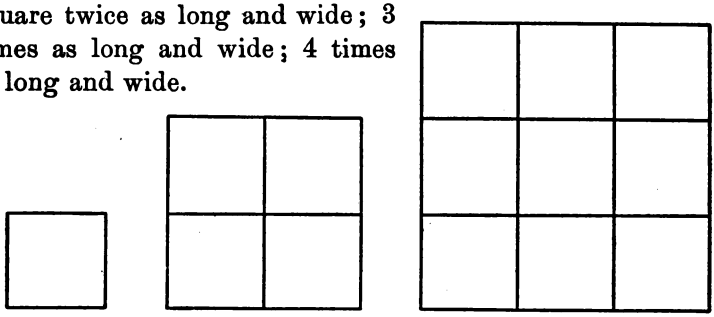
5. Three men working at the same rate per day did a piece of work for $90. A worked 5 days, B $6\frac{1}{2}$ days, and C $8\frac{1}{2}$ days. How much did each receive per day? How much did each receive for his work?

6. In a square-cornered lot 100 ft. long and 80 ft. wide there is an asparagus bed, also square-cornered, 50 ft. long and 16 ft. wide. What part of the lot does the bed occupy?

7. What part of his earnings does a man spend who works 6 da. a week, at $1.75 per day, and pays for his board $4 a week, and for other expenses $32.50 a quarter (13 wk.)?

8. Find the ratio of a square 1 in. long and 1 in. wide to a square 2 in. long and 2 in. wide.

9. Find the ratio of a square 4 in. long and 4 in. wide to a square twice as long and wide; 3 times as long and wide; 4 times as long and wide.



Oral Exercise

170. 1. In $\frac{2}{5}$ of a ream of paper there are 8 quires. How many quires are there in $\frac{1}{5}$ of a ream? in a whole ream?

2. 8 is $\frac{2}{5}$ of what number?

SOLUTION. Since $\frac{2}{5}$ of the number is 8, $\frac{1}{5}$ of it $= \frac{1}{2}$ of $8 = 4$, and $\frac{5}{5}$ of it $= 5$ times $4 = 20$. *Ans.* 20.

3. 6 hr. equal $\frac{3}{8}$ of what number of hours?

4. 12 da. equal $\frac{3}{4}$ of what number of days?

5. If the weight of $\frac{3}{4}$ of a bushel of potatoes is 45 lb., what is the weight of 1 bu.?

6. If I sell grapes for 60¢ a basket and thereby lose a sum equal to $\frac{1}{4}$ of the cost, what was the cost?

7. Bertha has been 6 hr. tacking $\frac{3}{7}$ of a comforter. At the same rate how many more hours will it take her to finish it?

8. A vessel having lost $\frac{1}{5}$ of her cable, has 200 ft. remaining. How many feet had she at first?

9. Kate is 14 yr. old, which is $1\frac{1}{6}$, or $\frac{7}{6}$, times the age of Alfonso. How old is Alfonso?

Oral Exercise

171. **1.** A and B are to share $100. B is to have $\frac{1}{4}$ as much as A. What is each one's part?

A's part is $\frac{4}{4}$ of itself; B's part with A's part will equal $\frac{5}{4}$ of A's part. Then $100 is $\frac{5}{4}$ of A's part.

2. A mother and her son together have $45; the son's part is $\frac{2}{3}$ as great as the mother's. What is each one's part?

3. When $6 received for the use of money is $\frac{3}{100}$ of the sum loaned, what is the sum loaned?

4. Of what sum of money is $4 $\frac{2}{100}$? is $6 $\frac{3}{100}$? is $18 $\frac{6}{100}$?

Written Exercise

172. **1.** $\frac{7}{8}$ lb. is $\frac{2}{25}$ of what?

2. $\frac{5}{12}$ ft. is $\frac{10}{7}$ of what?

3. $\frac{16}{25}$ A. is $\frac{4}{11}$ of what?

4. $6\frac{1}{4}$¢ $= \frac{2}{3}$ of what?

5. $3\frac{3}{8}$¢ $= \frac{9}{10}$ of what?

6. $16\frac{2}{3} = \frac{5}{8}$ of what?

7. If $\frac{3}{8}$ of a barrel of flour weighs $73\frac{1}{2}$ lb., what is the weight of a barrel?

8. A farmer received in 1 yr. $3575 as a royalty for oil produced on his farm. If this was $\frac{2}{9}$ of the value produced, what was the value produced?

What is the cost per pound:

9. Of veal when $5\frac{3}{4}$ lb. can be bought for 92¢?

10. Of butter when $4\frac{1}{8}$ lb. can be bought for $1.65?

11. Of sugar when $12\frac{1}{2}$ lb. can be bought for a dollar?

12. Of tea when a box of $13\frac{1}{3}$ lb. cost $10?

13. What is the cost per quire of paper when $4\frac{1}{2}$ quires can be bought for $1\frac{1}{8}$?

14. What is the cost per gross of pencils when $5\frac{1}{4}$ gross can be bought for $7\frac{7}{8}$?

Written Exercise

173. 1. Find the price per gallon, if $7\frac{1}{2}$ gal. of ink cost \$3.15.

2. Find the cost per pound, if $4\frac{1}{4}$ lb. of fish cost \$1.19.

3. After losing $\frac{1}{8}$ of his property a man had \$2400 left. How much money had he at first?

4. A sold $\frac{5}{9}$ of his farm and had 412 acres left. How many acres had he at first?

5. A, who owned $\frac{1}{3}$ of a yacht, sold $\frac{1}{2}$ of his share for \$550. What was the total value of the yacht at the same rate?

6. What number increased by $\frac{1}{4}$ of itself equals $291\frac{1}{3}$?

7. What number diminished by $\frac{1}{4}$ of itself equals $291\frac{1}{3}$?

8. A cargo of 14,500 bushels of No. 2 Red Winter wheat was sold in Liverpool for \$13,424. After deducting \$2740.50 for the cost of transportation and selling, what was the net price per bushel? How much was received for the wheat?

9. From a hogshead of molasses $31\frac{1}{4}$ gal. were taken, after which $\frac{4}{5}$ of the original quantity remained. How many gallons did the hogshead contain at first?

10. When $\frac{2}{3}$ of $\frac{7}{8}$ of a vessel is worth \$980, how much is the whole vessel worth?

11. In a college of 342 students there were $\frac{1}{8}$ as many women as men. What was the number of each?

12. A sold a pig for \$5.20, which was $1\frac{5}{8}$ of what it cost. How much did it cost, and how much was gained?

13. I lost \$2050 by a man's paying only $\frac{2}{5}$ of his debts. What was the total amount that he owed to me?

14. Four children sold their berries for \$2.60. One sold 5 qt., another 7 qt., another 6 qt., another 8 qt. What was the price per quart? the share of each child?

Oral Exercise

174. **1.** What is one of the four equal parts of 8? of 9?

One of the equal parts of a number is called an **aliquot part** of the number. $2\frac{1}{4}$ is an aliquot part of 9.

Of the following numbers find the aliquot parts and their multiples as indicated:

2. Of 10 find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{8}$.

3. Of 100 find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

4. Of 1000 find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

5. Of 200 find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

6. Of 50 find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

7. Of 30 find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$.

8. Of 60 find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{20}$.

9. Of 144 find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{24}$.

Express in fractional form in their lowest terms the ratios of the following numbers:

10. .5 to 1 **15.** 5 to 10 **20.** 50 to 100 **25.** 500 to 1000

11. .$2\frac{1}{2}$ to 1 **16.** $2\frac{1}{2}$ to 10 **21.** 25 to 100 **26.** 250 to 1000

12. .$7\frac{1}{2}$ to 1 **17.** $7\frac{1}{2}$ to 10 **22.** 75 to 100 **27.** 750 to 1000

13. .$3\frac{1}{3}$ to 1 **18.** $3\frac{1}{3}$ to 10 **23.** $33\frac{1}{3}$ to 100 **28.** $333\frac{1}{3}$ to 1000

14. .$6\frac{2}{3}$ to 1 **19.** $6\frac{2}{3}$ to 10 **24.** $66\frac{2}{3}$ to 100 **29.** $666\frac{2}{3}$ to 1000

30. $12\frac{1}{2}$ to 100 **36.** 625 to 1000 **42.** $116\frac{2}{3}$ to 100

31. $37\frac{1}{2}$ to 100 **37.** 875 to 1000 **43.** $83\frac{1}{3}$ to 1000

32. $62\frac{1}{2}$ to 100 **38.** $8\frac{1}{3}$ to 100 **44.** $166\frac{2}{3}$ to 1000

33. $87\frac{1}{2}$ to 100 **39.** $16\frac{2}{3}$ to 100 **45.** $833\frac{1}{3}$ to 1000

34. 125 to 1000 **40.** $83\frac{1}{3}$ to 100 **46.** $1166\frac{2}{3}$ to 1000

35. 375 to 1000 **41.** $62\frac{1}{2}$ to 1000 **47.** $137\frac{1}{2}$ to 1000

175. By using the aliquot parts of numbers, the work of multiplying and dividing can often be materially shortened.

Written Exercise

176. Illustrative Example. Overshoes are sold at the following prices, \$1, 50¢, \$.66⅔, and \$1.25. Find the cost of 60 pairs at each price.

WORK

Cost = \$60 at \$1

½ of \$60 = \$30, at 50¢

⅔ of \$60 = \$40, at 66⅔¢

5/4 of \$60 = \$75, at \$1.25

SOLUTION. 60 pairs at \$1 each will cost \$60. Since 60 pairs at \$1 each cost \$60, at 50¢ each (= \$½), they will cost ½ of \$60, = \$30; at \$.66⅔ (= \$⅔), they will cost ⅔ of \$60, = \$40; at \$1.25 (= \$1¼), they will cost 1¼ times \$60, = \$60 + \$15, = \$75.

What is the cost of 72 spoons at:

1. 50¢ apiece?
2. 25¢ apiece?
3. 75¢ apiece?
4. 12½¢ apiece?
5. 37½¢ apiece?
6. 62½¢ apiece?
7. 33⅓¢ apiece?
8. 66⅔¢ apiece?
9. \$1.16⅔ apiece?

Written Exercise

177. Illustrative Example. At \$1.68 a yard for crash what is the cost (1) of 62½ yd.? (2) of 133⅓ yd.?

WORK

(1) $\overset{21}{\cancel{\$168}} \times \frac{5}{\cancel{8}} = \$105$

(2) $\overset{56}{\cancel{\$168}} \times \frac{4}{\cancel{3}} = \$224$

EXPLANATION. 62½ yd. is ⅝ of 100 yd. 100 yd. at \$1.68 cost \$168; 62½ yd. will cost ⅝ of \$168 = \$105. *Ans.* \$105.

133⅓ yd. is ⅘ of 100 yd.; 133⅓ yd. will cost ⅘ of \$168 = \$224. *Ans.* \$224.

At \$3.36 a yard for cashmere what is the cost:

1. Of 50 yd.?
2. Of 12½ yd.?
3. Of 33⅓ yd.?
4. Of 250 yd.?
5. Of 37.5 yd.?
6. Of 16⅔ yd.?
7. Of .375 yd.?
8. Of 750 yd.?
9. Of 62½ yd.?

10. Find the cost of 54 yd. of cloth at 75¢ a yard.

11. Find the cost of 36 bushels of tomatoes at $87\frac{1}{2}$¢, and $87\frac{1}{2}$ bushels at 36¢ a bushel.

12. Find the balance due on an account for 39 yd. of velvet at $\$1.33\frac{1}{3}$ a yard, 25 yd. at \$1.50, and 66 yd. at $\$1.66\frac{2}{3}$, \$175 in cash having been paid on the account.

Written Exercise

178. Illustrative Example. How many pounds of tea at $\$1.33\frac{1}{3}$ a pound can be bought for \$25?

WORK

$$\frac{25 \times 3}{4} = 18\frac{3}{4}$$

SOLUTION. $\$1.33\frac{1}{3} = \$1\frac{1}{3}$, $= \$\frac{4}{3}$. At \$1 per pound \$25 will buy 25 lb. At $\$\frac{1}{3}$ per pound \$25 will buy 3 times 25 lb.; at $\$\frac{4}{3}$ per pound, \$25 will buy $\frac{1}{4}$ of 3 times 25 lb., which is $18\frac{3}{4}$ lb. *Ans.* $18\frac{3}{4}$ lb.

1. How many vases can be bought for \$9, the price being $12\frac{1}{2}$¢ each? $\$1.12\frac{1}{2}$ each? \$1.50 each? 60¢ each?

Find the number of pounds in a quarter of beef when:

2. Selling at 20¢ a pound it brings \$50.

3. Selling at $16\frac{2}{3}$¢ a pound it brings \$37.50.

4. Selling at $12\frac{1}{2}$¢ a pound it brings \$18.75.

5. Find the cost of 2 tubs of butter, each containing 35 lb., at 25¢ a pound, and 1 tub of 45 lb., at $37\frac{1}{2}$¢ a lb.

6. Find the cost of 2 boxes of cheese of 75 lb. each, at $12\frac{1}{2}$¢ a pound, and 1 box of 82 lb., at $16\frac{2}{3}$¢.

7. What is the cost of $12\frac{1}{2}$ yd. of silk at 90¢ a yard; $66\frac{2}{3}$ yd. at \$1.10, and $37\frac{1}{2}$ yd. at \$2.66 a yard?

What is the cost:

8. Of 6750 shingles at \$2.50 per M?

9. Of 15,250 bricks at $\$12\frac{1}{2}$ per M?

10. Of 18,750 ft. of lumber at $8\frac{1}{3}$¢ a foot?

11. Of 25 bbl. cement at \$3.50 per barrel?

12. A man failing in business paid only 40¢ on a dollar of his debts. Mr. Bush lost $2070 by him. How much was owed Mr. Bush?

13. After losing $\frac{3}{20}$ of his kite line, William still had 340 ft. left. How long was his line at first? How much did he lose?

14. A city merchant sells 120 bbl. of flour at 5.62\frac{1}{2}$ per barrel for a Western farmer, who pays the merchant $2\frac{1}{2}$¢ on a dollar for making the sale. How much does the farmer pay the merchant?

15. Express by fractions in their smallest terms the ratio of $1\frac{3}{4}$ da. to the days in a week; to the days in February, 1910; in April.

16. Express in a decimal the ratio of $\frac{7}{8}$ da. to 30 da.; of $\frac{5}{8}$ da. to 25 da.

17. If the ratio of the volume of water to ice is as 23 to 25, what space will the ice from 184 cu. ft. of water occupy?

PARTITIVE PROPORTION

179. Illustrative Example. A dwelling house was damaged by fire to the extent of $600. Two insurance companies made good the loss, one company paying $3 of the sum to $2 paid by the other. How much did each pay?

WORK

$$\frac{\$600 \times 3}{5} = \$360$$

$$\frac{\$600 \times 2}{5} = \$240$$

EXPLANATION. Since one company pays $3 to the other's $2 for every $5 paid by both, one will pay $\frac{3}{5}$, the other $\frac{2}{5}$. $\frac{3}{5}$ of $600 = $360; $\frac{2}{5}$ of $600 = $240.

Ans. $360; $240.

The process of separating a number into parts each having a ratio to the sum of the parts is called **partitive proportion.**

Oral Exercise

180. 1. Two men divide between them $2400 profits from a business, in the proportion of 3 to 5. Find the share of each.

2. Of 100 students, 3 are conditioned to 17 that are not. How many are conditioned?

3. Two carpenters build a fence, one working 5 days, the other 7 days. $36 being paid for the work, what is the share of each?

4. The income from a peach orchard is $180. Divide it among three persons in the proportion of 2, 3, and 4.

5. Apportion a tax of $350 to 3 lots of land containing, respectively, 2 A., 2 A., and 3 A.

Written Exercise

181. 1. Two herdsmen owned, one 372 cattle, the other 428 cattle which were kept at an expense of $1000. What was each man's share of the expense?

2. A charge of $50 for freight over two railroads, one a distance of 750 mi., the other of 500 mi., was paid the roads in proportion to the number of miles carried. How much was paid to each?

3. Two masons worked on a job, one 4 da., the other $3\frac{1}{2}$ da. If $34 was paid for the work, what was each mason's share?

4. Separate $26.40 into 3 parts in the proportion of 21, 22, and 23.

5. Four children picked berries which they sold for $2.60. A picked 5 qt., B 6 qt., C 7 qt., D 8 qt. How much money should each receive?

6. Divide an estate of $49,000 between two heirs so that one's portion is to the other's as $\frac{1}{2}$ is to $\frac{2}{3}$.

182. Messrs. Winn and Weeks are persons doing business together, and sharing the profits according to the money each puts in. They are called **partners** in business, and are said to be in **partnership**.

A partnership is called a company, a firm, a house, etc. Each partner of a firm usually contributes something that has a money value in the business; it may be money, goods, labor, skill, experience, or good will.

The gains and losses of the business are usually shared by the partners in proportion to the value and time of their investments.

Written Exercise

183. Illustrative Example. A and B form a partnership for the manufacture of shoes. A invests \$10,000 and B invests \$8000. Their profits for a year are \$4500. Find each partner's share of the profits.

WORK

\$10000
8000
\$18000

$\frac{10000}{18000} = \frac{5}{9}$, A's share of capital and profits.

$\frac{8000}{18000} = \frac{4}{9}$, B's share of capital and profits.

$\frac{5}{9}$ of \$4500 = \$2500, A's share.

$\frac{4}{9}$ of \$4500 = \$2000, B's share.

EXPLANATION. The entire capital is \$18,000. A's capital being $\frac{10000}{18000}$, or $\frac{5}{9}$, of the entire capital, his share of the profits is $\frac{5}{9}$ of \$4500, or \$2500. B's capital being $\frac{8000}{18000}$, or $\frac{4}{9}$, of the entire capital, his share of the profits is $\frac{4}{9}$ of \$4500, or \$2000.

Ans. A's, \$2500; B's, \$2000.

1. Ross and Flint formed a partnership, Ross putting in \$6000, Flint \$8000. They cleared \$4430. How much should each receive?

2. A and B who are partners in business make a profit in one year of \$3000, of which A is to have $\frac{1}{4}$ for managing the business. How much of the balance of profits should each receive if A contributes \$6000 and B \$3000 to the capital?

Written Exercise (*Continued*)

3. A firm has three partners, Messrs. Lloyd, Smith, and Warren, who invest as follows: Mr. Lloyd, $10,000; Mr. Smith, $15,000; Mr. Warren, $20,000. They gain $9276.39. Find each partner's share.

4. A vessel takes freight for A, B, and C as follows: for A, $3690 worth; for B, $8400 worth; for C, $9600 worth. During a storm $6925 worth of the cargo is thrown overboard. Apportion the loss among A, B, and C.

5. A loss of $11,500 by fire was paid by insurance companies in the proportion of the sums insured on the property by the companies, which sums were as follows: Company A, $10,000; B, $15,000; C, $12,000. Apportion the loss to the companies.

Written Exercise

184. Illustrative Example. A and B formed a partnership. A put in $2500 for 8 months, B put in $3200 for 10 months, and C $2000 for 12 months. They gained $1852.50. Find each partner's share of the gain.

WORK

$\$2500 \times 8 = \$20{,}000$ $\quad \frac{20000}{76000} = \frac{5}{19}$ of $\$1852.50 = \$487.50$, A's

$\$3200 \times 10 = \$32{,}000$ $\quad \frac{32000}{76000} = \frac{8}{19}$ of $\$1852.50 = \$780$, B's

$\$2000 \times 12 = \$24{,}000$ $\quad \frac{24000}{76000} = \frac{6}{19}$ of $\$1852.50 = \$585$, C's

$\$76{,}000$

EXPLANATION. A's $2500 for 8 months = 8 times $2500 = $20,000 for 1 month; B's $3200 for 10 months = 10 times $3200 = $32,000 for 1 month; C's $2000 for 12 months = 12 times $2000 = $24,000 for 1 month.

The entire capital is equal to $20,000 + $32,000 + $24,000 for 1 month.

A's capital being equal to $\frac{20000}{76000}$, or $\frac{5}{19}$, of the entire capital, his share of the gain is $\frac{5}{19}$ of $1852.50, or $487.50.

B's share of the gain is $\frac{8}{19}$ of $1852.50, or $780.

C's share of the gain is $\frac{6}{19}$ of $1852.20, or $585.

Ans. A's, $487.50; B's, $780; C's, $585.

Written Exercise

185. 1. Hale and Morse are in partnership. Hale invests $600 for 18 months, Morse $800 for 12 months. They gain $510. What is each partner's share of the gain?

2. A, B, and C engage in trade. A's investment is $300 for 7 months, B's is $500 for 8 months, C's is $200 for 12 months. They gain $850. How much should each partner receive?

3. F, G, and H contract to do a certain piece of work for $2870. F furnishes 20 men for 15 days, G 18 men for 14 days, H 30 men for 9 days. Find the sum due each contractor.

4. M and N formed a partnership for carrying on the coal business. M furnished a capital of $5000, N of $4000. At the end of 6 months G entered the firm with a capital of $6000. Their gain for the first year was $3600. How much should each partner receive?

5. The profits of the above firm for the next year amounted to $5250. How much should each partner receive?

6. A and B entered into partnership Jan. 1, A investing $3000 and B $2000. July 1, A added $200, and Aug. 1, B added $2500. Their gain at the end of the year was $4580. Find the share of each partner.

7. C and D formed a partnership with a capital of $12,000, of which C furnished $7000. After 18 months C withdrew $1000. At the end of 3 years the partnership was dissolved. Their profits for the 3 years were $7038. What was each partner's share?

8. A and B formed a partnership with a capital of $150,000, of which A contributed $75,000. After the first year A withdrew $25,000. The partnership continued for five years during which the profits were $15,000. What was each partner's share of these profits?

Written Exercise

186. Write 125 so that it shall express:

1. Thousands
2. Thousandths
3. Hundredths
4. Ten-thousands
5. Ten-thousandths
6. Hundred-thousandths
7. Units
8. Tenths
9. Millionths
10. Write 350 and 85 thousandths; 1 and 85 millionths.

Express as common fractions in their lowest terms:

11. 52 hundredths
12. 555 ten-thousandths
13. 750 thousandths
14. $16\frac{2}{3}$ hundredths
15. $162\frac{1}{2}$ hundredths
16. $12\frac{1}{2}$ tenths

Change to decimal fractions of equal value and add:

17. $\frac{4}{5} + \frac{1}{8} + \frac{6}{9}$
18. $\frac{5}{8} + \frac{5}{6} + \frac{5}{9}$
19. $\frac{6}{8} + \frac{6}{11} + \frac{4}{10}$
20. $\frac{4}{15} + \frac{5}{16} + \frac{1}{9}$
21. $\frac{7}{12} + \frac{8}{9} + \frac{1}{6}$
22. $\frac{1}{4} + \frac{1}{2} + \frac{5}{6}$
23. $3\frac{1}{3} + 2\frac{3}{4} + 1\frac{1}{12}$
24. $7\frac{1}{2} + 8\frac{5}{16} + 4\frac{7}{9}$
25. $3\frac{9}{125} + 1\frac{5}{9} + 9\frac{3}{8}$

Change to equivalent decimals and subtract:

26. $6\frac{4}{25} - 5\frac{1}{8}$
27. $1\frac{4}{5} - \frac{1}{6}$
28. $4\frac{3}{4} - 4\frac{1}{5}$
29. $9\frac{7}{40} - 8\frac{1}{10}$
30. $2\frac{3}{5} - 1\frac{9}{20}$
31. $4\frac{1}{2} - 1\frac{21}{100}$
32. $14\frac{3}{4} - .83$
33. $.875 - \frac{3}{8}$
34. $21\frac{1}{6} - 1\frac{2}{7}$
35. $21.806 - \frac{5}{6}$
36. $66\frac{2}{3} - 33\frac{1}{3}$
37. $4.28 - \frac{7}{8}$

Multiply:

38. 1.8 by 9
39. .59 by 100
40. .396 by .65
41. 40.21 by 9.23
42. .$324\frac{1}{2}$ by .75
43. .612 by 1000
44. 1.385 by $2.6\frac{1}{4}$
45. .0045 by .936
46. 1200 by .0037

Divide:

47. 3000 by .003
48. .0055 by 1000
49. .0925 by .25
50. 455 by .005
51. 4.96 by 62
52. 741.2 by .034
53. 6.0625 by .0025
54. 1.044 by .006
55. 33.333 by 11

Written Exercise

Review page 78.

187. Change:

1. 6 to tenths
2. 8 to hundredths
3. 7 to thousandths
4. .7 to thousandths
5. .15 to millionths
6. 4.5 to hundredths
7. 9.27 to ten thousandths
8. 32.46 to tenths
9. 5 and 32 hundredths to thousandths

Change to common fractions:

10. 1.4
11. .62$\frac{1}{2}$
12. .045
13. .068
14. .016$\frac{2}{3}$
15. .033$\frac{1}{3}$
16. .555
17. .066$\frac{2}{3}$
18. .07$\frac{1}{2}$
19. .056$\frac{1}{4}$
20. .142
21. .848
22. .750
23. .333$\frac{1}{3}$
24. .087$\frac{1}{2}$

Change the mixed decimals to mixed numbers:

25. 1.25
26. 11.12$\frac{1}{2}$
27. 9.6
28. 8.15
29. 6.816
30. 3.7$\frac{1}{2}$
31. 26.25
32. 1.666$\frac{2}{3}$
33. 2.32
34. 4.057
35. 2.105
36. 5.75
37. 5.0132
38. 7.0875
39. 1.833$\frac{1}{3}$

Change to equivalent decimals:

40. $\frac{1}{4}$
41. $\frac{3}{5}$
42. $\frac{17}{20}$
43. $\frac{3}{40}$
44. $\frac{7}{56}$
45. $\frac{13}{32}$
46. $\frac{5}{9}$
47. $\frac{8}{25}$
48. $\frac{19}{75}$
49. $\frac{7}{16}$
50. $\frac{25}{64}$
51. $\frac{4}{750}$
52. $\frac{15}{250}$
53. $\frac{5}{6}$
54. $\frac{7}{8}$
55. $\frac{2}{3}$
56. $\frac{11}{16}$
57. $\frac{7}{12}$

Change to equivalent mixed decimals, not beyond thousandths:

58. 1$\frac{3}{8}$
59. 5$\frac{2}{7}$
60. 4$\frac{6}{11}$
61. 16$\frac{2}{3}$
62. 8$\frac{1}{3}$
63. 30$\frac{1}{4}$
64. 41$\frac{2}{3}$
65. 7$\frac{3}{4}$
66. 56$\frac{1}{4}$
67. 66$\frac{2}{13}$
68. 8$\frac{9}{11}$
69. 16$\frac{5}{30}$
70. 6$\frac{5}{7}$
71. 7$\frac{7}{9}$
72. 18$\frac{6}{7}$
73. 14$\frac{2}{7}$
74. 13$\frac{7}{9}$
75. 15$\frac{3}{16}$

Written Exercise

188. 1. If .734 of the earth's surface is water, what part is land? How much more than half the surface is water?

2. A train going .5 mi. a minute starts at a given station at noon. Another train follows three hours later and goes .75 mi. a minute. At what time will the second train overtake the first?

3. An architect's pay for planning a house is $314.50. This is .075 of the cost of the house. Give its cost.

4. Find the cost of 43.55 sq. yd. of concrete walk at $1.90 a square yard.

5. How much will be received on a bill of $180 that is worth only $87\frac{1}{2}¢$ on a dollar?

6. By exposure to dampness an article absorbed 1.5 oz. of moisture. If the article then weighed 54 oz., what part of the whole weight was the moisture absorbed?

7. By drying, 54 oz. of seed corn lost 1.5 oz. in weight. What part of the original weight was lost?

8. Of .15 of a ship owned by a man he sells $\frac{1}{3}$ for $800. Find the value of the ship at the same rate.

9. Of $1260 loaned, .07 of the sum is paid for the use the first year, .06 the second year, and .05 the third year. How much in all is paid?

10. If David's wages are $1.75 a day, how many days must he work to pay for a suit of clothes costing $13?

11. If coffee bought at $14¢$ a pound is sold at a profit of $.12\frac{1}{2}¢$ per lb., how many pounds must be sold to make a profit of $3.50?

12. How many pounds are there in a quarter of beef which, selling at $8.33\frac{1}{3}$ per hundred pounds, brings $11.50?

189. The values of commodities are measured by money units. The following are the units of United States money with their relative values:

UNITED STATES MONEY

10 mills (m.)	= 1 cent (ct. or ¢)
10 cents	= 1 dime (d.)
10 dimes, or 100 cents,	= 1 dollar ($)
10 dollars	= 1 eagle (E.)

The gold dollar is the unit from which the other units are derived; it is called the standard unit. Its weight is 25.8 grains (Troy weight), 23.22 gr. of pure gold, and 2.58 gr. of alloy. The gold dollar is no longer coined.

The coins of the United States in common use are:

Gold		*Silver*	
Double eagle	= $20.00	Dollar	= $1.00
Eagle	= $10.00	Half dollar	= 0.50
Half eagle	= $ 5.00	Quarter dollar	= 0.25
Quarter eagle	= $ 2.50	Dime	= 0.10

Nickel 5-cent piece, and bronze 1-cent piece.

Gold coins are hardened by an alloy, 1 tenth copper with silver, the silver not to exceed 1 tenth the alloy. Silver coins are 1 tenth copper. The cent is 95 parts copper, 5 parts tin and zinc. The nickel is 75 parts copper and 25 parts nickel.

All gold coins are legal tender for any sum. Silver coins less than $1 in one payment are legal tender for $10 or less. Nickels and cents in one payment are legal tender for 25¢ or less.

The coins are coined at mints located in Philadelphia, New Orleans, Denver, and San Francisco.

Bank bills and treasury notes are largely used in place of coins. These represent $1, $2, $5, $10, $ 50, $100, $500, and $1000.

Oral and Written Exercise

Review pages 122–124.

190. Express the following aliquot parts of $1 as fractions of $1:

1. $50¢$	4. $10¢$	7. $33\frac{1}{3}¢$	10. $6\frac{1}{4}¢$
2. $25¢$	5. $5¢$	8. $16\frac{2}{3}¢$	11. $3\frac{1}{3}¢$
3. $12\frac{1}{2}¢$	6. $2\frac{1}{2}¢$	9. $8\frac{1}{3}¢$	12. $1\frac{2}{3}¢$

Find the cost of:

13. 360 yd. linen at $50¢$	21. 395 lb. sugar at $5¢$
14. 780 lb. tea at $33\frac{1}{3}¢$	22. 120 quires paper at $8\frac{1}{3}¢$
15. 836 hats at $25¢$	23. 160 grape fruit at $6\frac{1}{4}¢$
16. 192 files at $12\frac{1}{2}¢$	24. 690 lb. lime at $3\frac{1}{3}¢$
17. 960 lb. cheese at $20¢$	25. 550 papers at $2¢$
18. 660 collars at $16\frac{2}{3}¢$	26. 860 badges at $1\frac{2}{3}¢$
19. 200 lemons at $2\frac{1}{2}¢$	27. 2160 eggs at $2\frac{1}{2}¢$
20. 350 yd. cotton at $10¢$	28. 214,000 ft. board at $5¢$

Express the following prices in equivalent fractions of $1:

	a	*b*	*c*	*d*	*e*	*f*
29.	$37\frac{1}{2}¢$	$80¢$	$87\frac{1}{2}¢$	$40¢$	$90¢$	$70¢$
30.	$60¢$	$66\frac{2}{3}¢$	$75¢$	$62\frac{1}{2}¢$	$83\frac{1}{3}¢$	$18\frac{3}{4}¢$

191. Illustrative Example. Find the cost of 680 gal. of oil at $66\frac{2}{3}¢$ a gallon.

WORK

(1) $226\frac{2}{3}$

$\$\frac{680 \times 2}{3} = \$453\frac{1}{3}$

EXPLANATION. In (1) $680 = cost at $1, $226\frac{2}{3}$ = cost at $\frac{1}{3}$, $453\frac{1}{3}$ = cost at $\frac{2}{3}$.

In (2) $340 = cost at 50¢, $113\frac{1}{3}$ = cost at $16\frac{2}{3}¢$, $453\frac{1}{3}$ = cost at $66\frac{2}{3}¢$.

Ans. $453\frac{1}{3}$.

(2) $680 = cost at $1

$\frac{1}{2}$ of $680 = $340, cost at $50¢$

$\frac{1}{6}$ of $680 = $113\frac{1}{3}$, cost at $16\frac{2}{3}¢$

$453\frac{1}{3}$, cost at $66\frac{2}{3}¢$

Oral and Written Exercise

192. Supply the article and find the cost of:

1. 685 gal. @ $66\frac{2}{3}$¢
2. 450 qt. @ $37\frac{1}{2}$¢
3. 2160 ft. @ $26\frac{1}{4}$¢
4. 831 sq. yd. @ $83\frac{1}{3}$¢
5. 386 lb. at $62\frac{1}{2}$¢
6. 3125 bu. @ $87\frac{1}{2}$¢
7. 966 yd. @ 90¢
8. 798 ft. @ 70¢
9. 330 lb. @ 60¢
10. 868 lb. @ $63\frac{1}{8}$¢
11. 936 doz. @ $93\frac{3}{4}$¢
12. $66\frac{2}{3}$ qt. @ 15¢

Written Exercise

193. Illustrative Examples. Find the cost (1) of 3472 lb. of scrap iron at \$1.25 per C (hundred) pounds; (2) of 3472 bricks at $\$12\frac{1}{2}$ per M.

Work		Work
(1) 3472 = 34.72 C	(2)	3472 = 3.472 M
4)\$34.72, at \$1 per C		347.2, at \$100 per M
\$ 8.68, at $\$\frac{1}{4}$ per C		8)\$347.2
\$43.40, at \$1.25 per C		\$ 43.40, at $\$12\frac{1}{2}$ per M

Explanation. 3472 = 34.72 C = 3.472 M. In (1) we add the cost at \$1 and at 25¢. In (2) we find the cost at \$100 per M, then at $\$12\frac{1}{2}$ per M.

Supply the article and find the cost of:

1. 416 boxes @ \$15 per C
2. 939 lb. @ \$13 per C
3. 4500 ft. @ \$35 per M
4. 84,080 ft. @ \$70 per M
5. 382 lb. @ \$1.75 per C
6. 384 ft. @ \$20 per M
7. 84 ft. wire fence at \$2.50 per C
8. 783 mackerel @ \$15.60 per C
9. 31,080 lb. of coal @ \$6 per T.

Work of Example 9. $31.080 \div 2 \times \$6 = \$93.24$.

10. 3148 lb. coal @ \$5.60 per T.
11. 16,973 lb. hay @ \$15 per T.

Oral and Written Exercise

194. Illustrative Examples. (1) At the rate of 8 qt. of peas for \$1, how many quarts will \$4.50 buy? (2) At $87\frac{1}{2}$¢ a bushel, how many bushels will \$175 buy?

WORK

(1) $8 \times 4\frac{1}{2} = 36$

(2) $\frac{\overset{25}{\cancel{175}} \times 8}{\underset{1}{\cancel{7}}} = 200$

SOLUTION. (1) Since \$1 will buy 8 qt., \$4.50 will buy $4\frac{1}{2}$ times 8 qt., or 36 qt. *Ans.* 36 qt.

(2) Since \$$\frac{7}{8}$ buys 1 bu., \$175 will buy as many bushels as there are times \$$\frac{7}{8}$ in \$175, which is 200 times; or, Solution (2), since \$$\frac{7}{8}$ buys 1 bu., \$$\frac{1}{8}$ will buy $\frac{1}{7}$ bu., \$1 will buy $\frac{8}{7}$ of a bushel, and \$175 will buy 175 times $\frac{8}{7}$ bu. which is 200 bu. *Ans.* 200 bu.

Supply the missing article and find how much or how many can be bought for:

1. 50¢ @ 3 lb. for 25¢
2. \$3.25 @ 20 lb. for \$1
3. 65¢ @ 2 lb. for 5¢
4. \$1 @ 8 lb. for 10¢
5. \$5 @ 7 lb. for \$.25
6. \$1.50 @ 22 for 25¢
7. \$6.50 @ 75 lb. for \$1
8. \$3.25 @ 40 qt. for \$1
9. \$22 @ 25¢ per pound
10. \$74.25 @ 50¢ per gallon
11. \$584 @ $16\frac{2}{3}$¢ per yard
12. \$1.50 @ 2 doz. for 25¢
13. \$4.50 @ 18 for \$1
14. \$1.75 @ 6 for \$.25
15. \$36 @ 12 for \$6
16. \$22.50 @ \$5.00 per C
17. \$5.75 @ \$10 per M
18. \$333 @ $33\frac{1}{3}$¢ per foot
19. \$44.80 @ 20¢ per M
20. \$375 @ 75¢ per yard
21. \$6.50 @ \$1.25 per square yard
22. \$75 @ \$1.50 per T.

23 to **29.** How many yards can be bought for \$144 @ $83\frac{1}{3}$¢? @ $66\frac{2}{3}$¢? @ $37\frac{1}{2}$¢? @ $87\frac{1}{2}$¢? @ 60¢? @ 90¢? @ 40¢?

Oral and Written Exercise

195. Supply the commodity and find the cost of:

1. 11 yd. @ 5 ¢
2. 32 ft. @ 6 ¢
3. 43 lb. @ 7 ¢
4. 53 qt. @ 8 ¢
5. 24 gal. @ 9 ¢
6. 42 ft. @ 10 ¢
7. 62 yd. @ 12 ¢
8. 34 ft. @ 11 ¢
9. 12 gal. @ 9 ¢
10. 45 lb. @ 10 ¢
11. 24 qt. @ 6 ¢
12. 26 bu. @ 8 ¢
13. 12 bu. @ 80 ¢
14. 12 yd. @ 10 ¢
15. 45 pk. @ 9 ¢
16. $22\frac{3}{4}$ ft. @ 6 ¢
17. 48 gal. @ $8\frac{1}{3}$ ¢
18. 21 lb. @ 9 ¢
19. $22\frac{1}{2}$ yd. @ 10 ¢
20. $34\frac{1}{4}$ ft. @ 8 ¢
21. $41\frac{1}{2}$ gal. @ 12 ¢
22. $66\frac{1}{6}$ lb. @ 10 ¢
23. $22\frac{1}{8}$ yd. @ 8 ¢
24. $15\frac{1}{4}$ ft. @ 12 ¢
25. $91\frac{1}{2}$ gal. @ 10 ¢
26. $23\frac{3}{4}$ lb. @ 12 ¢
27. $34\frac{1}{4}$ yd. @ 50 ¢
28. $22\frac{3}{4}$ ft. @ 9 ¢
29. $29\frac{3}{4}$ gal. @ 10 ¢
30. $41\frac{1}{2}$ lb. @ 12 ¢
31. $22\frac{1}{2}$ yd. @ 15 ¢
32. $34\frac{1}{3}$ ft. @ 4 ¢
33. $24\frac{1}{2}$ gal. @ 3 ¢
34. $41\frac{3}{4}$ yd. @ 6 ¢
35. $52\frac{3}{4}$ yd. @ 10 ¢
36. $25\frac{1}{2}$ ft. @ 10 ¢
37. $16\frac{1}{2}$ lb. @ 5 ¢
38. 25 yd. @ 10 ¢
39. $42\frac{3}{4}$ ft. @ 12 ¢
40. $21\frac{1}{4}$ gal. @ 16 ¢
41. $90\frac{3}{4}$ lb. @ 10 ¢
42. $22\frac{1}{2}$ yd. @ 50 ¢
43. $28\frac{1}{2}$ ft. @ $33\frac{1}{3}$ ¢
44. $12\frac{1}{4}$ gal. @ 25 ¢
45. 49 pt. @ 20 ¢
46. $16\frac{1}{2}$ qt. @ $16\frac{2}{3}$ ¢
47. 52 yd. @ $12\frac{1}{2}$ ¢
48. $28\frac{1}{2}$ ft. @ 50 ¢
49. $19\frac{3}{4}$ qt. @ 50 ¢
50. $16\frac{1}{2}$ yd. @ 18 ¢
51. 32 ft. @ $12\frac{1}{2}$ ¢
52. 24 gal. @ $16\frac{2}{3}$ ¢
53. 45 oz. @ $33\frac{1}{3}$ ¢
54. 12 pairs @ 50 ¢
55. $14\frac{1}{2}$ lb. @ 8 ¢
56. 112 lb. @ 9 ¢
57. 121 lb. @ 12 ¢
58. $16\frac{1}{2}$ lb. @ 4 ¢
59. $22\frac{3}{4}$ lb. @ 10 ¢
60. 25 lb. @ 10 ¢
61. $83\frac{1}{3}$ yd. @ 9 ¢
62. $71\frac{1}{4}$ yd. @ 8 ¢
63. $56\frac{3}{4}$ yd. @ 12 ¢
64. $24\frac{1}{4}$ yd. @ 8 ¢
65. 53 yd. @ 16 ¢
66. 282 yd. @ 10 ¢
67. 512 gal. @ 50 ¢
68. 90 gal. @ 25 ¢
69. 666 gal. @ $33\frac{1}{3}$ ¢
70. 64 gal. @ 25 ¢
71. 58 gal. @ 20 ¢
72. 22 gal. @ $16\frac{2}{3}$ ¢

196. Review pages 33–35.

306 Broadway, New York, Aug. 1, 1908.

Mr. W. C. Castner,
143 W. 43 St., New York.

To O. H. Bromley, Dr.,
Sanitary Plumber.

1908						
Apr.	24	1 1 in. Gate Valve		2	30	
		1 1 in. Galvanized Keystone Union			75	
		2 1 in. × 5 in. Galvanized Nipples	@ $.15		30	
		1 1 in. × 9 in. Galvanized Nipples			23	
		1 ¾ in. × 9 in. Galvanized Nipples, right and left			21	
		1 ¾ in. × 9 in. Galvanized Coupling			13	
		1 Union Gasket, 2 in.			10	
		Faucet Packing			05	
		3½ hours Plumber and Helper	@ .85	2	98	
						7 05

Received payment,

Aug. 15, 1908. O. H. Bromley.

Above items are taken from the account of O. H. Bromley, creditor, against Mr. W. C. Castner, debtor, for plumbers' findings and work.

An itemized statement of indebtedness given by the creditor to the debtor is called a **bill.**

A bill as the above, the payment of which is acknowledged by the signature of the creditor or his agent, is called a **receipted bill.**

A bill should contain the name and address of both debtor and creditor, the date when the articles were sold, the name, quantity, and price of articles sold, and the total amount, called the **footing,** of the bill.

An itemized statement with date of goods sold at wholesale which the seller gives to the buyer at the time of sale or delivery, is called an **invoice.**

Written Exercise

197. Carry out, find the totals, and receipt the following:

1.

NEWPORT, N.H., Aug. 1, 1908.

MR. J. C. PURDY,

Bought of S. B. CLARK & SON,

Dealers in GROCERIES, ICE, and WOOD.

1909				
July	24	1 gal. Kerosene, 14¢, ½ lb. Tea, 30¢		44
		3 lb. Lard	@ $.12	36
		1 lb. Raisins, 14¢, 1 lb. B. Chocolate, 40¢		54
	25	1 doz. Bananas, 30¢, 1 lb. Coffee, 35¢		
	26	½ lb. Baking Powder . . .		25
		1 lb. Shredded Codfish . . .		13
		2½ lb. Confectioners' Sugar . .	@ .10	
		2 doz. Eggs	@ .23	
	27	5 lb. Sugar	@ .06	
		½ pk. Potatoes, 20¢, ½ pk. Peas, 20¢		
		1½ lb. Butter	@ .28	
		1 qt. Milk, 6¢, 1 loaf Bread, 10¢ .		
	29	4½ lb. Sugar	@ .06	
		1 doz. Bananas		30

2.

PORTLAND, ME., June 15, 1908.

MESSRS. BATES, FORBES & CO.,
Portland, Me.

Bought of COBB & FARNHAM,
WHOLESALE GROCERS.

30 lb. Coffee	@ $.28	
46 lb. Formosa Tea . . .	@ .75	
5 lb. Cocoa	@ .52	
227 lb. Granulated Sugar . .	@ .06	
12 cans Tomatoes	@ .13	
10 cans Peas	@ .14	

3. Copy and carry out the following invoice of hay and grain:

PROVIDENCE, R.I., July 14, 1909.

FRED B. SPENCER & CO.,

Sold to MR. A. B. SMALL,

Interest at 5% after 30 days. Westerley, R.I.

14,200 lb. T. Hay	@	$12.00	per T.
3640 lb. C. Hay	@	11.50	per T.
280 bu. Wheat	@	.95	a bu.
85 bu. Oats	@	.41	a bu.
72 bu. Corn Meal	@	.82	a bu.
70 bu. Rye Meal	@	.65	a bu.
150 bbl. Flour	@	4.50	a bbl.
180 bbl. Flour	@	4.30	a bbl.
100 bu. Rye Meal	@	.60	a bu.
100 bu. Corn Meal	@	.80	a bu.

4.

BANGOR, ME., Feb. 16, 1908.

MR. A. P. WILLIAMS,
Bangor, Me.

Bought of THE LAWRENCE NEWHALL COMPANY.

Terms: 30 days or cash less 2 per cent.

1125 ft. Spruce	@	$23.50	
1250 ft. C. Pine	@	40.00	
1890 ft. Maple	@	47.00	
1316 ft. Hemlock	@	42.80	
390 ft. Planed Hemlock	@	24.00	
1600 ft. Pine Boards	@	43.50	
1510 ft. W. Oak	@	78.50	
1700 ft. Hemlock	@	43.00	
1500 ft. Maple	@	48.00	

5. Find the balance of this monthly statement of account:

Mr. L. D. Gorman,
119 Arch St., Bridgeport, Conn.

A Statement of your Account with T. Allen & Son.

1909				
May	1	As per bill rendered	$11	50
	4	As per invoice	15	00
	10	As per invoice	7	50
	15	As per invoice	7	00
		Credits.		
May	2	Cash	10	00
	15	Cash	10	00
		Balance due		

Make out in proper form the following bills, invoices, etc., completing the extensions and finding the totals:

6. On May 1, 1909, Mr. T. V. Partridge, Portsmouth, N.H., bought of Robinson & Hall, Wholesale Grocers, Portland, Me., the following invoice of goods:

125 bbl. flour @ $4.75 per barrel.
65 bbl. salt @ $1.50 per barrel.
3144 lb. sugar @ $3\frac{1}{4}$¢ per pound.
126 lb. tea @ $28\frac{3}{4}$¢ per pound.

7. Davis S. Farr, Canton, Ohio, bought of Foster, Blake & Co., Wholesale dealers in Dry Goods at Cleveland, Ohio, on account, goods as follows:

Sept. 12, 1908. 1680 yd. denim @ $12\frac{1}{2}$¢ a yard.
Sept. 15, 1908. 480 yd. serge @ $37\frac{1}{2}$¢ a yard.
Sept. 20, 1908. 400 yd. cambric @ $6\frac{1}{4}$¢ a yard.
Sept. 26, 1908. 420 yd. delaine @ $16\frac{2}{3}$ a yard.

8. An invoice of dry goods was sold on 30 days' credit by H. A. Stewart & Co., Philadelphia, to John C. Steiger, Springfield, Mass., Aug. 15, 1908, as follows:

6 pieces delaine, 31, 34, 38, 32, 35, 36 yd. @ 16⅔ ¢ a yard.

6 pieces calico, 38, 35, 33¼, 36, 39½, 34 yd. @ 6¼ ¢ a yard.

20 pieces serge, 33¼, 34½, 35¼, 36, 39¼, 32¼, 38, 35, 33⅓, 36½, 34, 35, 38, 32¼, 36⅓, 39, 32, 35, 34, 31 yd. @ 37½ ¢ a yard.

12 pieces denim, 113, 112, 140, 118, 111, 109, 121, 116, 113, 108, 122, 125 yd. @ 12½ ¢ a yard.

9. On Sept. 16, 1908, B. Fox & Co. Brockton, Mass., sold to Bayard Morse & Co., St. Louis, Mo., the following invoice of shoes on 64 days' credit:

24 cases 24 pairs each, No. 6586, Men's Buff Congress, Globe tip, machine sewed, size 6, at $1.50.

10 cases 24 pairs each, No. 6588 Mens' Buff II's English Bals, Globe tip, M. S., size 6/11, @ $1.60.

20 cases 24 pairs each, No. 3230, Boys' Buff English Bals, seamless tip, creased vamp, sizes 1/5, @ $1.10.

14 cases 24 pairs each, No. 3140, Boys' Buff Congress, seamless, Globe tip, creased vamps, sizes 2/5, @ $1.40.

10. S. Knowlton bought of the York Supply Co.

10 Acme Gas Ranges, No. 22G, @ $ 12.66.

5 Essex Gas Ranges, No. 185F, @ $8.84.

6 Gas Radiators, No. 642G, @ $3.84.

4 Hot Plates, No. 6559, @ $1.65.

11. A. T. Wood, Boston, Mass., Wholesale Hardware Dealer, sold to John Burt, Westfield, Mass., goods as follows:

Feb. 2, $147.30; Feb. 6, $24.18; Feb. 9, $235.16; Feb. 12, $225.19; Feb. 17, $12.56; Feb. 22, $167.41; Feb. 26, $148.14.

Feb. 1, 1908, there was a balance against Mr. Burt of $218.17. He made the following cash payments: Feb. 3, $460; Feb. 13, $232; Feb. 24, $200. Make out a statement to March 1, 1908.

Written Exercise

198. 1. Express $\frac{15}{16}$ of a dollar in cents and mills.

2. Express \$.0875 + \$.625 + \$.0375 as the fraction of a dollar.

3. Subtract 1 ten-thousandth from 1 tenth.

4. Divide .1 by .0001, and .0001 by .1, and add the quotients.

5. By giving a bank his written promise to pay to the bank \$300 in 2 months, Mr. A obtained .99 of \$300 for his immediate use. What sum did he obtain?

6. A bushel, even measure, contains 2150.42 cu. in. If this is .782 of a heaped bushel, how many cubic inches are there in a heaped bushel?

7. A man sold a house for \$7248.80. If this was its original cost, and .04 more, what was its original cost?

8. At 65¢ a cubic yard, what is paid for excavating 482.75 cu. yd. of earth?

9. 72.375 A. of land cost \$3500. At the same rate, what is the cost of 1000 A.?

10. How long will it take a train running at the rate of 59.65 mi. per hour to cover 500 mi.?

11. Make a statement of account of Timothy Poole, debtor, to E. J. Wish & Co., for 1 automobile bought May 10, 1908, at \$3000, with credits of cash \$1500 paid June 1, cash \$850 paid July 1, 1908. Make the statement in due form to be submitted to the debtor July 1, 1908.

12. The Grafton Coal Company bought of the Pennsylvania Coal and Coke Co., Philadelphia, on Oct. 12, 1908, the following invoice of coal: 178 T. Globe @ \$5.05, 1100 T. Chestnut @ \$4.75, 400 T. Franklin @ \$7.85.

Measures of Weight

199. AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

200. TROY WEIGHT

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce
12 ounces = 1 pound

The standard unit of weight is the **Troy pound.**

1 lb. Troy = 5760 gr. 1 lb. Avoirdupois = 7000 gr.
175 lb. Troy = 144 lb. Av. 175 oz. Troy = 192 oz. Av.

Avoirdupois weight is used in weighing groceries and other coarser kinds of goods.

Troy weight is used in weighing silver, gold, precious stones, and other articles of great value.

The long ton of 2240 pounds Avoirdupois is used in weighing heavy articles as iron and coal at the mines, and goods on which duties are paid at the United States customhouses.

A cubic foot of water weighs 1000 oz. or 62½ lb. Avoirdupois.

The following units are in common use:

14 pounds = 1 stone
100 lb. of grain or flour = 1 cental
100 lb. dried fish = 1 quintal
200 lb. beef or pork = 1 barrel
196 lb. flour = 1 barrel
56 lb. butter = 1 firkin
280 lb. salt = 1 barrel
100 lb. nails = 1 keg

Apothecaries' Weight

201. DRY MEASURES

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb. or ℔)

202. LIQUID MEASURES

60 drops (gtt.) = 1 fluid dram (fʒ)
8 fluid drams = 1 fluid ounce (f℥)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gallon (cong.)

The grain, ounce, and pound are the same as in Troy weight.

Measures of Extension

203. LONG MEASURES

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards ($16\frac{1}{2}$ ft.)	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
5280 feet	= 1 mile

204. SQUARE MEASURES

144 (12^2) square in. (sq. in.)	= 1 square foot (sq. ft.)
9 (3^2) square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards ($272\frac{1}{4}$ sq. ft.)	= 1 sq. rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile

The standard unit of length is the *yard*, which is identical with the imperial yard of Great Britain.

205. SURVEYORS' MEASURES

7.92 inches	= 1 link
100 links	= 1 chain
80 chains	= 1 mile

The Gunter's chain is 4 rods. (66 ft. long)

206. MARINERS' MEASURES

6 feet	= 1 fathom (in depth)
120 fathoms	= 1 cable length
$7\frac{1}{8}$ cable lengths	= 1 common mile

A nautical mile (knot) is about 1.15 mile.

207. SURVEYORS' SQUARE MEASURES

10 square chains	= 1 acre	1 square mile	= 1 section
640 acres	= 1 square mile	36 sections	= 1 township

208. CUBIC MEASURES

1728 (12^3) cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 (3^3) cubic feet	= 1 cubic yard (cu. yd.)
$24\frac{3}{4}$ cubic feet	= 1 perch of masonry

209. WOOD MEASURES

16 cubic feet	= 1 cord foot (cd. ft.)
8 cord feet	= 1 cord (cd.)
128 cubic feet	= 1 cord

A cord of wood is usually a pile 8 ft. long, 4 ft. wide, 4 ft. high. A cord foot is a pile 1 ft. long, 4 ft. wide, 4 ft. high.

The following units are also used: 1 hand = 4 in., used in measuring the height of horses. 1 span = about 9 in., from the end of the thumb to the end of the little finger with hand extended.

210. TIME MEASURES

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
4 weeks	= 1 lunar month
52 weeks, 1 day, or 365 days	= 1 common year (c. yr.)
366 days	= 1 leap year (l. yr.)
100 years	= 1 century (cent.)
10 years	= 1 decade

In most business transactions 30 days are considered a month, and in some, 360 days are considered a year.

1st. January (Jan.) has 31 days
2d. February (Feb.) has 28 or 29 days
3d. March (Mar.) has 31 days
4th. April (Apr.) has 30 days
5th. May has 31 days
6th. June has 30 days
7th. July has 31 days
8th. August (Aug.) has 31 days
9th. September (Sept.) has 30 days
10th. October (Oct.) has 31 days
11th. November (Nov.) has 30 days
12th. December (Dec.) has 31 days

Thirty days have September,
April, June, and November:
All the rest have thirty-one,
Except February alone,
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

The time from any date until the corresponding date in the next month is called a **calendar month.**

The earth revolves around the sun in 365 da., 5 hr., 48 min., 46 sec. nearly, which is not quite $365\frac{1}{4}$ da. This is called the **solar year.**

The common year, 365 da., disregards the fraction, nearly $\frac{1}{4}$ day; hence, every fourth year February takes an extra day, making a year of 366 da., called **leap year.** But as the fraction is less than $\frac{1}{4}$, not quite a whole day is gained every fourth year. To rectify this, centennial years, except those divisible by 400, are considered common years.

Measures of Capacity

211. LIQUID MEASURES

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
63 gallons = 1 hogshead (hhd.)

212. DRY MEASURES

4 gills = 1 pint
2 pints = 1 quart
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

The standard unit of liquid measure is the gallon, which contains 231 cubic inches. 1 cu. ft. = $7\frac{1}{2}$ gal., approximately.

The barrel when used for measuring is $\frac{1}{2}$ hogshead or $31\frac{1}{2}$ gal. A pint of water weighs about 1 lb. Avoirdupois.

The standard unit of dry measure is the bushel, which contains 2150.42 cu. in., and equals about $1\frac{1}{4}$ cu. ft.

One bushel, which is the Winchester bushel, differs from the English Imperial bushel, which contains 2218.192 cu. in.

Miscellaneous Measures

213. ANGLES AND CIRCLES

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference (circ.)
90 degrees = 1 right angle

1 quarter of the circumference of a circle, 90°, is a **quadrant.**

214. PAPER

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

Paper is also commonly sold by the 100, 500, or 1000 sheets; also by the pound.

215. BOOKS

A book formed of sheets folded in 2 leaves is a folio.
4 leaves is a quarto.
8 leaves is an octavo.
12 leaves is a duodecimo or 12mo.
16 leaves is a 16mo.

216. COUNTING

12 ones = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (g. gro.)
20 ones = 1 score
2 ones are a pair
2 horses are a span
2 oxen are a yoke

Foreign Measures of Value

217. English Money

4 farthings (far.) = 1 penny (*d.*) = $.02 in United States money.
12 pence = 1 shilling (*s.*) = $.24 in United States money.
20 shillings = 1 pound (£) = $4.8665 in United States money.
The standard unit of value is the pound ($4.8665).

The coins of English money in general use are:

Gold, sovereign (£), ½ sovereign (10*s.*); *Silver*, ½ crown (2½*s.*) florin (2*s.*), shilling, 6-penny and 3-penny pieces; *Copper*, penny and ½-penny pieces.

The farthing is usually written as the fraction of a penny.

218. French Money

10 millimes (m.) = 1 centime (c.)
10 centimes = 1 decime (d.)
10 decimes = 1 franc (fr.)
The franc = $.193 United States money.
The standard unit of French money is the *franc*.

219. Canadian Money

10 mills (m.) = 1 cent (¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
Values and coins are as in United States.

The coins of French money in general use are:

Gold, 20-franc (called Napoleon), 10-franc, and 5-franc pieces; *Silver*, 5-franc, 2-franc, 1-franc, ½-franc pieces; *Nickel*, 25-centime piece; *Copper*, 10-centime and 5-centime pieces.

220. German Money

100 pfennigs (pf.) = 1 mark (M.) = $.2385 United States money.
3 marks = 1 Thaler = $.746 United States money.
The standard unit of value is the *mark*.

The coins of German money in general use are:

Gold, 20-mark and 10-mark pieces; *Silver*, 5-mark piece, 3-mark piece (Thaler), 2-mark, 1-mark, and 5-pfennig pieces; *Nickel*, 10-pfennig and 5-pfennig pieces; *Copper*, 2-pfennig and 1-pfennig pieces.

Oral Exercise

221. Illustrative Example. Change 30 mo. to years and months.

SOLUTION. 12 mo. = 1 yr.; 30 mo. equal 2 times 12 mo., and 6 mo. remain. Hence, 30 mo. = 2 yr. 6 mo. *Ans.* 2 yr. 6 mo.

222. A number, as 2 yr. 6 mo., consisting of two or more denominate numbers of different names, is called a **compound denominate number.**

The process of changing the form or denomination of a number without changing its value is called **reduction.**

Change to compound denominate numbers:

1. 66 mo.	**4.** 40 in.	**7.** 18 cd. ft.	**10.** 27 pk.
2. 22 ft.	**5.** 30 sq. ft.	**8.** 14 pt.	**11.** 63 qt.
3. 15 pt.	**6.** 17 gr.	**9.** 30 qt.	**12.** 37 ft.

223. Illustrative Example. Reduce 4 yd. 2 ft. 6 in. to inches.

WORK

4 yd. 2 ft. 6 in.
3
14 ft.
12
174 in.

EXPLANATION. Since 1 yd. = 3 ft., 4 yd. = 4 times 3 ft. = 12 ft. Adding the 2 ft., we have 14 ft. Since 1 ft. = 12 in., 14 ft. = 14 times 12 in. = 168 in. Adding the 6 in., we have 174 in.

Ans. 174 in.

In the above example the 4 yd. 2 ft. are changed to numbers of a lower denomination. The process is called **reduction descending.**

To change a compound number to a compound number of a lower denomination, we multiply the number of the highest denomination by the number which is required of the next lower denomination to make one of the higher, and to the product add the given number of the lower denomination. We multiply this sum in like manner, and so continue till the required denomination is reached.

Written Exercise

224. Reduce:

1. 6 gal. 3 qt. 1 pt. to pt.
2. 12 yd. 2 ft. 7 in. to in.
3. 5 sq. yd. 8 sq. in. to sq. in.
4. 23 lb. 7 oz. Avoir. to oz.
5. 26 da. 12 hr. to hr.
6. 25 cu. yd. 20 cu. ft. to cu. ft.
7. 6 gal. 0 qt. 1 pt. to pt.
8. 5 bu. 2 pk. 3 qt. to qt.
9. 41 hr. 30 min. to sec.
10. 16 qt. 1 pt. to gi.
11. 18 lb. Troy to gr.
12. 5 mi. 3 rd. 4 yd. to yd.
13. 2 T. 16 cwt. 40 lb. to lb.
14. 30 bbl. 100 lb. to lb. (salt).
15. 4 mi. 30 ch. to ch.
16. 120 fathoms to ft.
17. 180° to sec.
18. 10 da. 50 min. to sec.
19. 6 mo. 20 da. to da.
20. \$25, 87¢, 5 mills to mills.

Find the cost of:

21. 10 gal. 3 qt. of milk at 7¢ a quart.
22. 7 qt. 1 pt. of cream at 20¢ a pint.
23. 1 lb. 7 oz. oil of spearmint, Apoth. weight, at 25¢ an ounce.
24. 8 cd. 4 cd. ft. of wood at $66\frac{2}{3}$¢ a cord foot.
25. 27 days' work, 10 hours each, at $33\frac{1}{3}$¢ an hour.
26. $\frac{1}{2}$ T. of ice at $\frac{1}{2}$¢ a pound.
27. 5 bu. 3 pk. 7 qt. cranberries at 13¢ a quart.

Written Exercise

225. Illustrative Example. $\frac{1}{8}$ lb. of oil of cloves in 60 prescriptions gives how many grains to each?

WORK

$$\frac{1 \times 12 \times \cancel{8} \times \cancel{3} \times \cancel{20}}{\cancel{8} \times \cancel{60}_{\cancel{20}}} = 12$$

EXPLANATION. $\frac{1}{8}$ lb. is changed to Apoth. grains which are divided by 60.

Ans. 12 gr.

1. A balloon going $8\frac{3}{4}$ mi. an hour goes how many feet a minute?

Change:

2. $\frac{3}{4}$ lb. to ounces.
3. $\frac{7}{8}$ gal. to pints.
4. $\frac{9}{10}$ A. to square feet.
5. $\frac{11}{12}$ mi. to feet.
6. $\frac{7}{9}$ da. to hours.
7. £ $\frac{3}{8}$ to pence.
8. $\frac{8}{5}$ rd. to feet.
9. $1\frac{3}{11}$ sq. rd. to square feet.

Written Exercise

226. Illustrative Example. Change 424 ft. to yards, etc.

WORK

$3\overline{)424}$ —1 ft.

$5\frac{1}{2}\overline{)141}$, number of yd.

$11\overline{)282}$ —$\frac{1}{2}$ yd. = $3\frac{1}{2}$ yd.

25, number of rd.

EXPLANATION. There are 3 ft. in 1 yd.; 424 ft. divided by 3 ft. gives 141 with 1 ft. remaining. There are $5\frac{1}{2}$ yards in 1 rd.; 141 divided by $5\frac{1}{2}$ gives 25 with $3\frac{1}{2}$ yd. remaining. $\frac{1}{2}$ yd. with 1 ft. remaining = 2 ft. 6 in.

Ans. 25 rd. 3 yd. 2 ft. 6 in.

The process of changing a number to a number of higher denominations is called **reduction ascending.**

To change a given number to a number of higher denomination, we divide the given number by the number it takes of its denomination to make one of the next higher. We divide the quotient in the same way, and so proceed till the required denomination is reached. The last quotient with the remainders, if there are any, will be the result required.

Reduce to higher denominations:

1. 1648 in.
2. 1724 pt. (liquid).
3. 4271¢.
4. 2122 cu. ft.
5. 1426 gi.
6. 4486 oz. (Av.).
7. 4044 sq. in.
8. 4163 ft.
9. 60,000 sec.
10. 33,140 sq. ft.
11. 416,984 pwt.
12. 576 cd. ft.
13. 1573 cu. ft.
14. 3750 in.
15. 70,000 mills.
16. 3005 ch.
17. 5250 yd.
18. 963*d*.

Written Exercise

227. Illustrative Example. 10 gal. 2 qt. is what part of 1 hhd.?

WORK

10 gal. 2 qt. = 42 qt.
1 hhd. = 63 × 4 qt. = 252 qt.
42 qt. = $\frac{42}{252}$ hhd. = $\frac{1}{6}$ hhd.

EXPLANATION. 10 gal. 2 qt. changed to quarts = (10 × 4 qt.) + 2 qt. = 42 qt. 1 hhd. changed to quarts = 63 × 4 qt. = 252 qt. 42 qt. = $\frac{42}{252}$ hhd. = $\frac{1}{6}$ hhd. *Ans.* $\frac{1}{6}$ hhd.

What part of

1. 1 yd. is 2 ft. 3 in.?
2. 1 rd. is 4 yd. 2 ft. 1 in.?
3. 1 yd. is $2\frac{1}{4}$ in.?
4. 1 ft. is $\frac{3}{4}$ in.?
5. 1 T. is 18 cwt. 60 lb.?
6. 1 lb. Avoir. is $\frac{4}{5}$ oz.?
7. 1 cu. yd. is $\frac{5}{6}$ cu. ft.?
8. A circ. (360°) is 75°18′?

Written Exercise

228. Illustrative Example. Change .575 mi. to rods.

WORK AND EXPLANATION: .575 mi. = .575 of 320 rd. = 184 rd.

Change to compound numbers of lower denominations:

1. .235 mi.
2. .487 yd.
3. .83 bu.
4. .555 gal.
5. .15 rd.
6. .784 hr.
7. .5635 ft.
8. .937 T.

Written Exercise

229. Illustrative Example. Change 5 mo. 7 da. to the decimal of a year.

WORK

30)7.00, no. of da.
12)5.2333 ···, no. of mo.
.4361 ···, no. of yr.

EXPLANATION. 7 da. ÷ 30 (da.) = .2333, number of months; .2333 mo. with 5 mo. = 5.2333 mo. 5.2333 mo. ÷ 12 (mo.) = .4361, number of years. *Ans.* .4361+ yr.

Reduce to decimals of higher denominations:

1. 3 mo. 6 da.
2. 15 da. 6 hr.
3. 15 cu. ft.
4. 7 oz. to lb. Avoir.
5. 12 cwt. 65 lb.
6. 30 da. 12 hr.

Written Exercise

230. Illustrative Examples. (1) Add 2 mi. 172 rd. 6 ft.; 1 mi. 310 rd. 14 ft.; and 95 rd. 11 ft. (2) From 7 rd. 3 yd. 1 ft., take 1 rd. 4 yd. 2 ft.

WORK

ADDITION

(1)

2 mi.	172 rd.	6 ft.	
1 mi.	310 rd.	14 ft.	
	95 rd.	11 ft.	
4 mi.	258 rd.	$14\frac{1}{2}$ ft.	=
4 mi.	258 rd.	14 ft.	6 in.

SUBTRACTION

(2)

7 rd.	3 yd.	1 ft.		
1 rd.	4 yd.	2 ft.		
5 rd.	$3\frac{1}{2}$ yd.	2 ft.		
	$\frac{1}{2}$ yd.	= 1 ft.	6 in.	
5 rd.	4 yd.	0 ft.	6 in.	

EXPLANATION. We write the numbers, with units of the same denomination in the same column, and begin to add or subtract with the numbers of the lowest denomination.

(1) $11 + 14 + 6$ (ft.) $= 31$ (ft.) $= 1$ rd. $14\frac{1}{2}$ ft. We write the $14\frac{1}{2}$ ft. and add the 1 rd. with the rods. The sum is 578 rd. $= 1$ mi. 258 rd. We write the 258 rd., and add the 1 mi. with the miles.

Ans. 4 mi. 258 rd. 14 ft. 6 in.

(2) To subtract, as there is but 1 ft. in the minuend, we think of 1 yd. as changed to 3 ft., making, with the 1 ft., 4 ft. 4 ft. $-$ 2 ft. $=$ 2 ft. We write the 2 ft. We think next of 1 of the 7 rd. as changed to $5\frac{1}{2}$ yd., making, with the remaining 2 yd., $7\frac{1}{2}$ yd. $7\frac{1}{2}$ yd. $-$ 4 yd. $= 3\frac{1}{2}$ yd. We write $3\frac{1}{2}$ yd. The remaining 6 rd. $-$ 1 rd. $=$ 5 rd. But $\frac{1}{2}$ yd. $=$ 1 ft. 6 in. which added to 2 ft. equals 1 yd. 6 in.

Ans. 5 rd. 4 yd. 0 ft. 6 in.

1. Find the sum of 3 mi. 1 rd. 1 yd. 1 ft.; 5 mi. 1 rd. 2 yd. 1 ft.; 14 mi. 4 rd. 6 yd. 2 ft.; 4 mi. 5 rd. 2 yd. 2 ft.

2. Add 3 bu. 3 pk. 7 qt. 1 pt.; 99 bu. 6 qt.; 60 bu. 5 qt. 1 pt.; 2 pk. 5 qt. 1 pt.

3. Find the sum of 7 T. 15 cwt. 50 lb. 4 oz.; 5 T. 8 cwt. 45 lb. 12 oz.; 3 T. 4 cwt. 20 lb. 4 oz.; 9 T. 10 cwt. 10 lb. 10 oz.; and 18 T. 19 cwt. 90 lb. 15 oz.

Written Exercise

231. 1. Find the total contents of 5 bins of grain containing respectively, 6 bu. 3 pk. 2 qt. 1 pt.; 9 bu. 2 qt. 1 pt.; 50 bu. 2 pk.; 48 bu. 3 pk. 2 qt. 1 pt.; 156 bu. 1 pk.

2. I had 3 loads of coal delivered weighing respectively 18 cwt. 8 lb.; 1 T. 5 cwt. 6 lb.; 1 T. 12 cwt. 10 lb. What was the total weight?

3. Express the above loads in tons and pounds and find the cost at $6.75 a ton.

4. In 6 oil barrels the following quantities of oil remain: 15 gal. 2 qt. 1 pt.; 26 gal. 3 qt. 1 pt.; 19 gal. 3 qt.; 25 gal. 2 qt.; 2 gal. 3 qt.; 3 qt. 1 pt. Find the total amount, also its value at 10¢ a gallon.

5. From 20 mi. 95 rd. take 4 mi. 5 rd. 5 yd.

6. From 17 yd. 2 ft. take 15 yd. 2 ft. 6 in.

7. From 23 A. 70 sq. rd. take 16 A. 95 sq. rd.

8. A walk containing 180 sq. yd. $8\frac{1}{2}$ sq. ft. is to be paved with brick. After 134 sq. yd. 6 sq. ft. of the paving have been completed, how much remains to be paved?

9. The distance around a 4-sided field is 880 yd. The distance along 2 sides is 88 rd. What is the distance in rods along the other 2 sides?

10. From a barrel of petroleum which contained $44\frac{1}{2}$ gal., 37 gal. 3 qt. 1 pt. have been drawn out. How much still remains in the barrel?

11. A woman worked at her home for 1 hr. 50 min., and at her neighbor's for 9 hr. 40 min. How much of a day of 12 hours remained?

12. Find the total weight of 1 T. 12 cwt. 40 lb. 13 oz.; 71 T. 56 lb. 11 oz.; 16 cwt. 14 oz.; 94 T. 11 cwt. 66 lb. 10 oz.

Written Exercise

232. Illustrative Example. What is the time in years, months, and days from May 11, 1897 to Aug. 7, 1908?

WORK

1908	8	7
1897	5	11
11	2	26

EXPLANATION. August being the 8th and May the 5th month in the year, we use 8 and 5 for the months, and making the later date the minuend, subtract the earlier date from it. 30 days are considered a month, 360 days, 1 year. *Ans.* 11 yr. 2 mo. 26 da.

Find the time in years, months, and days between:

1. Jan. 1, 1883 and June 23, 1892.
2. Apr. 25, 1897 and Sept. 8, 1903.
3. Jun. 12, 1902 and Nov. 17, 1907.
4. Jan. 4, 1901 and April 30, 1905.
5. Feb. 24, 1898 and Aug. 31, 1908.
6. Oct. 26, 1906 and Aug. 20, 1907.

Written Exercise

233. Illustrative Example. Find the exact number of days from April 13, 1907 to Sept. 26, 1907.

WORK

17	April
31	May
30	June
31	July
31	Aug.
26	Sept.
166	

EXPLANATION. There are 17 more days in April, 31 days in May, 30 in June, 31 in July, 31 in August, and 26 in September, making a total of 166 days. The exact number of days can also be obtained thus: between April 13 and Sept. 26 there are 5 months 13 days; as 3 of the months have 31 days, the whole number of days will be 16 da. $+5 \times 30$ da. $= 166$ da. *Ans.* 166 days.

Find the exact number of days from:

1. Apr. 7, 1906 to Dec. 3, 1906.
2. Feb. 8, 1905 to July 27, 1905.
3. Aug. 20, 1906 to Jan. 20, 1907.
4. Sept. 16, 1904 to Feb. 22, 1905.
5. May 30, 1906 to July 4, 1906.

Written Exercise

234. Illustrative Example. Multiply 5 gal. 3 qt. 1 pt. by 9.

	Work	
5 gal.	3 qt.	1 pt.
		9
52 gal.	3 qt.	1 pt.

Explanation. 9 times 1 pt. = 9 pt. = 4 qt. 1 pt. We write the 1 pt. and reserve the 4 qt. 9 times 3 qt. = 27 qt., with the 4 qt. reserved = 31 qt., = 7 gal. 3 qt. We write the 3 qt., and reserve the 7 gal. 9 times 5 gal. = 45 gal., with the 7 gal. reserved = 52 gal. We write 52 gal. *Ans.* 52 gal. 3 qt. 1 pt.

Multiply:

1. 15 gal. 2 qt. 1 pt. by 5
2. 26 bu. 3 pk. 1 qt. by 14
3. 29 mi. 95 rd. by 11
4. £5 4*s.* 6*d.* by 9
5. 23 A. 7 sq. rd. by 3
6. 2 cwt. 80 lb. 4 oz. by 6
7. 19 da. 12 hr. 4 min. by 13
8. 14° 40′ 52″ by 8.

Written Exercise

235. Illustrative Example. Divide 126 bu. 3 pk. 4 qt. into 5 equal parts.

	Work		
5)126 bu.	3 pk.	4 qt.	
25	1	4	

Explanation. ⅕ of 126 bu. = 25 bu., with 1 bu. undivided. We write 25 bu. 1 bu. = 4 pk. 4 pk. + 3 pk. = 7 pk. ⅕ of 7 pk. = 1 pk., with 2 pk. undivided. 2 pk. = 16 qt. 16 qt. + 4 qt. = 20 qt. ⅕ of 20 qt. = 4 qt. *Ans.* 25 bu. 1 pk. 4 qt.

Divide:

1. 23 wk. 6 da. 14 hr. by 6
2. 44 mi. 37 rd. 4 yd. by 8
3. 50 bu. 2 pk. by 18
4. 21 T. 12 cwt. 80 lb. by 50
5. 39 gal. 3 qt. by 4
6. 4 A. 70 sq. rd. by 16

7. An automobile runs 456 mi. 18 rd. in 6 da. What is the average per day?

8. A board 12 ft. 9 in. long is cut into 9 equal lengths. Find the length of each.

9. Divide 18 A. 66 sq. rd. into 60 equal house lots. How much land has each?

Written Exercise

236. Illustrative Example. Divide 180 lb. Av. by 2 lb. 4 oz.

Work	Explanation.
180×16 oz. $= 2880$ oz.	180 lb. changed to ounces = 2880 oz. 2 lb. 4 oz. changed to ounces = 36 oz. The quotient of $2880 \div 36 = 80$. *Ans.* 80.
2 lb. 4 oz. $=$ 36 oz.	
$2880 \div 36 = 80$	

Change to the lowest given denomination and divide:

1. 28 A. 20 sq. rd. by 1 A. 14 sq. rd.

2. 56 sq. yd. 2 sq. ft. 36 sq. in. by 19 sq. ft.

3. 14 mi. 116 rd. by 13 yd. 2 ft.

4. 60 lb. by 1 lb. 14 oz.; by 3 lb. 12 oz.

5. How many pellets of 5 gr. each can be made from 2 lb. of soda?

6. How long will it take a train going at the rate of 36 mi. 160 rd. an hour to cover 438 mi.?

7. How many packages each containing 7 lb. 8 oz. can be made out of 1 bu. (60 lb.)?

8. Into how many 45-min. periods can a forenoon from 9 to 12 o'clock be divided?

9. How many steps of 2 ft. 8 in. each are there in 1 mi.?

10. How many books at 1*s.* 6*d.* each can be bought for £5 6 *d.*?

11. How many loads of coal averaging 1875 cwt. are there in 73 T. 250 lb.?

12. How many tons (2000 lb.) are contained in 20 long tons (2240 lb.)?

13. How many years of 365 da. are 4 yr. of 365 da. 5 hr. 48 min. 46 sec. and how many hours, minutes, and seconds over?

Written Exercise

237. 1. A grocer bought canned goods, at $2.50 per dozen quart cans, and paid $140 for the lot. How many quarts were there?

2. Allowing for a shrinkage of $\frac{3}{5}$ in the process of canning the above lot, how many bushels of fruit were used?

3. At $.06 a quart, how many bushels of beans can be bought for $15.60?

4. At 4¢ a quart, how much is received for 6 bu. 2 qt. 1 pt. of hazelnuts?

5. How many barrels, each holding $2\frac{3}{4}$ bu., will be required to pack 305 bu. 1 pk. of turnips?

6. At $2\frac{1}{2}$¢ per pound, what is the value of 1 bbl. of flour?

7. At the rate of 30¢ a bushel, 80 lb. to the bushel, what is the value of a ton of coal?

8. In 6 T. 960 lb. of wheat there are how many bushels of 60 lb. each? Find its cost at 90¢ per bushel.

9. How much silver is needed to make 1 gross of spoons of 15 pwt. each? What is its value at 51¢ an ounce?

10. What is the total weight of gold in 3 watch cases containing 1 oz. 4 pwt.; 1 oz. 1 pwt. 12 gr.; 19 pwt. 19 gr.?

11. At the rate of $16.50 per pound Troy, what is the cost of 1 oz.?

12. Reduce 188,000 gr., apothecaries' weight, to pounds.

13. How many badges of 7 pwt. 3 gr. each can be made out of 6 lb. of silver?

14. The distance from A to Z is traveled in 70 hr. 42 min. 33 sec. on boat, and by automobile in $\frac{1}{10}$ the time. What is the automobile's time?

Written Exercise

238. 1. What is the profit on 56 gal. 3 qt. of milk bought at 20¢ a gallon and sold at 7½¢ a quart?

2. How much is received for 50 gal. of oysters at 35¢ a pint?

3. How many gallons of perfumery will sell for $660, the price being $3.20 a quart?

4. How many goblets weighing 7 oz. 15 pwt. can be made from 60 lb. of silver?

5. Out of 2 lb. 10 oz. of gold how many 2½-oz. watch cases can be made, and how many 3-pwt. rings from what remains?

6. Sound moves 1120 ft. per second. How long will it be in going 3 mi.?

7. A wheel 7 ft. 4 in. in circumference turns how many times in going .125 of a mile?

8. From Portland to Boston is 110 mi. How many times will an engine wheel 18 ft. 6 in. in circumference turn in going to Boston and returning?

9. In how many days can 12 doz. garments be stitched, allowing 2 hr. 15 min. to each, and 8 working hours to the day?

10. 6 mi. 156 rd. of railway is fenced on both sides. What is the cost at 5¢ a running foot?

11. 1 mi. of railway laid with rails 70 ft. long requires how many rails?

12. Find the cost of fencing a lot whose sides are: 36 ch. 2 rd. 6 l.; 35 ch. 3 rd. 9 l.; 34 ch. 20 l., at $1.55 per rod.

13. What is the profit on a 43-gal. barrel of kerosene bought at 11¢ and sold at 15¢ a gallon?

14. Find the cost at $16 a ton of 4 loads of hay weighing 1600 lb.; 1 T. 150 lb.; 1875 lb.; 2750 lb.

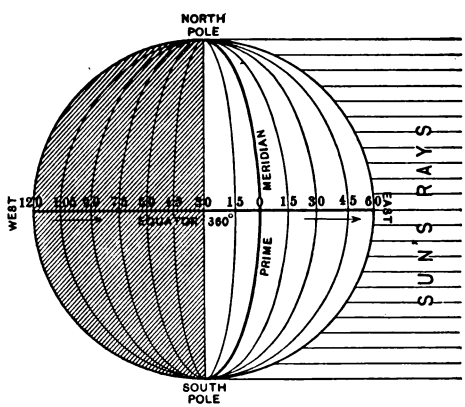
239. Repeat the table for circular measure, Art. 213.

A day of 24 hr. is the time the earth takes to rotate, or turn, once upon its axis.

By the earth's rotation, every point in its circumference of 360° is turned daily toward the sun. This relation of the earth to the sun determines the solar time of day of any place.

When a place or point on the surface of the earth is turned directly toward the sun so that it receives the sun's vertical rays, it is noon, 12 o'clock, at that place. The place is then said to be at its *meridian*.

All points in the same north-and-south line have the same meridian. Such a north-and-south line is called a *line* of *longitude*.



Distances east and west on the earth are reckoned from one chosen meridian line, called the *prime*, or *first meridian*.

The meridian of Greenwich near London is a prime meridian. Places east of the prime meridian are said to be in east longitude, and places west in west longitude.

NOTE. This subject should be illustrated by the use of a globe.

Written Exercise

240. 1. Since any place passes through 360° of longitude in 24 hr., how many degrees does it pass through in 1 hr.?

2. If it passes through 15° in 1 hr., how many minutes of longitude does it pass through in 1 min. of time? How many seconds of longitude does it pass through in 1 sec. of time?

241. From the written exercise, page 160, we see that 15° *of longitude make a difference of* 1 *hr. in time*, 15′ *a difference of* 1 *min., and* 15″ *a difference of* 1 *sec. in time.*

The earth turns from west to east. This makes the sun appear to move from east to west. So, when it is noon on our meridian, it is *past* noon at all places *east* of us, and *before* noon at all places *west* of us. By knowing the meridian of a place east or west of ours, we can find its time.

Oral Exercise

242. Illustrative Examples. St. Petersburg is 30° east of Greenwich; Philadelphia is 75° west of Greenwich; San Francisco is $122\frac{1}{2}$° west of Greenwich. When it is noon at Greenwich, what time is it (1) at St. Petersburg? (2) at Philadelphia? (3) at San Francisco?

WORK AND EXPLANATION. (1) $(30 \div 15 = 2.)$ The difference in time is 2 hr. At St. Petersburg, which is east of Greenwich, it is 2 hr. past 12 o'clock, or 2 o'clock P.M.

(2) $(75 \div 15 = 5.)$ At Philadelphia, which is west of Greenwich, it is 5 hr. before 12 o'clock, or 7 o'clock A.M.

(3) $(122\frac{1}{2} \div 15 = 8\frac{1}{6}.)$ At San Francisco, which is west of Greenwich, it is $8\frac{1}{6}$ hr., or 8 hr. 10 min. before 12 o'clock, or 3 o'clock 50 min. A.M.

243. 1. It is noon at Boston. What time is it 15° east of Boston? 30° east? 90° east? 30° west? 45° west?

2. When it is sunrise at any place, is it sunrise earlier or later at a place 30° east, and how much? at 50° west?

3. In what direction from each other are all places that have sunrise at the same time?

What is the difference in solar time of places on the following meridian lines:

4. 15° E., and 30° E., long.?
5. 15° W., and 45° W., long.?
6. 45° E., and 30° E., long.?
7. 35° E., and 20° E., long.?
8. 15° W., and 15° E., long.?
9. 30° W., and 10° E., long.?

10. What is the difference in longitude of places that differ in solar time 1 hr.? 2 hr.? 4 hr.? 3 hr.? 1 min.? 3 min.? 1 sec.? 2 sec.?

11. When it is noon in San Francisco, is it forenoon or afternoon in Boston? in London and Paris? in the Hawaiian Islands?

12. A commercial traveler's watch keeps the time of his home. He takes it to Portland, Me., about 20° east, also to Portland, Ore., about $33\frac{1}{3}$° west, of his home. At which place will it be too slow? how much? How much too fast will it be at the other place?

13. My watch keeps the time of New Orleans, which is about 90° west longitude. Where I am now it is an hour slow. Which way have I traveled? What is my longitude? I am in a large United States city. Find it on the map, p. 166.

14. If I travel from New Orleans in the opposite direction and find my watch 2 hr. fast, what is my longitude?

15. My watch keeps Greenwich time, and at Toulon in France it is 24 minutes slow; what is the longitude at Toulon? Look at a map and see if this is correct.

Table of Longitudes

Berlin, 13° 23′ 33″ E.	Melbourne, 144° 58′ 35″ E.
Bombay, 72° 48′ 56″ E.	New Orleans, 90° 3′ 28″ W.
Boston, 71° 3′ 30″ W.	New York, 74° 0′ 3″ W.
Canton, 113° 16′ 30″ E.	Paris, 2° 20′ 22″ E.
Chicago, 87° 36′ 45″ W.	Philadelphia, 75° 10′ 5″ W.
Denver, 104° 58′ 0″ W.	San Francisco, 122° 24′ 32″ W.
London, 0° 5′ 48″ W.	St. Petersburg, 30° 19′ 21″ E.
Manila, 120° 58′ 6″ E.	Washington, 77° 3′ 6″ W.

Tokyo, 139° 44′ 30″ E.

Written Exercise

244. Illustrative Example. When it is noon at New York, what is the solar time in London according to the above table?

WORK

$$\begin{array}{r} 74^\circ\ 0'\ 3'' \\ 5\ 48 \\ \hline 15)73^\circ\ 54'\ 15'' \\ \hline 4\ 55\ 37 \end{array}$$

EXPLANATION. Both places being in west longitude, to find their difference in longitude we subtract that of London from that of New York, and find the difference to be 73° 54′ 15″. Since 15° of longitude make a difference of 1 hr. in time, 15′ of longitude 1 min. in time, and 15″ of longitude 1 sec. in time, by dividing the difference in longitude by 15 we find the difference in time, which is 4 hr. 55 min. 37 sec.

Ans. 55 min. 37 sec. past 4 o'clock P.M. in London.

What is the difference in solar time:

1. Between Boston and San Francisco?
2. Between Denver and Washington?
3. Between New York and London?
4. Between London and Berlin?
5. Between Chicago and Berlin?
6. Between New Orleans and St. Petersburg?

7. At 2 P.M. in Paris what is the time at Melbourne? at Philadelphia?

Written Exercise

245. Illustrative Example (1). The difference in time between New York and Paris is 5 hr. 5 min. $21\frac{2}{3}$ sec. Find the difference in their longitude.

WORK

$$\begin{array}{r} 15^\circ\ 15'\ 15'' \\ 5\ \ 5\ \ 21\frac{2}{3} \\ \hline 76^\circ\ 20'\ 25'' \end{array}$$

EXPLANATION. Since 15° of longitude make a difference of 1 hr. in time, 15′ of longitude 1 min. in time, and 15″ of longitude 1 sec. in time, by multiplying 15° 15′ 15″ by the corresponding numbers expressing the time, we obtain the difference in longitude.

Ans. 76° 20′ 30″.

Find the difference in longitude between Greenwich and:

1. Baltimore, the difference in time being 5 hr. 6 min. 26 sec.

2. Cambridge, Mass., the difference in time being 4 hr. 44 min. 30 sec.

3. Canton, China, the difference in time being 7 hr. 33 min. 6 sec.

4. Manila, P. I., the difference in time being 8 hr. 3 min. 52.4 sec.

5. Quebec, Can., the difference in time being 4 hr. 44 min. 52 sec.

6. Tokyo, Japan, the difference in time being 9 hr. 18 min. 58 sec.

Written Exercise

246. Illustrative Example (2). At Para, Brazil, a watch that keeps Paris time is 3 hr. 23 min. 20.9 sec. fast. What is the longitude of Para?

WORK

15°	15′	15″
3	23	20.9
50°	50′	13″
2	20	22
48°	29′	51″

EXPLANATION. The difference in longitude is 50° 50′ 13″. But the watch is fast and Para is in West longitude, while Paris is in East longitude. So we subtract the Paris longitude from the difference.

Ans. The longitude of Para is 48° 29′ 51″ west.

1. The time of two ships, A and B, at sea differs by 3 hr. 20 min. What is the difference in their longitude? If the longitude of A is 45° 30′ east and her time is later than B's, what is B's longitude?

2. It is 5 o'clock 15 min. P.M. at a certain place named X, and 8 P.M. at another place named Y. Which place is further east? and how much?

3. At noon at Liverpool 2 persons land from different steamers. The watch of each has the time of the place he came from. By one it is 9 o'clock 50 min. P.M., by the other 8 o'clock 10 min. A.M. Give the direction whence each came. Find the difference in longitude of the two places.

4. Explain how news from London dated 8.30 P.M. can be received in New York at 3.30 P.M. the same day.

5. A ship sails from longitude 42° 10′ 15″ west to longitude 20° 4′ 16″ east. What change should a passenger make in his watch?

6. At 2.30 P.M. a ship in longitude 30° 20′ 15″ west sends a wireless message to another ship. It is received at 2 P.M. What is the longitude of the receiving ship?

7. At 9 o'clock A.M. a steamer in longitude 50° 10′ 15″ west sends a wireless telegram to the Nantucket light-ship 70° west longitude. What is the time of receiving it at the light-ship?

NOTE. Sailing westward around the earth, a ship gains 1 da. on the sun; sailing eastward, it loses 1 da. To adjust its calendar, at an irregular line called the *international date line*, corresponding to the 180th meridian, it carries its date forward 1 da. if going westward, and back 1 da. if going eastward.

STANDARD TIME

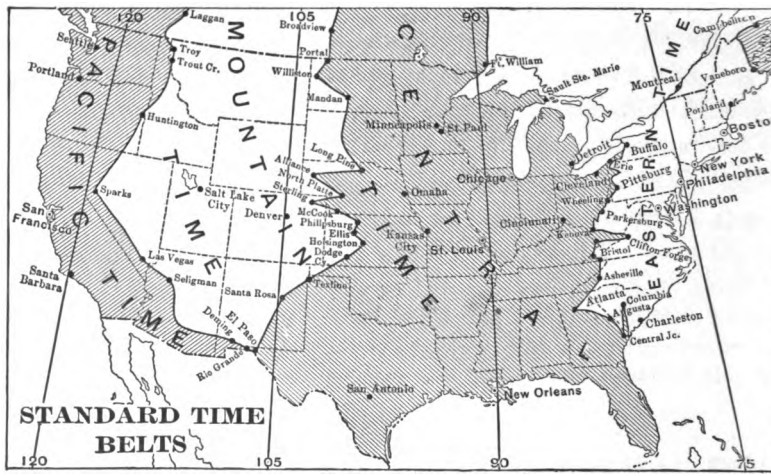
247. In 1883 the railroads in the United States and Canada decided to run their trains by the time of the five meridians 15° apart, the 60th, 75th, 90th, 105th, and 120th. Time belts extending, nominally, $7\frac{1}{2}°$ on each side of these several meridians, use each the same time. This is called **standard time.**

The map on p. 166 shows the time belts in the United States. The irregularities are due to each railroad's selecting the most convenient stations on its road for changing from the standard time of one belt to that of the other.

248. By standard time, the belt that has the time of the 60th meridian is called the *Atlantic* time belt, the others in order from east to west are the *Eastern*, the *Central*, the *Mountain*, and the *Pacific*.

Eastern time is exactly 1 hour earlier than Atlantic time, Central time 1 hour earlier than Eastern time, Mountain time 1 hour earlier than Central time, and Pacific time 1 hour earlier than Mountain time.

Standard time has been adopted by most civilized countries, the time meridian chosen being, usually, some multiple of 15 from the prime meridian of Greenwich.



Oral and Written Exercise

(See Map and Arts. 247–248.)

249. **1.** The stock quotations in New York were telegraphed from that city to Chicago, San Francisco, Boston, Denver, and New Orleans at 10.30 A.M. standard time. Allowing no time for this transmission, give the time the telegram reached each city.

2. Allow 30 min. for transmission, and tell the time when a dispatch sent at 8 A.M. standard time in San Francisco reaches St. Louis, Philadelphia, Washington, Portland, Me.

3. At 2.15 P.M. St. Paul time, give the time at Baltimore, Quebec, Galveston, Los Angeles.

4. Find the difference in standard and solar time at Chicago; at San Francisco.

250. The shortest distance between points is a **straight** line.

Two or more lines that have the same direction are called **parallel** lines.

The outside of anything is called its **surface.**

A flat surface is called a **plane.**

The representation of a plane surface bounded by lines is called a **plane figure.**

The sum of the bounding lines of a plane surface is called its **perimeter.**

Written Exercise

251. 1. 1 mile equals how many rods? how many yards? how many feet?

2. Change .25 mile to rods; 59.4 in. to rods.

3. At what rate per minute does a train go, if it runs 35 mi. per hour?

4. Measure the length of each side of your schoolroom and find the perimeter of the floor surface.

5. Find the distance around the schoolhouse lot in rods and feet.

6. What is the cost of 18 yd. of rubber hose at $16\frac{2}{3}$¢ a foot?

7. The two parallel sides of Mr. Vernon's house lot are 57 yd. 2 ft. apart. Give the width of the lot in feet.

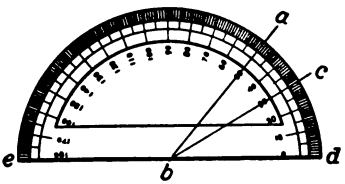
8. What is the cost of netting for the two ends of a tennis court measuring 18 yd. each, at $.15 per running foot?

9. How many steps of 2 ft. 6 in. does a person take in walking 31.2 miles?

10. Mt. St. Elias is 3 mi. 2184 ft. high. Give its height in feet.

11. Mt. Blanc lacks 96 ft. of being 3 mi. high. Pike's Peak is 1633 ft. lower than Mt. Blanc. How high is Pike's Peak?

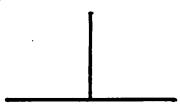
252. The difference in direction of two lines that meet, or tend to meet, is called an **angle**. The lines are called the **sides** and their meeting point the **vertex** of the angle.



PROTRACTOR

To measure angles we use a **protractor**, which is a half circle whose circumference is marked off into 180°, and each degree into 60′ (Art. 213). Any part of the circumference of the circle is called an **arc**.

The arc *cd* on the protractor is 30°, so the angle of the lines *bd* and *bc*, whose vertex is at the center of the circle, is 30°. The arc *ad* is 50°, and so the angle of the lines *ab* and *bd*, which is read angle *abd*, is 50°.


TWO RIGHT ANGLES

An angle of 90° is called a **right angle**.

Any angle less than 90° is called an **acute angle**.

Any angle greater than 90° is called an **obtuse angle**.

Oral Exercise

253. **1.** Repeat the table for Angles and Circles, Art. 213.

2. In the protractor above, what kind of angle is angle *abc*? angle *abe*? angle *cbd*? angle *cbe*?

3. What kind of angle do the hour and minute hands of the clock form at 9 o'clock? at 10 o'clock? at 5 o'clock? at 8 o'clock? at 7 o'clock?

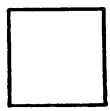
4. Through how many degrees does the hour hand of the clock move in 6 hr.? in 3 hr.? in 1 hr.? in 90 min.? in $1\frac{1}{4}$ hr.? in 12 hr.?

5. The hour and minute hands of a clock form an angle of how many degrees at 3 o'clock? at 2 o'clock? at 1 o'clock? at 1:30 o'clock?

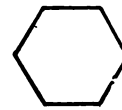
254. Any plane figure bounded by straight lines is a **polygon.**

A polygon having all its sides and all its angles equal is called a **regular polygon.**

EQUILATERAL TRIANGLE


SQUARE

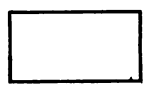
PENTAGON


HEXAGON

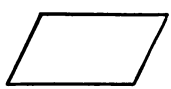
A polygon of 3 sides is called a **triangle**; of 4 sides, a **quadrilateral**; of 5 sides, a **pentagon**; of 6 sides, a **hexagon**; of 8 sides, an **octagon**; of 9 sides, a **nonagon**; of 10 sides, a **decagon**; of 12 sides, a **dodecagon.**

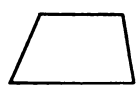
The quadrilaterals are named as follows:

SQUARE


OBLONG

RHOMBUS


RHOMBOID


TRAPEZOID

TRAPEZIUM

The **square** has 4 right angles and all its sides equal.

The **oblong** has 4 right angles, but only its opposite sides equal.

The **rhombus** has no right angles, but all its sides equal.

The **rhomboid** has no right angles and only its opposite sides equal.

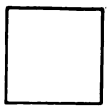
The **trapezoid** has only two of its opposite sides parallel.

The **trapezium** has no two of its sides parallel.

A quadrilateral having both its opposite sides equal is called also a **parallelogram.**

A straight line connecting the vertices of a quadrilateral is called its **diagonal.**

RECTANGLES

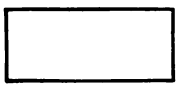


SQUARE

255. The two quadrilaterals (square and oblong) that have 4 right angles are called **rectangles.**

A square each of whose sides is 1 inch long is called 1 square inch.

What should a square be called each of whose sides is 1 foot long? 1 yard long? 1 rod long?



OBLONG

The contents of a surface reckoned in square units is called its **area.**

256. Illustrative Example. Find the area of a rectangle 4 inches long, 2 inches wide.

EXPLANATION. This figure represents a rectangle 4 inches long and 2 inches wide, drawn on a scale of $\frac{1}{2}$ inch to 1 inch. Each of the rows contains 4 sq. in., hence, the whole contains 2 times 4 sq. in., or 8 sq. in. *Ans.* 8 sq. in.

sq.in.	sq.in.	sq.in.	sq.in.
sq.in.	sq.in.	sq.in.	sq.in.

To find the area of a rectangle, we multiply together the numbers of like units of its length and width. The product represents the number of corresponding square units.

Written Exercise

257. 1. The dimensions of a ceiling are as follows: length, 22 ft.; width, 14 ft. Find its dimensions in square feet. Find the cost of plastering it at 45¢ per square foot.

2. Find the cost at 25¢ a square foot for slate blackboards 4 ft. wide to cover the available space of your schoolroom.

3. A floor 40 ft. by 32 ft. contains how many square yards?

4. What is the value at $72 an acre of a tract of land 62 rd. long by 40 rd. wide?

Written Exercise

258. 1. At 45¢ a square yard, how much does it cost to grade and turf a lawn that measures 65 ft. by 70 ft.?

2. Find the difference in the area of a lawn 50 ft. square and a lawn 100 ft. by 25 ft.

3. If it takes 60 blocks to pave 1 sq. yd., how many blocks will it take to pave a street 54 ft. wide, 1 mi. long?

4. My sitting-room floor measures 16.5 ft. long by 15 ft. wide. How many square yards does it contain?

5. What is the cost of linoleum at $62\frac{1}{2}$¢ a yard to cover a floor that is 5 yd. long, 4 yd. wide?

6. At $1.90 a square yard, what is the cost per rod for a sidewalk 4 ft. wide?

7. A school lot measures 210 ft. by 108 ft. and an adjoining park is 250 ft. square. Find the sum of their areas.

8. How many yards of curbing will be required to surround two lots, one 50 ft. square, the other 100 ft. long, 25 ft. wide?

Find the number of square feet of lighting and of floor surface in the following schoolrooms, and the relation which the lighting bears to the floor surface:

9. Room 31 ft. by 28 ft.; 4 windows of 2 sashes, 6 panes 18 in. by 12 in.

10. Room 34 ft. by 28 ft.; 6 windows of 2 sashes, 9 panes 15 in. by $13\frac{1}{2}$ in.

11. Room 64 ft. by 59 ft.; 11 windows, 2 sashes, 6 panes 15 in. by 12 in.

12. Room 31 ft. by 30 ft.; 6 windows, 2 sashes, 12 panes 14 in. by 10 in.

APPLICATIONS OF SURFACE MEASURE

Flooring

259. Illustrative Example. Find the cost of flooring a room 15 ft. by 40 ft. with boards at $75 per M feet.

15	Work	Explanation. The area of the floor is (15 x 40) sq. ft., or 600 sq. ft. 600 sq. ft. is $\frac{6}{10}$ of 1000 sq. ft. If 1000 sq. ft. cost $75, $\frac{6}{10}$ of 1000 sq. ft. cost $\frac{6}{10}$ of $75, which is $45. *Ans.* $45.
40	$600 \div 1000 = \frac{6}{10}$ of 1 M	
600	$\frac{6}{10}$ of $75 = $45	

1. Find the cost of flooring a dining room with quartered oak, dimensions 15 ft. 6 in. by 18 ft., at 25¢ a square foot for material and labor.

2. How much will the material cost for a chamber floor 11 ft. 6 in. by 14 ft. at $38 per M? Adding 6¢ a square foot for laying and nails will make the total cost how much?

Carpeting

260. Illustrative Example. How many yards of carpeting 28 in. wide will be needed for a floor 14 ft. × 12 ft. 5 in.? (Find the answer approximately by comparing the area of the floor with the area of 1 yd. of carpet.)

Work

$14 \times 12.5 \times 1$ sq. ft. $= 175$ sq. ft.

28 in. $= \frac{7}{9}$ of 1 yd.

$\frac{7}{9}$ of 1 sq. yd. $= \frac{7}{9}$ sq. yd.

$\frac{7}{9}$ of 9 sq. ft. $= 7$ sq. ft.

175 sq. ft. ÷ 7 sq. ft. $= 25$

Explanation. The product of the dimensions of the room gives the area of the floor, 175 sq. ft. The product of the dimensions of 1 sq. yd. gives the area of 1 yd. of carpet. Divide the floor area by the area of 1 yd. of carpet. The quotient equals the number of yards required. *Ans.* 25 yd.

1. Find the cost of carpeting a chamber 11 ft. 6 in. × 12 ft. with straw-matting 1 yd. wide at 35¢ a yard.

2. A hall measures 11 ft. × 14 ft. How much will it cost to carpet it with material $\frac{3}{4}$ of a yard wide at $1.60 a yard?

261. Illustrative Example. How many yards of carpeting 30 in. wide will be required for a room 14 ft. 6 in. wide and 19 ft. long, the carpet to be laid lengthwise?

WORK

14 ft. 6 in. $= \frac{29}{2}$ ft.

30 in. $= \frac{5}{2}$ ft.

$\frac{29}{2}$ ft. $\div \frac{5}{2}$ ft. $= 5\frac{4}{5}$ or, 6 strips

$$\frac{6 \times 19}{3} = 38$$

EXPLANATION. Divide the width of the room, $\frac{29}{2}$ ft., by the width of the carpet $\frac{5}{2}$ ft. The quotient is $5\frac{4}{5}$. Therefore 6 strips, each 19 ft. long, or 114 ft., or 38 yd., will be needed. *Ans.* 38 yd.

1. At $\$1.66\frac{2}{3}$ a yard what will be the cost of carpeting a room 6 ft. 6 in. $\times$ 10 ft. 4 in., the carpet to be laid lengthwise of the room and to be 27 in. wide?

2. How many yards of carpet 30 in. wide will be needed for a room 21 ft. 3 in. $\times$ 18 ft. 3 in., the carpet to be laid most economically? What will be the cost of the carpet at $\$2.25$ a yard?

PLASTERING

262. Illustrative Example. What will be the cost at 40¢ a square yard of plastering the walls and ceiling of a room 11 ft. 6 in. $\times$ 11 ft. 6 in., and 11 ft. 6 in. in height, allowing for $\frac{1}{2}$ of the surface in 4 doors, each 7 ft. $\times$ $3\frac{1}{2}$ ft., and 4 windows, each 6 ft. $\times$ 3 ft.?

SOLUTION. Find the total area of walls and ceiling. Subtract from this all allowance for surfaces of doors and windows. Reduce the remainder to square yards and multiply by the price per square yard.

1. How much will it cost to plaster a sewing room that is 10 ft. 6 in. long, 9 ft. 4 in. wide, and $9\frac{1}{2}$ ft. high at 42¢ a square yard, allowance being made for one half the surface of one window 3 ft. $\times$ $6\frac{1}{2}$ ft., and of one door 3 ft. 6 in. $\times$ 6 ft. 6 in.?

Roofing

263. 1. What will be the cost of shingles at $2.75 per M to cover the two sides of a roof, each side measuring 32 ft. long and 17½ ft. from eaves to peak, if the shingles are laid 4 in. to the weather, estimating 1000 shingles per square (*i.e.* 10 × 10 (sq. ft.) = 100 sq. ft.)?

2. What will be the cost of shingling the above roof if the shingles are laid 4½ in. to the weather? 5 in. to the weather?

Papering

264. 1. At 32¢ a roll, what will be the cost of paper for a room 10 ft. wide, 12 ft. long, and 9 ft. high, allowing for 2 doors and 2 windows?

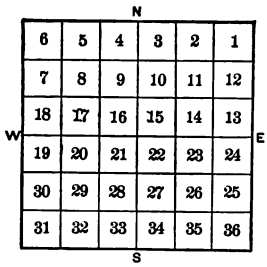
Note. Wall paper is sold in rolls usually containing about 4 sq. yd. Paper hangers find the total area of the walls and ceiling, subtract 20 sq. ft. for each door and window, and divide the remainder by 33 sq. ft. for the approximate number of rolls. Parts of a roll are not sold.

2. Find the cost, at 30¢ a roll, of papering the walls of a room 14 ft. 6 in. × 13 ft. 4 in. and 12 ft. high, including bordering at 10¢ a yard, and hanging at 15¢ a roll, allowing for 4 windows and 3 doors.

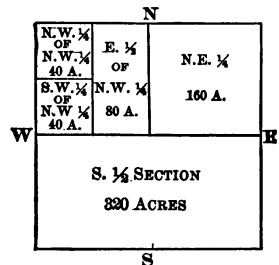
United States Public Land Measure

265. The United States public lands, before being offered for sale, are surveyed into tracts, 6 mi. × 6 mi. each, called townships. The townships are subdivided into sections of 1 sq. mi., 640 A., there being 36 of these sections in a township. Sections are subdivided in turn into quarter sections of 160 A. each, and the quarters into "halves" or eighties of 80 A. each, or quarters or forties of 40 A. each.

The method of numbering sections in a township and the subdivisions of a section are illustrated on p. 175.



SECTIONS OF A TOWNSHIP



DIVISIONS OF A SECTION

The lines bounding a township extend due north and south, and east and west, and a line on a parallel of latitude is always established as a **base.** A line of townships extending north and south is called a **range.** The ranges are designated by their numbers east or west of the principal meridian, and the townships in each range by their number north or south of the base line.

Lots which for any reason are irregular in form are designated as Lot 1, 2, 3, 4, etc., of a particular section. City and village plots are subdivided into blocks, and these again into smaller lots.

1 township	= 36 sq. mi. =	23,040 A.
1 section	= 1 sq. mi. =	640 A.
1 half section	= ½ sq. mi. =	320 A.
1 quarter section	= ¼ sq. mi. =	160 A.
1 half-quarter section	= ⅛ sq. mi. =	80 A.
1 quarter-quarter section	= 1/16 sq. mi. =	40 A.

1. How much must be paid for the N.E. quarter of section 11 of a South Dakota township at $3 per acre?

2. If a speculator buys the northern half of a section of land and sells at various times the N.E. ¼ of N.W. ¼, the N.W. ¼ of N.E. ¼, the S.E. ¼ of N.W. ¼, and the N.E. ¼ of N.E. ¼, how many acres has he left? Draw a diagram to show what part of his half section he has left.

3. An agent bought a section of land at $5 an acre. He sold the south ½ section at $7 an acre, the N.E. ¼ of N.E. ¼ at $8 an acre, and the N.E. ¼ of N.W. ¼ at $6.50 an acre. How many acres did he sell? What part of the section had he left?

Surveyors' Measure

266. For measuring land, surveyors use a measure called a chain (ch.), which is 4 rd. long and is divided into 100 links (l.). A square chain should be how many square rods in area? In 1 A. there are how many square rods? How many square chains?

Since 10 sq. ch. equal 1 A., square chains may be changed to acres by moving the decimal point one place to the left, and acres may be changed to square chains by moving the decimal one place to the right.

267. Illustrative Example. A tract of land is 142 ch. by 48 ch. 17 l. Find the number of acres it contains.

Explanation. 48 ch. 17 l. = 48.17 ch. 142 × 48.17 sq. ch. = 6840.14 sq. ch. = 684.014 A. *Ans.* 684.014 A.

1. How many acres are there in a lot 56 ch. 24 l. long and 38 ch. 16 l. wide?

2. A pasture measures 68 ch. by 47 ch. 20 l. It contains how many acres?

3. At $75 an acre what is the value of a piece of land 106 ch. 34 l. × 87 ch. 44 l.?

268. To find one dimension of a rectangle, the area and the other dimension being given.

Illustrative Example. A farm 160 rods long costs $3750 at $75 an acre. What is its width?

Work

$3750 ÷ $75 = 50, the number of acres.

50 × 160 sq. rd. = 8000 sq. rd.

8000 ÷ 160 = 50, the number of rods.

Therefore it is 50 rd. wide.

Explanation. To find the number of acres in the area we divide $3750 by $75, and find there are 50 acres. 50 acres = 8000 sq. rd. The area is 8000 sq. rd. Divide 8000 by the 160 in the given dimension and we obtain 50, the number of rods in the other dimension. *Ans.* 50 rd. wide.

Written Exercise

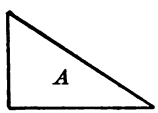
269. 1. The area of a field is 20 acres. If its length is 64 rd., what is its width?

2. Find the cost of fencing, at $11.25 a rod, a 36-acre field that is 96 rd. long.

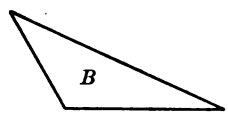
3. How long is a walk 12 ft. wide, if its area is 84 sq. yd.?

Triangles

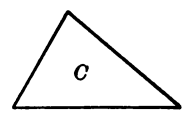
270. A **triangle** is a figure having 3 sides and 3 angles.



Right-angled Triangle

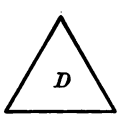


Obtuse-angled Triangle

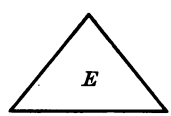


Acute-angled Triangle

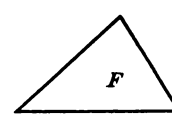
A **right-angled** triangle has one right angle.
An **obtuse-angled** triangle has one obtuse angle.
An **acute angled** triangle has all its angles acute.



Equilateral Triangle

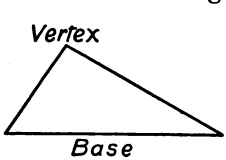


Isosceles Triangle



Scalene Triangle

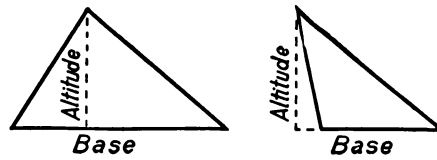
An **equilateral** triangle has 3 equal sides and 3 equal angles.
An **isosceles** triangle has 2 equal sides.
A **scalene** triangle has all its sides unequal.



In these six kinds of triangles A, B, and C are named from their angles. D, E, and F are named from their sides.

The **base** of a triangle is the side upon which it is supposed to rest.

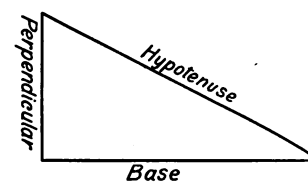
The angle opposite to the base is called the **vertex.**



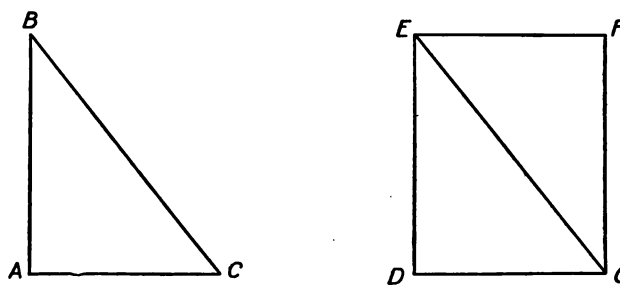
The **altitude** of a triangle is the perpendicular distance from the base, or the base produced, to the highest point in the triangle.

The **hypotenuse** of a right-angled triangle is the side opposite the right angle.

The **perpendicular** of a right-angled triangle is the line drawn at right angles to the base.



Area of a Triangle



271. To find the area of a triangle. Cut two triangles of the size of ABC and place them as shown in $DEFG$.

What is the name of the figure $DEFG$?

To how many triangles of the size of ABC is $DEFG$ equal? We may conclude therefore that:

The area of a triangle is equal to half the area of a rectangle of the same base and altitude.

272. Illustrative Example. Find the area of a triangle with a base of 16 rd. and an altitude of 20 rd.

EXPLANATION. The number of square rods in the area of a rectangle with a base of 16 rd. and an altitude of 20 rd. is $20 \times 16 = 320$. The number of square rods in the area of a triangle of the same base and altitude is $\frac{1}{2}$ as many as in the rectangle, that is, $\frac{16 \times 20}{2} = 160$. *Ans.* 160 sq. rd.

1. There are how many square feet in a triangular platform having a base of $9\frac{1}{2}$ ft. and an altitude of 10 ft. ?

2. Find the area of a triangle that has a base of 20 ft. and an altitude of 13 ft.

3. Find the area of a triangle whose base is $12\frac{1}{2}$ ft. long and altitude $4\frac{2}{3}$ ft.

4. Find the area of a triangle whose base is 10 yd. 2 ft. long and altitude 3 yd. 1 ft.

5. What must be the height of a triangle that contains 2250 sq. ft. and whose base is 180 ft. in length?

6. How many square feet are there in a triangular flower bed whose base is 19 ft. and the height 9 ft. 6 in. ?

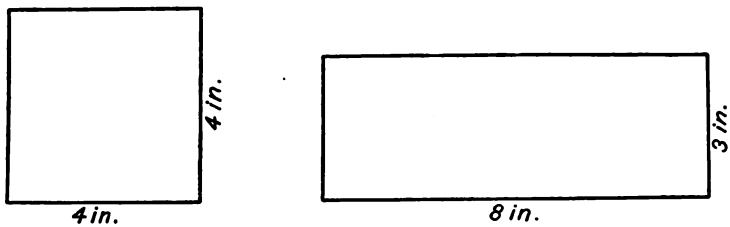
7. Mary is to crochet a half square breakfast shawl, each of its two equal sides being $1\frac{1}{4}$ yd. long. How many square feet will she crochet?

8. Find the surface of an octagonal spire, each of whose eight faces is $19\frac{1}{2}$ ft. at the base and $51\frac{2}{3}$ ft. high.

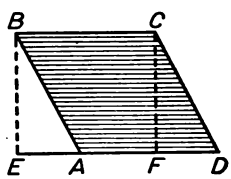
9. Formulate a rule for finding the area of a triangle when the base and the altitude are given.

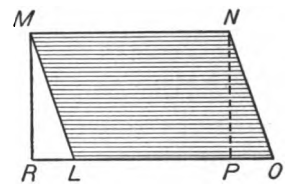
10. Formulate a rule for finding either the base or altitude of a triangle, the area and either the base or altitude being given.

Area of Quadrilaterals



273. To find the area of a rhombus and of a rhomboid. What is the area of each of these rectangles?





It will be seen by the diagram that the rhombus *ABCD* is equal to the rectangle *EBCF* of the same base and altitude.

If the shaded part, *NPO*, of the rhomboid *LMNO* is cut off, it will be found to fill exactly *MRL*, thus giving what kind of a figure?

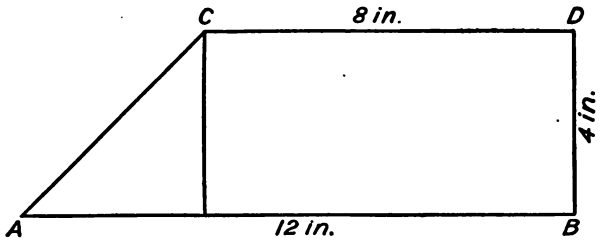
Compare the base of the rhomboid with the base of the rectangle. Compare the altitudes.

This proves that the area of a rhomboid is equal to that of a rectangle of the same base and altitude. The same principle may be applied to finding the area of a rhombus.

1. How many square feet are there in a rhomboid whose base measures $4\frac{1}{2}$ ft. and whose height is $1\frac{2}{3}$ ft.?

2. The base of a rhombus is 14 ft. and its altitude 11 ft. What is its area?

3. What is the area of a parallelogram whose base is 36 ft. and altitude 16 ft.?

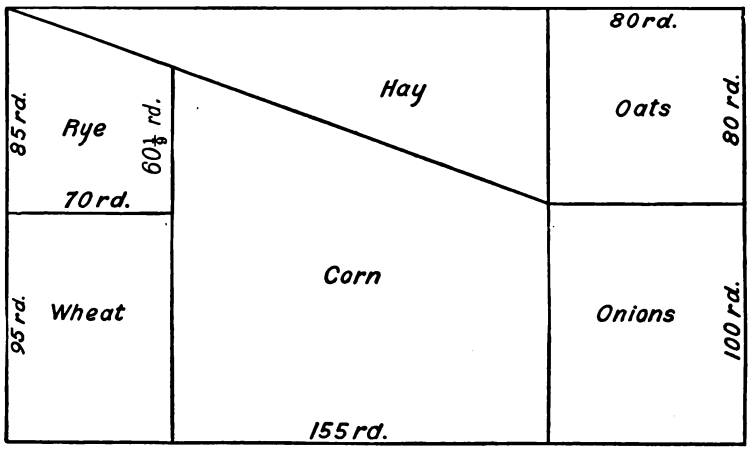


274. To find the area of a trapezoid.

If you draw a perpendicular from C to the base AB, you will have a rectangle and a triangle. Find the area of each and add the results, thus obtaining the area of the trapezoid $ABCD$.

1. What is the area of a trapezoid, the longer of the two parallel sides being 120 ft., the shorter 110 ft., and the altitude 75 ft. ?

2. A plank 25 ft. long is $1\frac{1}{2}$ ft. wide at one end and 1 ft. wide at the other. What is the area of one side of this plank ?

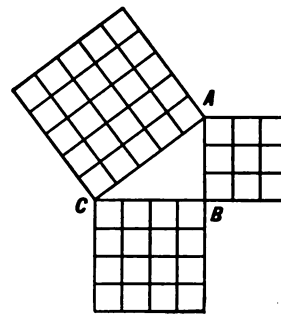


3. Find the number of acres in the farm of which this is a plan.

Find the area in acres of the field (see p. 181) used for:

4. Corn. 6. Oats. 8. Hay.
5. Onions. 7. Rye. 9. Wheat.

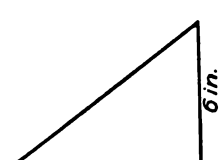
Right Triangles



275. ABC is a right-angled triangle whose sides are 3 in., 4 in., and 5 in. respectively. The square formed upon the hypotenuse, AC, contains how many square inches? The square formed upon the base, BC, contains how many square inches? The square formed upon the perpendicular, AB, contains how many square inches? How does the number of square inches upon AB and BC together compare with the number of square inches upon the hypotenuse, AC?

The length of the hypotenuse of a right-angled triangle equals the square root of the sum of the squares of the other two sides.

276. Illustrative Example.



The base of a right-angled triangle is 8 in. and the perpendicular 6 in. What is the hypotenuse?

$8^2 = 64$

$6^2 = 36$

$\sqrt{100} = 10$ Therefore the hypotenuse is 10 in. *Ans.*

In the diagram (page 182) compare the number of squares upon the base BC with the number upon AB and AC. It will be found that the number upon BC is the same as the difference in number between those on AC and AB. This will also hold true in comparing the number of squares upon the perpendicular AB with the difference in number of those upon AC and BC. Therefore,

The square root of the difference of the squares of the hypotenuse and the other side will give the base or the perpendicular.

Written Exercise

277. Illustrative Example. The hypotenuse is 5 in., the perpendicular 3 in. What is the base?

$$5^2 = 25$$
$$3^2 = 9$$
$$\sqrt{16} = 4 \quad \textit{Ans.}\ 4 \text{ in.}$$

1. What is the perpendicular if the base of a triangle is 8 in. and the hypotenuse 10 in.?

2. Find the hypotenuse if the base is 21 in. and the perpendicular 20 in.

3. What must be the length of a ladder to reach the top of a house 24 ft. high, the foot of the ladder being placed 18 ft. from the house?

4. If a ladder 26 ft. long is placed with its foot 10 ft. from a wall, how high up the wall will it reach?

5. Two boys start from the corner of a square field and walk along the two sides. How far apart will they be when one has walked 130 yd. and the other 144 yd.?

6. Two ships sail from a harbor, one 56 mi. due east, and the other 90 mi. due south, in a day. How far apart are they at the end of the day?

7. A ship sails 72 mi. on a northeast course and then 21 mi. on a southeast course. How far does this bring her from her starting point?

8. Find the diagonal of the floor of a room 24 ft. long, 18 ft. wide.

9. Find the height and area of an isosceles triangle whose base is 78 ft. and each of the other sides 89 ft.

10. Find the height and area of an equilateral triangle, each side of which measures 10 in.

11. Find how long the rafters must be for a barn 40 ft. wide, the ridgepole being 18 ft. above the level of the eaves, and the eaves projecting 2 ft. from the walls.

12. A rectangular park is 40 rd. long and 36 rd. wide. Find the length in rods of a walk between its opposite corners.

In the following find the length of the missing side:

	No.	Base	Perpendicular	Hypotenuse
13. Right-angled triangle	1	12	9	—
14. Right-angled triangle	2	—	12	20
15. Right-angled triangle	3	18	32	—
16. Right-angled triangle	4	15	—	25
17. Right-angled triangle	5	21	—	35

Measurement of Circles

278. A plane figure bounded by a curved line every point of which is equally distant from a point within called a center is a **circle.**

The curved line that bounds a circle is called the **circumference.**

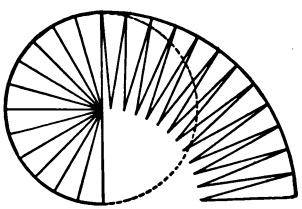
Any straight line extending between two opposite points in the circumference and passing through the center is called a **diameter.**

Any straight line from the center to any point in the circumference is called a **radius**. A radius is one half a diameter.

In every circle the circumference very nearly equals *the diameter multiplied by* 3.1416. Test the accuracy of this statement by measuring the diameter and circumference of various circular objects, *e.g.* the bases of bottles, boxes, etc.

3.1416 may be represented by the Greek letter π (pī), the diameter by the abbreviation d, and the radius by r. Then $\pi \times d$ or πd, or $2\pi r$ is the formula for finding any circumference.

A circle may be considered as made up of triangles the sum of whose bases is the circumference of the circle and whose height is the radius.



Since the circumference $= 2\pi r$, the area of a circle $= 2\pi r \times \frac{r}{2}$, or πr^2.

The area of a circle equals the product of the circumference by half the radius, or the radius squared $\times$ 3.1416.

Written Exercise

279. What is the area of a circle

1. Whose circumference is 18.8496 ft. and radius 3 ft.?
2. Whose circumference is 23.562 ft. and diameter $7\frac{1}{2}$ ft.?
3. What is the length of the circumference of a 7-in. stovepipe?
4. What is the area of a circular table top that has a diameter of 2 ft. $9\frac{1}{2}$ in.?
5. The distance around a circular path is 157.08 ft. What is the area of the inclosed plot?

Find the area of a circle when

6. The diameter = 6 rd.
7. The radius = 4 ft. 6 in.
8. The circumference = 460 ft.
9. The circumference = 680 ft.

10. Over how many square feet can a cow feed when tethered so that her head reaches 30 ft. from the stake?

11. The trunk of a tree in California has a circumference of 106 ft. What is the distance through and what is the area of a cross section?

12. How many feet of surface has a circular mirror inside a frame that has a breadth of 3 in., and whose outside edge is the circumference of a circle whose diameter is 4 ft. 6 in.?

13. How many square feet are there in the frame of the mirror?

14. What is the circumference of a wagon wheel that has a radius of 18 inches? How many revolutions of this wheel will cover a mile?

15. How many square feet of sheet iron are there in a piece of 6-inch stovepipe 2 ft. 6 in. long?

16. Formulate a rule for finding the area of a circle when the diameter is given.

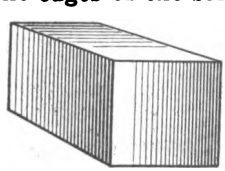
Measure of Volumes

280. A figure or object that has length, breadth, and thickness is called a **solid.**

A solid that is bounded on all sides by rectangles is called a **rectangular solid.**

The rectangles by which a rectangular solid is bounded are called the **faces** of the solid. Taken together the faces make the surface of the solid.

The lines formed by the meeting of any two faces are called the **edges** of the solid.



Rectangular Solid

A rectangular solid that is bounded by six equal squares is called a **cube.**

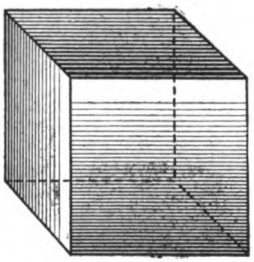
A cube having edges 1 inch in length is a 1-inch cube and contains 1 cubic inch.

A solid having each edge 1 foot long contains a cubic ——.

A solid having edges 1 yard long is called a cubic ——.

The standard unit of measurement for the contents of a solid is the cubic inch, from which are derived the cubic foot and the cubic yard.

The contents of a solid reckoned in units of solid measurement is called its **volume.**



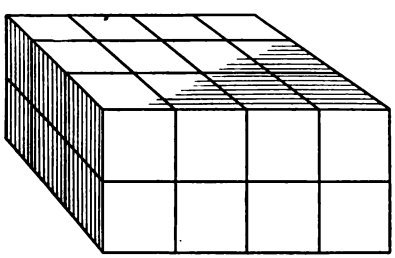
CUBE

The dimensions of a solid are its **length, breadth,** and **thickness.**

VOLUMES OF RECTANGULAR SOLIDS

Written Exercise

281. Illustrative Example. What is the volume of a block of concrete 4 ft. long, 3 ft. wide, and 2 ft. thick?



EXPLANATION. The area of the top face of the block is $4 \times 3 \times 1$ sq. ft., or 12 sq. ft. The upper half of the block is 1 ft. thick. The upper half contains 12 cu. ft., *i.e.* $4 \times 3 \times 1$ cu. ft. $= 12$ cu. ft. But the whole block is 2 ft. thick. Then the whole block contains $2 \times 4 \times 3$ cu. ft. $= 24$ cu. ft. *Ans.* 24 cu. ft.

1. How many cubic feet of concrete will make a block 8 ft. long, 3 ft. wide, and $1\frac{1}{2}$ ft. thick?

2. How many cubic feet of air are there in a room 35 ft. $\times$ 45 ft. $\times$ 12 ft.?

3. What is the weight of the water in a tank 26 ft. long, 16 ft. wide, and 5 ft. deep, if 1 cubic foot weighs $62\frac{1}{2}$ lb.?

4. What is the weight of a block of granite 8 ft. long, $1\frac{1}{2}$ ft. wide, and $1\frac{1}{4}$ ft. thick, at 165 lb. to the cubic foot?

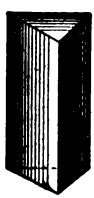
5. What must be the length of a beam 16 in. by 22 in., to contain 115½ cu. ft.?

NOTE. Divide the volume 115½ by the product of the two dimensions given.

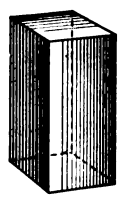
6. Formulate a rule for finding the volume of a rectangular solid when the dimensions are given.

PRISMS, CYLINDERS, PYRAMIDS, CONES, SPHERES

282. A **prism** is a body having two equal parallel polygons as bases and its other faces parallelograms.



TRIANGULAR PRISM

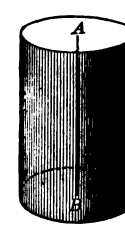


SQUARE PRISM

A prism is **triangular, quadrangular, square, pentagonal,** etc., according as its bases have *three* sides, *four* sides, *five* sides, etc.

The equal and parallel polygons are the **bases** of the prism, and the perpendicular distance between the bases is the **altitude** or **height.** The parallelograms taken together form its **convex surface.**

283. A **right** or **circular cylinder** is a solid bounded by a uniformly curved surface and having for its bases circles that are parallel to each other. In this book *cylinder* means *circular cylinder.*



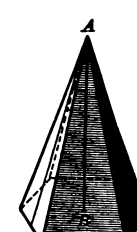
CYLINDER

284. A **pyramid** is a solid bounded by one polygon, the base, and three or more triangles that terminate in one point at the top, called the vertex.

Pyramids, like prisms, are named from their bases, as **triangular, square, hexagonal,** etc.

The perpendicular distance from the vertex to the base of a pyramid is its **altitude.**

The **slant height** of a pyramid is a straight line from the vertex to the center of one side of the base.



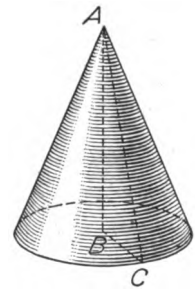
PYRAMID

285. A **cone** is a body whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex** of the cone.

The **altitude** of a cone is the shortest distance from the vertex to the center of the base, as AB.

The **slant height** is the shortest distance from the vertex to the perimeter of the base, as AC.

The **convex surface** of a cone may be thought of as made up of an infinite number of triangles.



CONE

286. A **sphere** is a solid bounded by a curved surface, every point of which is equally distant from a point within, called the center.

SPHERE

A straight line passing through the center of a sphere and terminating at both ends in the surface is called its **diameter.**

One half the diameter of the sphere, or the distance from the center to the surface, is called its **radius.**

The greatest distance around a sphere is called the **circumference** of the sphere.

SURFACES OF SOLIDS

287. Convex surface of a prism or a cylinder. If a prism or a cylinder is 1 in. high, its convex surface contains as many square inches as there are inches in the perimeter of the base. If the height is increased to 2, 3, or any number of inches, the convex surface will be increased in the same proportion. Hence,

To find the convex surface of an upright prism or a cylinder, we multiply the perimeter of the base by the height.

Written Exercise

288. 1. What is the convex surface of a cylinder whose diameter is 3 ft. and whose altitude is 6 ft.?

2. Find the convex surface of a triangular prism whose base measures 8 in. on each side and whose altitude is 15 in.

3. Find the entire surface of a cylinder that is 8 ft. in height and that has a radius of 2 ft.

4. What is the convex surface of a square prism whose sides are each 3.5 ft. and whose altitude is 6 ft.?

289. Convex surface of a pyramid or a cone. The convex surface of a regular pyramid or of a cone is composed of triangles whose bases form the perimeter of the base of the solid, and whose height is the slant height of the solid. Hence,

To find the convex surface of a regular pyramid or of a cone, we multiply the perimeter of the base by the slant height and divide the product by two.

1. What is the convex surface of a cone having a base 4 in. in diameter and a slant height of 9 inches?

2. Find the convex surface of a rectangular pyramid whose base is 15 inches square and whose slant height is 25 inches.

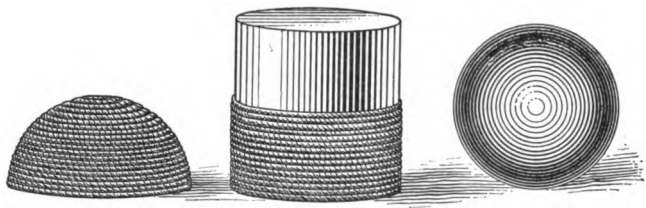
3. At 25¢ a square yard, what is the cost of painting a church steeple, the base of which is a hexagon 6 ft. on each side, and whose slant height is 75 ft.?

4. How many square yards of canvas are required to make a conical tent 19 ft. in diameter and 12 ft. high?

5. What is the surface of an octagonal pyramid whose base is 13 in. on a side, and whose slant height is 12 in.?

6. Find the convex surface of a cone whose base is 15 ft. in diameter, and whose slant height is 15 ft.

290. Convex surface of a sphere. If we wind a sphere with cord and wind a cylinder whose radius equals the radius of the sphere, and whose height equals the diameter, we find that it



takes as much cord for the cylinder as for the sphere. In the picture, for convenience, only half of each is wound.

The convex surface of a sphere equals the convex surface of a cylinder of the same diameter and height. Hence, it equals the diameter $\times \pi d$ (Art. 287), *or the square of the diameter* $\times 3.1416$, *or* πd^2.

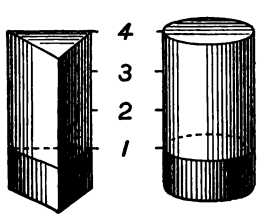
1. What is the convex surface of a sphere that has a radius of 4 in.?

2. The earth is a sphere of 8000 mi. diameter. What is its area?

3. There is a gilded ball on top of a tower. The radius of the ball is 2 ft. How much did it cost to gild it at 10¢ a square foot?

Volumes of Solids

291. Volume of a prism or a cylinder. Suppose that the area of the base of the triangular prism is 6 sq. in., the volume of the lower shaded part, which is 1 in. high, will be 6 cu. in.



Suppose it is shaded 2 in. high, what will be the volume of the shaded part? 3 in. high? 4 in. high?

The same reasoning will give the volume of a cylinder.

The volume of a prism or of a cylinder is equal to the product of the area of the base by the altitude.

1. What is the volume of a prism the area of whose base is 16 sq. in., and whose height is 9 in.?

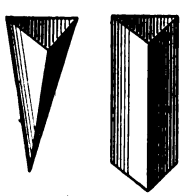
2. Find the volume of a cylinder with a radius of 6 in. and a height of 17 in.

3. How many bushels of wheat will a bin 9 ft. square and 10 ft. high contain?

4. How many gallons of water will a cylindrical reservoir contain, if it is 13 ft. in diameter and 20 ft. high?

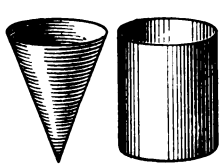
5. Find the value, at $.65 a bushel, of corn that will fill a bin the bottom of which measures 9 ft. by 6 ft., and which is 5 ft. deep.

292. Volume of a pyramid. If we construct a hollow prism and a hollow pyramid of the same base and height, and fill the prism with sand, we find that the pyramid can be filled three times with the same amount. Therefore, we may conclude that:



The volume of a pyramid equals one third the product of the area of the base and height.

293. Volume of a cone. Similarly we may find that:



The volume of a cone equals one third the product of the area of the base and height.

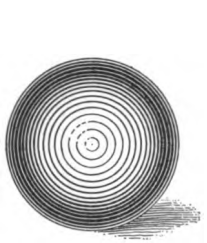
NOTE. It is desirable that these solids should be constructed by the class.

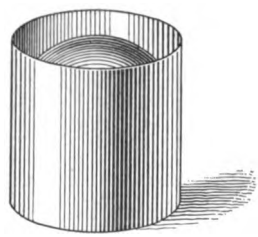
1. What is the volume of a cone, the diameter of whose base is 5 ft. and whose altitude is 9 ft.?

2. Find the volume of a regular square pyramid, the area of whose base is 64 sq. in. and whose altitude is 17 in.

3. Find the cubical contents of a cone, the radius of whose base is 11 in. and whose slant height is 14 in.

294. Volume of a sphere. If we place this sphere in this hollow cylinder of the same diameter and height, and fill in the





spaces with sand, we find that the sand fills one third of the cylinder after the sphere has been removed. Therefore, we may conclude that:

A sphere equals two thirds the volume of a cylinder of the same diameter and height.

Therefore, since the volume of a cylinder equals $\pi r^2 \times 2r$ (Arts. 278 and 291), the volume of a sphere equals $\frac{2}{3} \times \pi r^2 \times 2r = \frac{4}{3}\pi r^3$.

1. Find the volume of a sphere whose radius is 6 ft.

2. What is the volume of a sphere whose diameter is 2.5 ft.?

Written Exercise

295. 1. How many cubic inches are there in a triangular prism, each side of whose base measures 4 in. and whose height is 8 in.? Find also the convex surface and total surface of this prism.

2. How many cubic feet are there in a pyramid 2 ft. high with a base 1 ft. square? Find also the convex surface.

3. A regular hexagonal prism contains how many cubic inches if its height is 6 in. and its base measures 16 square inches?

4. How many cubic inches are there in a cylinder whose height is 6 in. and the diameter of the base 4 in. ?

5. What is the volume of a cone of the same base and the same height as the cylinder just described ?

6. How many gallons of oil can be stored in a cylindrical iron vat 4 ft. in diameter and 9 ft. deep ?

7. How many square feet of sheet iron are there in a piece of stovepipe 6 in. in diameter and 3 ft. long, allowing 1 in. for lapping at the joint ?

8. How many cubic inches are there in a bushel measure, cylindrical in form, 18.5 in. in diameter and 8 in. deep ?

9. How many square inches are there in the surface of a ball 4 in. in diameter ?

10. How many cubic inches are there in this ball ?

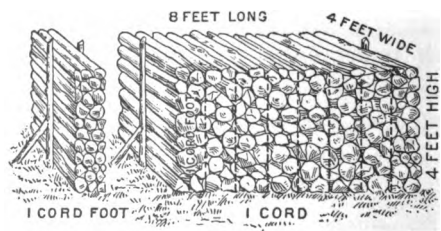
Wood Measure

Oral and Written Exercise

296. Wood or bark is usually cut for marketing into 4-ft. lengths. A pile of wood or bark, 4 ft. high and 8 ft. long, contains 8 cd. ft. or 1 cd. It contains how many cubic feet ?

The following illustration represents 1 cd. and 1 cd. ft.

See the table of Wood Measures, page 145.



1. How many cords of bark are there in a pile 4 ft. high, 4 ft. wide, and 8 ft. long ?

2. At $6 a cord what is the cost of 6 cd. ft. ? of 12 cd. ft. ?

At \$8 a cord for wood, find the cost of a load measuring:

3. 4 ft. by 4 ft. by 4 ft. 5. 8 ft. by 4 ft. by 1 ft.

4. 4 ft. by 4 ft. by 2 ft. 6. 9 ft. by 4 ft. by 4 ft.

7. What is the cost at \$7.50 per cord of a load of wood 8 ft. by 4 ft. by 2 ft.? 12 ft. by 8 ft. by 4 ft.?

8. If wood is cut in lengths of 6 ft. and piled 4 ft. high, how long must the pile be to contain a cord?

9. If wood cut 3 ft. 9 in. long was sold for 4 ft. wood, at \$7 a cord, what deduction should be made in the bill for what was sold for 30 cd.?

10. Formulate a rule for measuring wood or bark.

Lumber Measure

297. Lumber, *i. e.* boards, plank, and timber, is measured and sold by the board foot (bd. ft.), which is a square foot 1 in. thick, and contains 12 × 12, or 144, cu. in., or $\frac{1}{12}$ of 1 cu. ft.

Note. Unless otherwise specified, all square lumber is reckoned as at least 1 in. thick; all over 1 in. to 1¼ in. is reckoned as 1¼ in. thick; all over 1¼ to 1½ in. is reckoned as 1½ in. thick; all over 1½ in. to 2 in. is reckoned as 2 in. thick; all over 2 in. thick, is reckoned according to its thickness.

298. Illustrative Example. There are how many board feet in 20 planks 18 ft. long, 10 in. wide, and 2 in. thick?

Explanation. Each foot in length if it were 1 in. thick would contain $\frac{1}{12}$ of 1 bd. ft. But each plank is 18 ft. long and 2 in. thick and there are 20 planks.

Therefore, we multiply $\frac{1}{12}$ by 18, by 2, and by 20, $\frac{20 \times \overset{3}{\cancel{18}} \times 10 \times \cancel{2}}{\underset{\cancel{2}}{\cancel{12}}} = 600.$

Ans. 600 bd. ft.

1. Find the cost of the following bill of lumber:

20 pieces 10 ft. by 8 in. by 6 in., at \$25 per M (thousand feet).
40 pieces 12 ft. by 10 in. by 12 in., at \$30 per M.
28 pieces 8 ft. by 14 in. by 14 in., at \$36 per M.
3 pieces 18 ft. by 16 in. by 16 in., at \$28 per M.

2. What is the value of 12 boards, each 14 ft. long, 10 in. wide at one end and 8 in. wide at the other at $6\frac{1}{2}¢$ per foot?

NOTE. Take the average width. Boards are reckoned as 1 in. thick.

3. What is the cost of a pine plank 16 ft. long, 30 in. wide, and 3 in. thick, at \$60 per M?

4. At \$35 per M, what is the cost of 2-in. plank to floor a wharf that is 28 ft. wide and 132 ft. long?

5. Give the contents of the following pieces of lumber:

3 boards 12 ft. by 12 in. by 1 in.
2 boards 16 ft. by 6 in. by 1 in.
6 boards 14 ft. by 8 in. by 1 in.
4 2-in. plank 16 ft. by 8 in.
6 joists 20 ft. by 6 in. by 6 in.
3 joists 10 ft. by 6 in. by 4 in.

6. Find the cost of boards $\frac{7}{8}$ of an inch thick required to lay a platform 75 ft. by 48 ft., at \$18 per M.

See note, under Art. 297.

BRICKWORK

299. Bricks of common size are 8 in. long, 4 in. wide, and 2 in. thick.

In laying walls the mortar occupies about $\frac{1}{5}$ of the space, so that 22 bricks to a cubic foot is a fair estimate.

300. Illustrative Example. Find the number of bricks required for a wall 120 ft. long, 6 ft. high, and 24 in. thick.

EXPLANATION. Find the cubical contents of the wall and multiply by 22, the number of bricks to a cubic foot. $120 \times 6 \times 2 \times 22 = 31{,}680$, *Ans.*

1. What would be the cost of bricks at \$10.50 per M to lay one half of the above wall?

2. How many bricks 4 in. by 8 in. by 2 in. will be required to pave a court 30 ft. long and 10 ft. wide if they are laid with the long edge up?

3. Find the cost at $2.50 per M, for laying the brick walls of a cistern that is 12 ft. by 15 ft. inside measure, the walls being 9 ft. high and $1\frac{1}{2}$ ft. thick.

4. To fill out the corners of the above cistern, how many times the thickness of the wall must be added to the total length of the 4 sides?

NOTE. In estimating the amount of stock required for a building, a deduction of one half of the openings for doors and windows is usually made.

5. A building of buff brick is 42 ft. by 36 ft. outside. The walls are 18 ft. high and average $2\frac{1}{2}$ ft. thick. There are 8 windows, each $4\frac{1}{2}$ ft. by 8 ft., and 1 door, 9 ft. 10 in. by 8 ft., for which areas a deduction of $\frac{1}{2}$ is to be made. What is the cost of the brick at $16.50 per M?

6. Find the number of bricks in a chimney 30 ft. tall, 16 in. square outside, single thick, no allowance for mortar.

7. What would be the total cost of the chimney, using brick at $10 per M, labor at $10.50, mortar at $1.35 per M, bricks and hauling the bricks at $2?

STONEWORK

301. Stonework is estimated by the **perch**, which is equivalent in volume to a wall a rod ($16\frac{1}{2}$ ft.) long, $1\frac{1}{2}$ ft. wide, and 1 ft. thick, and which contains $24\frac{3}{4}$ cu. ft. In ordinary calculation, the perch is reckoned as 25 cu. ft.

302. Illustrative Example. At $2.50 a perch what is the cost of building a field wall 140 ft. long, 3 ft. high, 2 ft. thick?

EXPLANATION. The product of the dimensions divided by the number of cubic feet in a perch gives the number of perches. The quotient multiplied by the cost per perch, gives the total cost.

$$\frac{140 \times 3 \times 2 \times \$2.50}{25} = \$84.00. \quad \textit{Ans.}\ \$84.00.$$

Written Exercise

303. 1. What will be the cost at $1.00 a perch (25 cu. ft.) for a retaining wall 2 ft. wide at the base, $1\frac{1}{2}$ ft. wide at the top, 3 ft. deep, and 32 ft. long?

NOTE. Average the width.

2. How many perches (25 cu. ft.) are there in a stone pier 18 ft. high and 22 ft. × 14 ft. at the base and 20 ft. × 12 ft. at the top?

3. What will be the cost, at $8.00 a perch, of building a stone cellar 50 ft. long, 36 ft. wide inside, the walls being 14 ft. high and 3 ft. thick, allowance being made for two doors 6 ft. by 8 ft., and six windows 4 ft. by 2 ft.?

MEASURES OF CAPACITY

304. In estimating the capacity of bins, cisterns, boxes, etc., certain units of measure are used of which the following are most commonly employed.

1 T. of coal (2000 lb.)	= about 35 cu. ft.
1 bbl. ($31\frac{1}{2}$ gal.)	= about $4\frac{1}{2}$ cu. ft.
1 bu. (2150.42 cu. in.)	= about $1\frac{1}{4}$ cu. ft.
1 bu. heaped measure	= about $1\frac{1}{2}$ cu. ft.
1 gal. of water (231 cu. in.)	= about $\frac{4}{5}$ cu. ft. = $8\frac{1}{3}$ lb. weight.
1 pt. of water	= about 1 lb.
1 cu. ft. = about $\frac{4}{5}$ of a bushel	= about $7\frac{1}{2}$ gal.
1 cu. ft. of water = 1000 oz.	= $62\frac{1}{2}$ lb.

In measuring bulky fruits and vegetables, as apples and potatoes, the measures are heaped. In measuring small fruits, grain, etc., the measures are evened, or stricken with a straight edge.

Written Exercise

305. 1. How many bushels of potatoes can be put into a bin 6 ft. by 4 ft. by 5 ft.?

2. How many bushels of grain can be put into the above bin?

3. How much coal by weight can be put into a coal wagon whose body is 8 ft. long, 4 ft. wide, and 18 in. deep?

4. A cistern holding 1000 gal. is 6 ft. deep and 4 ft. wide. How long is it?

5. What is the weight of the water in a five-gallon jar that is half full?

6. What must be the depth of a grain box 18 in. by 30 in. to hold 6 bu. of oats?

7. A can of kerosene oil 16 in. by 14 in. by 9 in. contains how many gallons?

8. A jar weighs 8 lb. when empty and 60 lb. when filled with water. How many gallons does it hold?

9. There are how many gallons of water in a tank 10 ft. in diameter if the water is 6 ft. deep? What will be the weight of the water?

10. In a swimming tank 18 ft. wide, 30 ft. long, and having an average depth of water of $5\frac{1}{2}$ ft., there are how many gallons of water? Find the weight of the water.

MISCELLANEOUS PROBLEMS

306. 1. Find the capacity in bushels of a bin 12 ft. long, 7 ft. wide, and 6 ft. deep.

2. How much will it cost to carpet a room $17\frac{1}{2}$ ft. long by $13\frac{1}{4}$ ft. wide, with carpet 27 inches wide, at \$1.80 per yard?

3. Find the area in square feet of a triangular signboard whose base is $16\frac{1}{3}$ ft. and whose altitude is 10 ft.

4. There are how many square yards in the walls, floor, and ceiling of a room 20 ft. by 16 ft. and 11 ft. high?

5. Multiply $\frac{1}{2}$ of $7\frac{1}{4}$ ft. by $5\frac{3}{5}$.

6. Divide 675 rd. by .005 rd.

7. Change 14,962 inches to rods, yards, etc.

8. Add 18 yd. 2 ft. 11 in.; 14 yd. 6 in.; 4 ft. 4 in.; 9 in.

9. From 5 hr. 30 min. 13 sec. take 20 min. 20 sec.

10. If it takes 23 hr. 20 min. 45 sec. for 1 man to complete a piece of work, how long will it take for 4 men to do it?

11. Add $\frac{2}{3}$ yd., $\frac{3}{5}$ ft., $\frac{1}{10}$ in.

12. Add $63\frac{3}{10}$ rd., $3\frac{1}{3}$ yd., $2\frac{5}{6}$ ft., $5\frac{3}{5}$ rd., $7\frac{5}{12}$ ft.

13. From a piece of suiting containing $26\frac{3}{4}$ yd. there were sold as follows: 1 suit $6\frac{3}{4}$ yd., 2 suits $5\frac{1}{4}$ yd. each, 1 suit $6\frac{1}{3}$ yd. Find how much the remainder sells for at \$3.75 a yard.

14. Find $\frac{3}{11}$ of $18\frac{1}{4}$ lb.

15. Find $\frac{5}{8}$ gal. $\div \frac{1}{8}$.

16. 4 tons of flour will fill how many barrels (196 lb. each)?

17. What is the cost of laying a brick wall 6 ft. by 60 ft., at \$20 per M, if it takes 6 bricks to a square foot of surface?

18. What is the cost of a farm 200 rd. long and 140 rd. wide at \$46 an acre?

19. Find the entire surface of a 6 in. cube.

20. If a man can build $16\frac{2}{3}$ rd. of wall in 6 da., how many rods can he build in $4\frac{1}{2}$ da.?

21. A man owning $\frac{5}{6}$ of an acre of land sold $\frac{1}{2}$ of his share for \$300. What was the value of the whole acre?

22. A piece of land containing $1\frac{1}{4}$ A. is 20 rd. long. How much will it cost to fence the piece at 65¢ a yard?

23. At 35¢ a square yard, find the cost of painting the walls of a room $23\frac{1}{2}$ ft. long, 15 ft. wide, and $11\frac{1}{2}$ ft. high.

24. A 30-yd. piece of carpeting had $5\frac{1}{4}$ yd., $6\frac{1}{2}$ yd., and $8\frac{1}{3}$ yd. cut from it. How much remained?

25. From a 2-ft. cube of wood how many 6-in. cubes could be cut, allowing no waste?

26. A woodpile 6 ft. high and 4 ft. wide contains 10 cd. How long is it?

27. A lot of land 80 ft. by 120 ft. sells for $1000. What is the price per square foot?

28. $\frac{2}{3}$ of the distance from P to B is 60 mi. What is $\frac{13}{15}$ of the distance?

29. An athletic field contains $1\frac{3}{8}$ A. It is 195 ft. wide. How long is it?

30. A tank $12\frac{1}{2}$ ft. long, 10 ft. wide, contains 1410 cu. ft. How deep is it, and how many gallons will it hold?

31. What is the value, at $8\frac{1}{4}$ a cord, of a pile of wood 16 ft. long, 4 ft. wide, and 12 ft. high?

32. How much will it cost to fence a field $\frac{1}{8}$ mi. square at $1.25 a rod?

33. An express train that covers 85 mi. in 2 hr. travels how many feet per minute?

34. Find the time required to row a distance of $12\frac{1}{2}$ mi. at the rate of $1\frac{3}{4}$ mi. in 15 min.

35. Find the cost, at $18 per M, of 2 pieces of timber 6 in. × 8 in. × 28 ft.; of 2 pieces of timber 6 in. × 8 in. × 32 ft.

36. If 8.5 yd. of cloth cost $38.25, how much will $12\frac{1}{4}$ yd. cost?

37. 15 bu. of grain cost $9. How much can be bought for $100 at the same rate?

38. An engine consumed 340 lb. of coal in 10 hr. How many tons will the same engine consume in 30 da. of 8 hr. each?

39. How many lots, each containing 40 sq. rd., can be made from an $11\frac{1}{2}$-A. tract of land?

40. Change .16$\frac{2}{3}$ lb. to a common fraction of a pound.

41. Change $\frac{7}{12}$ gross to a decimal of a gross.

42. How many shingles will be required for a roof that is 24 ft. long and 18 ft. from eaves to center on each of 2 sides?

43. What is the area of a triangle 24 rd. on the base, and having an altitude of 20 rd.?

44. From a pile of wood 40 ft. long, 8 ft. high, and 4 ft. wide was sold at different times $2\frac{1}{2}$ cd., $\frac{3}{4}$ of a cord, $4\frac{1}{4}$ cd. At \$8 a cord what was the value of the amount sold and the value of the remainder?

45. If I own $\frac{2}{3}$ of a lot of land and $\frac{2}{5}$ of my share is worth \$150, what is the value of the whole lot?

46. Divide 4 ten-thousandths of an inch by 30.

47. Divide 650 ft. by 25 thousandths of a foot.

48. From $\frac{3}{4}$ of a rod take 6 ft. 4 in.

49. 35 A. 125 sq. rd. is to be divided into 50 lots. What will be the size of each lot?

50. What is the value of 3 bu. 3 pk. 1 qt. of beans at \$2.25 a bushel? How much will the entire quantity sell for at 9¢ a quart?

51. At 28¢ a square foot, what would be the value of a lot of land containing $40\frac{1}{2}$ sq. rd.?

52. If a pile of wood containing 100 cd. is piled 8 ft. high and cut 4 ft. in length, how long is the pile?

53. At \$45 per M for floor boards how much will the material cost for a floor 25 ft. square?

54. Find the difference in years, months, and days between Dec. 13, 1892, and Aug. 4, 1907.

55. July 12, 1907, Mr. George Pepper bought of the Independent Coal and Wood Company 11 T. of furnace coal at \$6.75 per ton, 6 T. of stove coal at \$7.50 per ton, $\frac{1}{2}$ cd. of hard wood at \$8 per cord, and $\frac{1}{4}$ cd. of pine wood at \$6 per cord.

Mr. Pepper was also charged 45¢ a ton extra for putting the coal into the cellar. Make out a bill for the above from the Independent Coal and Wood Company to Mr. George Pepper.

56. In taking 5744 steps of an average length of 2 ft. 10 in., what distance is traveled?

57. How does $\frac{2}{5}$ of 60 ft. compare in size with $\frac{3}{10}$ of 60 ft.? with $\frac{3}{20}$ of 60 ft.?

58. What part of a day is 18 hr. 30 min.?

59. What is the cost of 3.008 T. of hay at the rate of $13.50 per ton?

60. What is the value of 10,250 ft. of boards at the rate of $44 per M?

61. If 348 people are thrown out of work for 3 wk. and 3 da. in consequence of a fire, what is the entire loss in wages if each one averages a daily wage of $2.25?

62. How much coal is there in a bin 5 ft. × 6 ft., the coal being $4\frac{1}{2}$ ft. deep?

63. What is the cost of 3200 sheets of paper at $10.50 per ream?

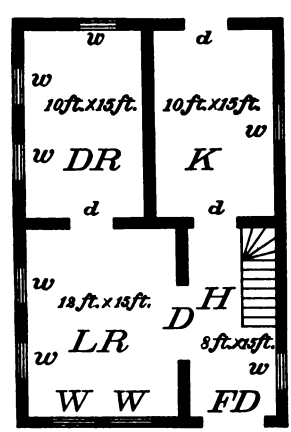
This is the plan of the first floor in a cottage. The rooms are 9 ft. high.

Rooms. *LR*, Living Room; *DR*, Dining Room; *K*, Kitchen; *H*, Hall.

Doors. *FD*, the front door, 5 ft. by 8 ft.; *D*, 6 ft. by 7 ft. *d*, *d*, *d*, $3\frac{1}{2}$ ft. by 7 ft.

Windows. *W*, *W*, $3\frac{1}{2}$ ft. by $7\frac{1}{2}$ ft. The others, *w*, 3 ft. by 6 ft.

Baseboards. All 9 in. high.



64. Find the number of square feet in the baseboard of the living room, passing under all the windows.

65. Find the number of square feet in the windows and doors of the living room.

66. How many square feet are in the walls and ceiling of the kitchen, deducting for baseboards, doors, and windows?

67. At 40¢ per square yard, find the cost of plastering the dining room walls, deducting half the area of the doors and windows.

68. What is the cost of kalsomining the ceiling of the living room and dining room at 9¢ per square yard?

69. How many square feet of glazing are there in the above plan, *W*, *W* having each 4 panes 42 in. by 18 in., and the others having each 4 panes 31 in. by 15 in.?

70. How many yards of carpet, 1 yd. wide, will be required to carpet the living room? How many yards $\frac{3}{4}$ of a yard wide will be required, no allowance being made for matching figures?

71. How much will it cost to excavate a cellar for the cottage 22 ft. wide and 33 ft. long to a depth of 6 ft. at 25¢ per cu. yd.?

72. What would be the cost of laying the foundation walls 8 ft. high and 18 in. thick at 35¢ a perch of 25 cu. ft., no allowance being made for corners or openings?

73. How many rolls of paper 18 in. wide, 3 strips to a roll, will be required to paper the living room, deducting the width of the windows and doors, and reckoning a whole roll for any part of a roll required? (Art. 264.)

74. What will be the cost of wall paper for the living room at 65¢ a roll?

75. How many rolls of paper 18 in. wide, 3 strips to a roll, will be required to paper the dining room, deducting for doors and windows, as in Ex. 73? Find the cost of this paper at 50¢ a roll.

76. If an 18 in. border of hard wood is laid around the living room floor, how many square feet of hard wood are laid?

NOTE. The pupil will be aided in this example by first drawing a plan of the flooring.

77. What must be the dimensions of carpeting to fit the remainder of the floor of the dining room exclusive of the hard-wood border, and how many breadths of Wilton $\frac{3}{4}$ of a yard wide, and laid lengthwise, will be required? How much will the carpet cost at \$1.625 per yard?

78. How many yards of bordering 9 in. wide will be required for a rug measuring before the border is put on 3 ft. long and $2\frac{1}{2}$ ft. wide?

NOTE. The corners must be counted twice, as half of them will be cut off when attached to the rug.

79. How many square yards of linoleum will be required for the hall, deducting for the staircase 10 ft. in length and $4\frac{1}{2}$ ft. in width?

80. Make similar problems about carpeting dining room and laying linoleum in the kitchen.

81. If a piazza is built 5 ft. wide on the front of the house in the plan on p. 203, and 6 ft. wide on the side, how many square feet are there in the floor of the piazza, reckoning the corner also, the house measuring outside 32 ft. in length and 22 ft. in width?

82. The cottage represented on p. 203 stands upon a lot 12 rd. 2 ft. long, and 6 rd. 1 ft. wide. What was the cost of the land at 5¢ a square foot?

83. What was the cost of grading the above lot at \$1.75 per square of 100 sq. ft.?

84. At 50¢ each, find the cost of planting trees 25 ft. apart on the boundary of the lot, beginning at a corner.

307. Read the following fractions: $\frac{5}{100}$, .05, $\frac{26}{100}$, .26, $\frac{37}{100}$, .37, .04, $\frac{4}{100}$, .07, $\frac{7}{100}$, .32, $\frac{32}{100}$.

In the above fractions, what is the common denominator?

Fractions like $\frac{5}{100}$, .26, $\frac{37}{100}$, .04, $\frac{7}{100}$, etc., which have 100 for a denominator, are also written thus, 5 per cent (the phrase *per cent* means *hundredths*) or 5 % (% means *per cent*), 26 per cent or 26 %, 37 per cent or 37 %, 4 per cent or 4 %, etc.

.32, $\frac{32}{100}$, 32 per cent, and 32 % of any quantity are equal.

It is often convenient to reason in hundredths — reducing other fractions to fractions with 100 for a denominator.

Oral and Written Exercise

308. Illustrative Example. Reduce $\frac{5}{8}$ to hundredths and also write it as per cent.

WORK

$\frac{1}{8}$ of $\frac{100}{100} = \frac{12\frac{1}{2}}{100} = .12\frac{1}{2}$

$\frac{5}{8} = 5 \times .12\frac{1}{2} = .62\frac{1}{2} = 62\frac{1}{2}$ %

EXPLANATION. $\frac{1}{8}$ of any quantity is equal to $\frac{12\frac{1}{2}}{100}$ of it, or $.12\frac{1}{2}$ of it; $\frac{5}{8}$ of a quantity = $.62\frac{1}{2}$, or $62\frac{1}{2}$% of it. *Ans.* $62\frac{1}{2}$%.

Reduce to hundredths and per cents:

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{7}{10}$.

2. $\frac{1}{8}$, $\frac{11}{10}$, $\frac{1}{12}$, $\frac{5}{6}$, $\frac{3}{6}$, $\frac{2}{7}$, $\frac{5}{7}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{9}$, $\frac{1}{11}$.

3. $\frac{2}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{2}{11}$, $\frac{7}{11}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{11}{12}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{6}{7}$, $\frac{7}{9}$.

4. Out of a lot of 40 melons, 8 were unsalable. What part of the lot was unsalable? How many hundredths of the lot were unsalable?

5. Out of $100 a man lost $10. What part of his money did he lose? What per cent?

6. A drover had 200 lambs and sold $\frac{1}{4}$ of them. How many hundredths of them did he sell? What per cent? How many lambs did he sell?

Oral Exercise

309. Illustrative Example. A nurseryman set out 2820 trees, but lost 15 % of them by drought. How many did he lose?

WORK

(1)
$$\begin{array}{r} 2820 \\ .15 \\ \hline 141\ 00 \\ 282\ 0 \\ \hline 423.\not{0}\not{0} \end{array}$$

EXPLANATION (1). $100\% = 2820$.
$1\% = \frac{1}{100}$ of $2820 = 28.20$.
$15\% = 15 \times 28.20 = 423$.
In practice we say .15 of $2820 = 423$, because 15 % of a quantity is the same as .15 of it.

(2) $15\% = \frac{3}{20}$

$$\begin{array}{r} 141 \\ \hline 2\,|\,0\,)\,282\,|\,0 \end{array} \quad 3 \times 141 = 423$$

(2). 15 % of a quantity is the same as .15, or $\frac{15}{100}$, or $\frac{3}{20}$, of the quantity.
$\frac{1}{20}$ of $2820 = 2820 \div 20 = 141$.
$\frac{3}{20}$ of $2820 = 3 \times 141 = 423$.

(3) $15\% = \frac{15}{100} = \frac{3}{20}$

$$\begin{array}{r} 141 \\ \hline \not{2}\not{8}\not{2}\not{0} \times 3 \\ \hline \not{2}\not{0} \end{array} = 423$$

The reasoning in (3) is the same as in (2), but the work is performed by cancellation.

Ans. 423.

In solving problems, the per cent given may be used as above, either as a decimal or as a common fraction. Use whichever is the quicker method.

Find:

1. 1 % of $600, $300, $1000, $4000, $50, $5.
2. 2 % of $600, $300, $1000, $4000, $50, $5.
3. 3 % of $600, $300, $1000, $4000, $50, $5.
4. 4 % of $200, $20, $2, $10, $15, $25.
5. 20 % of $1, $10, $100, $1000, $200, $20, $2000.
6. 25 % of $1, $10, $100, $1000, $200, $20, $2000.
7. 75 % of $1, $10, $100, $1000, $200, $20, $2000.

Written Exercise

310. **1.** Express as decimals: 6%, 8%, 25%, 9%, 4%, 18%, 20%, 10%, 13%, 11%.

2. Express each of the above as a common fraction in the lowest terms.

Expressions like $\frac{1}{2}$ of 1 per cent, $\frac{1}{4}$ of 1 per cent, in which the per cent is less than 1 per cent, are written decimally thus: .00$\frac{1}{2}$ ($\frac{1}{2}$ of 1 per cent), .00$\frac{1}{4}$ ($\frac{1}{4}$ of 1 per cent).

Express decimally:

3. $\frac{1}{2}$ of 1 per cent, $\frac{3}{4}$ of 1 per cent, $\frac{5}{8}$ of 1 per cent.

4. $\frac{1}{6}$ of 1 per cent, $\frac{1}{6}$ of 1 per cent, $\frac{1}{8}$ of 1 per cent.

Expressions like $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, $8\frac{1}{3}\%$, etc., are written decimally thus: .12$\frac{1}{2}$, .16$\frac{2}{3}$, .08$\frac{1}{3}$, etc. As common fractions they are written thus: $\frac{12\frac{1}{2}}{100}$, $\frac{16\frac{2}{3}}{100}$, $\frac{8\frac{1}{3}}{100}$.

Express as decimals and as common fractions:

5. $9\frac{1}{2}\%$, $33\frac{1}{3}\%$, $12\frac{1}{2}\%$, $16\frac{2}{3}\%$.

6. $4\frac{2}{3}\%$, $8\frac{1}{3}\%$, $62\frac{1}{2}\%$, $6\frac{1}{4}\%$.

7. Express $6\frac{1}{4}\%$ as a common fraction in its lowest terms.

SOLUTION: $6\frac{1}{4}\% = .06\frac{1}{4} = \frac{6\frac{1}{4}}{100} = 6\frac{1}{4} \div 100 = \frac{25}{4} \div 100 = \frac{25}{4} \div \frac{400}{4} = \frac{25}{400} = \frac{1}{16}$.

WRITTEN WORK: $\frac{6\frac{1}{4}}{100} = \frac{25}{4} \div \frac{400}{4} = \frac{25}{400} = \frac{1}{16}$. *Ans.* $\frac{1}{16}$.

Express the following as common fractions in the lowest terms:

8. $2\frac{1}{2}\%$, 10%, 25%, $62\frac{1}{2}\%$, $6\frac{1}{4}\%$.

9. $12\frac{1}{2}\%$, $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, $6\frac{2}{3}\%$, $16\frac{2}{3}\%$.

10. 40%, 75%, $8\frac{1}{3}\%$, 20%, 50%.

11. $37\frac{1}{2}\%$, $87\frac{1}{2}\%$, $83\frac{1}{3}\%$, 30%, 60%.

12. 90%, 111%, 150%, 125%, 6%.

311. Memorize the following table of equivalents, to be given in any order:

$1\ \% = \frac{1}{100}$	$10\ \% = \frac{1}{10}$	$33\frac{1}{3}\ \% = \frac{1}{3}$	$70\ \% = \frac{7}{10}$
$2\ \% = \frac{2}{100}$ or $\frac{1}{50}$	$12\frac{1}{2}\ \% = \frac{1}{8}$	$37\frac{1}{2}\ \% = \frac{3}{8}$	$75\ \% = \frac{3}{4}$
$4\ \% = \frac{1}{25}$	$15\ \% = \frac{3}{20}$	$40\ \% = \frac{2}{5}$	$80\ \% = \frac{4}{5}$
$5\ \% = \frac{1}{20}$	$16\frac{2}{3}\ \% = \frac{1}{6}$	$50\ \% = \frac{1}{2}$	$83\frac{1}{3}\ \% = \frac{5}{6}$
$6\frac{1}{4}\ \% = \frac{1}{16}$	$20\ \% = \frac{1}{5}$	$60\ \% = \frac{3}{5}$	$87\frac{1}{2}\ \% = \frac{7}{8}$
$6\frac{2}{3}\ \% = \frac{1}{15}$	$25\ \% = \frac{1}{4}$	$62\frac{1}{2}\ \% = \frac{5}{8}$	$90\ \% = \frac{9}{10}$
$8\frac{1}{3}\ \% = \frac{1}{12}$	$30\ \% = \frac{3}{10}$	$66\frac{2}{3}\ \% = \frac{2}{3}$	

The per cent should be found in the shortest way by using the per cent either as a decimal or as a common fraction.

Oral Exercise

312. I. To find any per cent of a number.

Illustrative Example. What is $37\frac{1}{2}\ \%$ of \$328?

WORK

(1) \$328 × $.37\frac{1}{2}$ = 1 64 + 22 96 + 98 4 = \$123.00

(2) $37\frac{1}{2}\ \% = \frac{3}{8}$; $\frac{1}{8}$ of \$328 = \$41; $\frac{3}{8}$ of \$328 = 3 × \$41 = \$123, *Ans.*

(3) $\frac{3}{\cancel{8}} \times \$\overset{41}{\cancel{328}} = \$123$

1. What is $2\ \%$ of \$60? $3\ \%$ of \$100? $7\ \%$ of \$40?

2. What is $6\frac{1}{4}\ \%$ of 32? $8\frac{1}{3}\ \%$ of \$48? $12\frac{1}{2}\ \%$ of 64?

3. By selling goods at $25\ \%$ above cost how much was gained if the cost was \$12? \$40? \$16? \$20? \$32?

4. By selling goods at $10\ \%$ below cost what was the loss if the cost was \$30? \$20? \$50? \$40?

5. Mr. A. had \$800 in the bank. He drew out $87\frac{1}{2}\ \%$ of it. How much did he draw out?

Written Exercise

313. 1. A farmer had 968 sheep and sold 25% of them. How many did he sell?

2. A man planted a field of corn and gave $37\frac{1}{2}\%$ of the crop to the owner for the use of the land. He raised 2432 bu. of corn. How many bushels did the owner of the land receive?

3. Mr. Sanborn bought 600 cd. of wood at \$5 a cord. He sold $66\frac{2}{3}\%$ of it at \$7 a cord, the remainder at cost. What sum was received for all?

4. A field is 20% of a mile long and 14 rd. wide. Find the number of square rods in the field.

5. What is the cost of $83\frac{1}{3}\%$ of 144 T. of hay at \$15 per ton?

6. What is $1\frac{1}{2}$ times 800 lb.? What is 150% of 800 lb.?

7. John Bates had \$1700 in the bank. He withdrew 60% of it at one time and 40% of it at another time. What sum was left in the bank?

8. A farmer raised 560 bu. of potatoes for market. He sold $14\frac{2}{7}\%$ of the lot at 50¢ a bushel, and the rest at 70¢ a bushel. How much money did he receive?

9. Mr. Rowe bought a horse for \$275 and sold it at 20% profit. What was his gain?

10. A real estate dealer bought 90 A. of land at \$160 an acre and sold it at $12\frac{1}{2}\%$ profit. What was the selling price?

11. A trader bought flour at \$8 a barrel and sold it at a loss of 5%. How much did he lose on 24 bbl.?

12. A house and lot cost \$3800. Find the selling price to gain $6\frac{1}{4}\%$.

13. A furniture dealer sells a lot of school furniture for $4200. Of this $15\frac{1}{2}\%$ is profit. What is the amount of his profit?

14. There are 26 letters in the English alphabet. Of these $23\frac{1}{13}\%$ are vowels. How many are vowels?

15. A man whose salary is $3200 per year spends $14\frac{1}{2}\%$ of it for house rent. What is the amount paid for rent?

16. $83\frac{1}{3}\%$ of 3600 bu. of potatoes were sold for 80¢ a bushel. How much was received for them?

17. From X to Y is 480 mi. If there is an electric car line for 75% of the distance, how long is it?

18. A music reader contains 354 pages, and $33\frac{1}{3}\%$ of the book is given up to exercises. There are how many pages of exercises?

19. On an investment of $5500 a speculator makes a profit of $7\frac{1}{2}\%$. What is his profit?

20. From a case that contains 225 sheets of tin, 5% were used at one time, 10% at another, and 5% at another. How many sheets were used in all?

21. A sheet of lead 24 ft. long, 5 ft. wide, and $\frac{1}{8}$ of an inch thick weighed at the rate of 6 lb. per surface foot. What was its total weight? If a sheet of the same length and width weighed 50% as much, it would be how thick and would weigh how much?

22. If bronze consists of 10% tin and the remainder copper, there are how many pounds of tin in 850 lb. of bronze?

23. Find $\frac{1}{8}\%$ of 8400 A.; of 9648 A.; of 3240 A.

24. Find $\frac{1}{5}\%$ of $500; of $2500; of $6255.

25. Find $\frac{1}{4}\%$ of 1296 lb.; of 2448 lb; of 7208 lb.

26. Find $\frac{1}{3}\%$ of $750; of $2400; of $9603.

Oral Exercise

314. II. To find what per cent one number is of another.

1. What part of 25 is 5? How many hundredths of 25 is 5? What per cent of 25 is 5?

2. What part of 15 is 3? What per cent of 15 is 3?

3. What part of 42 is 7? What per cent of 42 is 7?

4. What per cent of

30 is 15?	\$48 is \$12?	150 is 50?
45 is 9?	\$36 is \$9?	100 is 70?
60 is 12?	\$42 is \$7?	27 is 9?
72 is 8?	\$56 is \$8?	32 is 8?
96 is 12?	\$81 is \$9?	50 is 2?

5. What per cent of 40 gal. is 10 gal.?

6. What per cent of 1 gro. is 4 doz.?

7. What per cent of time is lost, if 6 da. out of 20 are lost?

8. A milkman buys milk at 5¢ and sells it at 7¢ a quart. What per cent does he gain?

9. Fred had 90 chickens, but during a cold rain storm 18 of them died. What per cent of his chickens did he lose?

10. A tree that was 15 ft. high last year grew 9 ft. this year. By what per cent did its height increase?

11. What per cent of 50 is 40? of 40 is 50?

315. Illustrative Example. What per cent of \$600 is \$150?

WORK

$\frac{150}{600} = \frac{1}{4} = 25\,\%$

Or, .25 or 25 %

600)150.00

1200

3000

3000

EXPLANATION. Since \$150 is $\frac{150}{600}$ of \$600 and $\frac{150}{600}$ is $\frac{1}{4}$, or .25, then \$150 is 25 % of \$600.

In the second method, the common fraction is changed to a decimal by dividing the numerator by the denominator. *Ans.* 25%.

Written Exercise

316. What per cent of

1. \$75 is \$15?
2. \$1000 is \$25?
3. \$900 is \$200?
4. 325 yd. is 75 yd.?
5. 365 bu. is 125 bu.?
6. 1200 A. is 240 A.?

7. A farmer raised 100 bu. of potatoes and sold 40 bu. What part of the whole did he sell? What per cent of the whole?

8. There are 320 rd. in a mile. 280 rd. is what per cent of a mile?

9. A cow that cost \$45 was sold for \$60. Find the gain; the gain per cent.

10. Goods bought for \$2200 were sold for \$2310. What was the gain per cent?

11. If I buy land at \$640 an acre and sell at \$5 a square rod, what per cent do I gain?

12. A painter bought an old wagon for \$25. After painting it he sold it for \$35. What was the per cent of profit?

13. A boy bought a two-bladed knife for \$.45. After breaking one blade he sold it for \$.25. What per cent did he lose?

14. A farmer gave a boy 4 % of all the strawberries that he could pick to pay for his work. The boy received 9 baskets. How many baskets did he pick?

15. I gained \$150 on steel sold for \$750. What was the per cent of gain?

16. What per cent of an acre is a lot of land 33 ft. square?

17. A merchant's profits on the sale of some goods were \$65. The total amount of the sale was \$585. What was his per cent of profit?

18. The following statement gives the quantity of wool imported into the port of New York for the week ending January 12, as reported by the custom house.

Find, to the nearest tenth of a per cent, what per cent the amount of wool imported from each country is of the total amount imported.

	BALES		BALES
England	630	Panama	101
Belgium	81	Netherlands	168
Germany	2444	Scotland	628
China	1342		

19. Concerning the exports of pig iron from Great Britain, the following table will show by how much the shipments of 1906 exceeded those of 1905 to the chief countries.

Find the per cent of increase for each country to the nearest tenth of a per cent.

Find the per cent of increase in the total amount for 1906 over the total for 1905.

	TONS	
	1906	1905
Germany and Holland	447,087	152,603
America	114,351	52,716
France	65,691	26,954
Belgium	54,976	20,553
Italy	100,303	75,837
Norway and Sweden	78,068	56,697
Scotland	410,649	346,039

20. A quantity of white paper was made from 2500 lb. of cotton rags and 700 lb. of linen rags. What per cent of the paper was linen?

21. A farm consists of 230 A. of woodland, 135 A. of pasture, 50 A. of marsh land, and 85 A. of tillage land. What per cent of the whole farm is each part?

22. A rectangular lot is 1250 ft. long and 900 ft. wide. What per cent is the width of the length?

23. A man bought a horse for $175 and sold it so that he gained $15. What per cent did he gain?

24. B had $560 in the bank and withdrew $125.44. What per cent did he withdraw?

25. A merchant agreed to deduct $24 from a bill amounting to $800. What per cent did he deduct?

26. The area of Spain is 190,000 sq. mi. and the area of Italy is 92,000 sq. mi. What per cent is the area of Italy of the area of Spain?

27. A piece of pine weighing 46 lb. when soaked a week weighed 72 lb. What was the gain per cent in weight?

28. $2000 was paid for some goods. Half of them were sold for $645 and the rest for $555. What per cent was lost?

29. In 1905 the French navy had 326 ships, the German navy had 206 ships. What per cent was the number of German ships of the French ships?

Oral Exercise

317. III. To find a number when a per cent of it is given.

1. If $\frac{2}{3}$ of a number is 12, what is $\frac{1}{3}$ of it?

2. If $\frac{3}{5}$ of a number is 18, what is $\frac{1}{5}$ of it? What is $\frac{5}{5}$, or the whole number?

3. If $\frac{6}{100}$ of a number is 24, what is $\frac{1}{100}$ of it? What is $\frac{100}{100}$ of it, or the whole number?

Read this question, using the term per cent instead of hundredths.

4. If 10 % of a number is 30, what is 1 % of it?

5. If $62\frac{1}{2}$ %, or $\frac{5}{8}$, of a number is 25, what is the number?

6. If $87\frac{1}{2}$ % of a number is 56, what is the number?

Written Exercise

318. Illustrative Example. Find the number of which \$320 is 40 %.

WORK

(1) \$8 is 1 %
40)\$320

\$8 is 1 %
100
\$800 is 100 %

EXPLANATION. Since \$320 is 40%, 1% is $\frac{1}{40}$ of \$320, or \$8; and 100%, or the whole number, is 100 × \$8, or \$800.

In the second method 40% is reduced to the common fraction $\frac{2}{5}$. *Ans.* \$800.

(2) \$160 is $\frac{1}{5}$
2)\$320 is $\frac{2}{5}$

\$160 is $\frac{1}{5}$
5
\$800 is $\frac{5}{5}$

Find the number of which:

1. 150 is 25 %
2. 267 is 37½ %
3. \$900 is 75 %
4. \$560 is 87½ %
5. \$450 is 90 %
6. 620 is 80 %

7. The cost of some goods was \$150, which was 83⅓ % of the selling price. What was the selling price?

8. Damaged goods were sold for \$450, which was 10 % below cost. Find the cost.

9. 8 % is lost by selling goods for \$460. What was paid for the goods?

10. I bought eggs at 30 ¢ a dozen and sold them at 3 ¢ each. Find the gain per cent.

11. Cost \$90; selling price \$99; find the gain per cent.

12. Selling price \$105; gain \$15; find the gain per cent.

13. Selling price $750; gain 25 %; find cost.

14. I sold property for $2560, which was 20 % below cost. What was the cost?

15. A lot of land 10 rods square is what per cent of $2\frac{1}{2}$ acres?

16. By selling goods at $11\frac{1}{9}$ % below cost I lost $190. Find the cost. Find the selling price.

17. In a certain school there are 200 girls, and this number is 40 % of the whole number of pupils. How many pupils are there in the school?

18. A boy sold a second-hand bicycle for $11 and made a profit of 10 %. How much did he pay for the bicycle?

19. A man is now receiving a salary of $27 a week. This is $12\frac{1}{2}$ % more than he formerly received. What was his former salary?

20. A boy weighs 88 lb., which is 10 % more than he weighed one year ago. What was his weight a year ago?

21. A dealer lost 10 % by selling paper at 27 cents a package. Find the cost of the paper.

22. A city lot was sold for $3500, which was $16\frac{2}{3}$ % less than it cost. How much was paid for it?

23. The population of a city this year is 550,000, which is 10 % more than it was last year. What was last year's population?

TRADE DISCOUNT

319. A very common application of percentage is made in transactions like the following:

1. Abbe & Sawyer, Hardware Dealers, buy of the Empire Saw Works 10 doz. back saws listed in the catalogue at $16 per dozen, subject to a discount of 40 % off the catalogue price. Since 40 % of $16 = $6.40 and $16 − $6.40 = $9.60, the actual cost of the saws is $9.60 a dozen, and of the 10 doz. $96.

2. Small Bros., Tailors, buy of Case & Locke, wholesalers, a lot of suitings amounting to $784. This bill is subject to a discount of 2% for cash within 10 days, thus making the actual amount paid $784, less $15.68, or $768.32.

Any deduction from a customary or fixed price, or from any amount due, is called a **discount.**

A deduction from the list price of goods is called a **trade discount** or a **commercial discount.**

The price remaining after deducting all discounts is called the **net price.**

Discount is usually reckoned at a given rate per cent called the **rate of discount.**

320. Illustrative Example. A bill of $364 was discounted at 5½%. What was the discount and the amount paid?

WORK

$364, amount of bill
.05½, rate of discount
182
1820
$20.02, discount
$364.00
20.02
$343.98, amount due

EXPLANATION. 5½% of $364 = $20.02, the amount of discount. $364 less $20.02 = $343.98, the amount due.

Ans. $20.02; $343.98.

321. Illustrative Example. I bought 5000 envelopes at $10.50 per M, with discounts of 10% and 5% off. What was the cost?

	WORK	
$10.50	$52.50	
5	5.25	
$52.50	$47.25	$47.25
.10	.05	2.36
$5.2500	$2.3625	$44.89

EXPLANATION. 5000 envelopes at $10.50 per M cost 5 × $10.50 = $52.50. 10% of $52.50 = $5.25, the first discount. $52.50 − $5.25 = $47.25, the first remainder. 5% of $47.25 = $2.36, the second discount. $47.25 − $2.36 = $44.89, cost.

Ans. $44.89.

Oral Exercise

322. 1. Find the sale price for:

Elgin silver case watches at $10.50, at 10 % discount.
Elgin gold case watches at $27, at 20 % discount.
Gold-filled 2 years watches at $12, at 25 % discount.
Pocket books (leather) at $3.50, at 15 % discount.
Plated tablespoons at $3.60 per dozen, at 2 % discount.
Eight-day alarm clocks at $2.25 each, at 20 % discount.

2. Find the cost to the customer of the furs in the following advertisement:

CHRISTMAS SALE OF FURS

Natural Squirrel Sets, $15.00, at 10 % discount.
Blended Squirrel Sets, $15.00, at 20 % discount.
Sable and Isabella Fox Sets, $25.00, at 5 % discount.
Japanese Mink Sets, $21.00, at $33\frac{1}{3}$ % discount.
Black Caracul Sets, $10.00, at 10 % discount.
Russian Pony Sets, $22.50, at $14\frac{2}{7}$ % discount.
Eastern Mink Sets, $35.00, at 20 % discount.
Royal Ermine Sets, $75.00, at 10 % discount.

3. All articles here advertised are offered at a discount of 10 % off the value. Find the sale price of each.

Women's Desks, value $16.75
Fancy Rockers, value $6.75
Fancy Tables, value $8.75
Hall Chairs, value $7.50
Brass Beds, value $60.00
Plant Stands, value $9.00
Costume Poles, value $12.50
Morris Chairs, value $13.00
Phonograph Records, value $5.00
Desk Chairs, value $7.75
White Enamel Desks, value $14.00

Written Exercise

323. **1.** How much is paid for a set of Howe Wagon Scales, listed at $270, with discounts of 10% and 5% and an extra discount of 5%?

2. How much is paid cash for 6 T. Jupiter steel at $2\frac{1}{4}¢$ per pound, and $\frac{3}{4}$ T. sleigh shoe steel at $2\frac{15}{100}¢$ per pound, 2% discount for cash in 10 days?

3. What is the selling price of a boiler feed pump listed at $75, 60% off?

4. A 3% discount on a bill of goods amounted to $75. What was the amount of the bill?

5. A bill for 1000 copies of Longfellow's Complete Poems, less a 3% discount, was $1600.50. What was the amount of the bill and the discount?

6. What was paid on an introductory order for 1200 copies of the Normal Geography listed at 90 cents each, with $\frac{1}{5}$ off for introductory order? ($\frac{1}{5}$ = what per cent?)

7. In collecting a bill for $257 an attorney discounted 3% for a cash settlement. How much did he pay over?

8. What is the rate of discount on a $1.50 article sold at $1.25?

9. How much was paid for a rug marked $175, with discounts of $12\frac{1}{2}$% and 2%?

10. Brass cribs are marked $33.90, with a discount of $8\frac{1}{3}$%. Find the actual selling price.

11. Mahogany library tables are listed at $29.90, with a discount of $16\frac{2}{3}$%. How much will two tables cost me?

12. Sideboards of polished quartered oak, value $31.75 each, are offered for sale at a discount of $33\frac{1}{3}$%. What is the sale price?

13. A $5000 automobile was bought for cash at 10% off. How much was paid for it?

14. A typewriter is sold for $80, with a discount of 30% for schools. Find the cost of 6 such typewriters for a high school.

15. A dealer orders a half gross of Handy tool kits listed at $1.50, with a discount of 30% and 10%. He sells the lot at $2.00 each. What is his profit?

16. I bought goods listed at $4500 at a discount of 10% and 20%. What was the net cost?

17. Goods listed at $960 were bought at $\frac{1}{8}$ and 5% off. What was the net cost?

18. The list price of a lot of hardware was $1500. It was bought at $33\frac{1}{3}$% and 10% off, and sold at a net gain of 20%. Find the selling price.

19. Find the cost of 150 cases of boots at $24 a case, less $8\frac{1}{3}$% discount.

20. I am offered goods listed at $1800 at 20% and 10% off, or at one discount of 30%. Which is the better price for me, and how much better?

21. I marked goods that cost $900 at 20% profit, but I discounted $8\frac{1}{3}$% from the marked price for cash. What was the selling price?

22. Mr. Underwood sold a bill of goods for $800 subject to a discount of $11\frac{1}{9}$%. What was the net sum received for the goods?

23. Find the cost of 9 dozen hats at $18 a dozen, less a discount of $6\frac{1}{4}$%.

24. I marked clothing costing $1600 at 40% advance, but made 20% discount on the asking price. What was the actual selling price?

324. Every town and city needs to raise money for schools, for the construction and repair of streets, roads, and bridges, for the care of the poor, for salaries of officials, etc.

Every county needs money to pay the salaries of the county commissioners, the sheriff, and other county officers, also for building and keeping in repair the courthouse, jail, and other county buildings.

Every state needs money for the purpose of paying the salaries of the governor, the lieutenant governor, members of the legislature, and other officers, also for the support of normal schools, insane asylums, and other state institutions, likewise for public buildings.

Money raised for these purposes is called a **tax**.

The citizens assembled in town meeting, or representatives chosen by the citizens assembled in city council, or in a county board or a state legislature, determine by vote what sum of money shall be raised to meet the expenses of the town, the city, the county, or the state government. This sum is called the **tax levy**.

Taxable property is separated into two classes: furniture, cattle, bonds, etc., which are movable; and houses and lands, which are immovable. The first is called **personal property**, and the second **real estate**.

Taxes levied on personal property and real estate are called **property taxes**.

Officers called **assessors** estimate the value of each piece of real estate, recording their estimates against the owners' names. The *sum* of all these estimates is called the *valuation of real estate*. The value of personal property owned by each citizen is estimated and recorded in like manner; and the *sum* of these estimates is the *valuation of personal property*.

The *sum* of the valuations on real estate and personal property is the **total valuation**.

Taxes are always reckoned as a per cent of the total valuation of the property; for example:

1. Mr. A owns $5000 worth of property. The tax rate is $1\frac{7}{10}\%$, *i.e.* $17 on each thousand dollars' worth of property, and his property tax amounts to $85. Mr. B, who owns $50,000 worth of property, pays at the same rate a property tax amounting to $850. A person who owns property valued at $50,000 ought to pay a larger tax than one who owns property valued at only $5000.

Some persons have no property, but they enjoy all the benefits of schools, libraries, roads, etc., as much as those who have; therefore they ought to pay something toward the expenses of the town or city and state.

Consequently every male inhabitant over 20 years of age is subject to a small tax in the town or city in which he resides, regardless of whether he has any property or not. Since the tax is for an individual, a head, or poll, it is called a **poll tax.**

In Example 1 above, Mr. A and Mr. B pay the same amount of poll tax, $2 each, but Mr. S, who owns no property, also pays the same poll tax, $2.

325. Illustrative Example. In the Valuation Lists of the Real and Personal Estates of the Inhabitants of the Town of Hallowell, as assessed May 1, 1909, were found the following statements:

Number of Polls	Personal Property	Real Estate	Total Valuation
2644	$1,742,510	$5,431,060	$7,173,570

State Tax	County Tax	Town Expenses	Rate on $1000
$7875.00	$4418.93	$88,640.97	$13.00

To be received on account of Bank and Corporation Tax from State, $2,390.49.

EXPLANATION. An examination of the figures on p. 223 shows that the expenses of the town, including the amounts paid for State Tax, $7875.00, County Tax, $4418.93, Town Expenses, $88,640.97, make a total of $100,934.90.

The town will receive $2 from each Poll Tax payer; $2×2644=$5288. This added to the amount received from the State Bank & Corporation Tax, $2390.49, makes a total of $7678.49, which may be deducted from the total amount to be raised. $100,934.90 less $7678.49 leaves $93,256.41, the total which is to be raised by taxation.

To find the rate of taxation we find what per cent $93,256.41, the amount to be raised by taxation, is of $7,173,570, the total valuation. We find it to be $1\frac{3}{10}\%$, or $1.30 on each $100, or $13 on each $1000.

326. Illustrative Example. In a tax record book was found a statement regarding the property and tax of Paul Thompson, whose tax bill is here shown:

Payment of the Tax is hereby demanded. Payable at the Collector's Office, City Hall. Poll Taxes are due and payable on demand of Collector. If not paid before October 1st a summons will be issued, for which a charge of twenty cents additional will be made. An error in the name of the person taxed does not defeat the collection of the tax.

Prompt payment will save costs and arrest.

Any failure to receive tax bill does not invalidate a tax, or proceedings for the collection of the same.

A. T. JACOBS, *Collector of Taxes.*

August 1, 1909.

BRING THIS BILL AT PAYMENT

CITY *of* HAVERHILL

Page 47 Line 11 Residence 10 Front St., Ward 2

Paul Thompson

Your City, County and State Tax for 1908 is:

On POLL, - - - - - -	2	00
PERSONAL ESTATE, -	68	90
REAL ESTATE, - -	66	30
TOTAL TAX, - - -	$ 137	20

Received Payment,

A. T. Jacobs

Collector of Taxes.

Rate, $13.00 per $1000.

Office Hours from 8.30 A. M. to 12 M., and from 1.30 to 4 P.M. Saturdays, 8.30 A. M. to 1 P. M.

The following is the statement of the value of Mr. Thompson's property and the amount of his tax at a rate of $1\frac{3}{10}\%$:

Name	Poll	Real Property	Personal Property	Total	Tax
Thompson, Paul		House, $3,000			
		Barn, 300			
	$2	Land, 3 A. 1,800			
		$5,100	$5,300	$10,400	$137.20

To find Paul Thompson's total tax we multiply the total amount of property, $10,400, by the rate (per cent) of taxation, $.01\frac{3}{10}$. $\$10{,}400 \times .01\frac{3}{10} = \$135.20$, the property tax, to which is added $2, his poll tax. $\$135.20 + \$2 = \$137.20$, total tax.

A person who is a resident of a town or city pays a tax in that town or city on his personal property, and on all the real estate which he owns in that town or city, but upon any real estate belonging to him in another town or city he pays a tax in the town or city in which it is situated.

Taxes are usually computed on less than the full value of the property. The value that is taken as a basis for reckoning the taxation is called the **assessed valuation**, *e.g.* a house lot having a valuation of $4200 may be taxed for only $2800, or $\frac{2}{3}$ of its value.

Written Exercise

327. 1. W. A. Peters owns in the town of Westfield the following property, valued as follows: Personal property, $2400; real estate, $4700. The personal property is assessed for full value, the real estate for $\frac{3}{4}$ of its value. What is his total tax at the rate of $11.50 per thousand with $2 poll tax? ($11.50 per thousand is $.1\frac{15}{100}\%$.)

2. Samuel Reed pays a tax of $57.80 on property at a rate of $17 on a thousand. For what was the property assessed?

3. The American Machine Co. owns a manufacturing establishment. The property is assessed for $\frac{1}{2}$ its value, and the total tax at the rate of $16 on a thousand is $1426. What is the total value of their property?

4. The taxable property of the town of L. is valued at $2,344,680. The amount to be raised by taxation is $25,791.48. What is the tax rate?

5. The tax raised in a city is $248,000. The rate is 14 mills on the dollar. What is the valuation?

6. What is the rate of taxation in a city when $52 is paid on a house and lot worth $4500, assessed for $\frac{2}{3}$ of its value?

7. In the town of F—— there are 5151 poll tax payers. The poll tax is $2. The total valuation of the town is $3,412,500. The total amount to be raised by taxation is $44,427. What is the tax rate and what is the total tax of Mr. Patrick Garvey, who owns a homestead worth $5400 and a store worth $8000, if his property is assessed at $\frac{3}{4}$ its value?

8. At a tax rate of $20 on a thousand, what is the tax on property valued at $6500?

9. Mrs. Vance pays a property tax on property valued at $124,000. At the rate of $8\frac{1}{2}$ mills on the dollar, what is her total tax?

10. On property assessed for $11,000 the taxes are $156. What is the rate of taxation?

11. To raise a tax amounting to $4,500,000, on a valuation of $225,000,000, what must be the rate of taxation?

12. 59 A. of pasture land valued at $75 an acre is taxed for $\frac{2}{3}$ of its value at the rate of $6\frac{1}{2}$ mills. What is the amount of the tax?

13. If a house valued at $8000 is taxed at the rate of 25 mills, for $180, what is the assessed valuation?

328. The expenses of the national government are not paid from a tax upon property, but are paid from:

(1) **The Internal Revenue,** — a tax on the right to manufacture or sell liquors, tobacco, etc.

(2) **Customs or Duties,** which are taxes on goods imported from foreign countries.

Merchandise brought into this country may be:

(1) On the **free list,** that is, exempt from duty.

(2) Subject to an **ad valorem duty,** which is reckoned at a certain per cent on the cost of the goods in the country from which they are imported.

(3) Subject to a **specific duty,** which is a fixed charge according to number, quantity, weight, etc., without reference to the cost.

(4) Subject to both an **ad valorem and a specific duty.**

The duty is 67 ¢ on a ton of bituminous coal. Is this duty specific or ad valorem?

Name the kind of duty in the following:

Silk bindings pay a duty of 50 %.

Brushes pay a duty of 40 %.

Pig iron pays a duty of $4 a ton.

Manufactured glass pays a duty of 45 %.

Iron ore pays a duty of 40 ¢ a ton.

Cologne pays a duty of 60 ¢ a pound and 45 %.

Eggs pay a duty of 5 ¢ a dozen.

The **customhouse** is a building belonging to the United States, where all business connected with incoming and outgoing vessels and their cargoes is transacted with officers of the government.

A **port of entry** is a place where a customhouse has been established. It is usually a seaport, but may be any place on or near the boundary where merchandise is brought into the

country. The duties levied on imported merchandise are paid at the customhouse.

Any avoidance of payment of the duty on goods is called **smuggling**.

The rates of duty to be paid on the various kinds of merchandise imported are fixed by an Act of Congress, usually referred to as "The Tariff Act." To learn the rate of duty on the various kinds and qualities of merchandise, an importer is obliged to refer to the schedules contained in the last "Tariff Act."

NOTE 1. In estimating specific duties, certain allowances are made, as for weight of boxes, casks, etc., called *tare;* and for waste of liquors, called *leakage.* The weight of goods before allowances are made is the *gross weight,* and the weight after deducting allowances is the *net weight.*

NOTE 2. The value of imported goods appears from the invoice, which is a list of the goods with statement of the quantities and of the prices in the country from which they were imported.

Written Exercise

329. Illustrative Example. What is the duty on 10 casks of molasses containing 63 gal. each, at 3¢ a gallon, an allowance of 15 gal. being make for leakage?

WORK

63 gal. in 1 cask
10
630 gal. in all casks
15 gal., allowance for leakage.
615 gal., left

$.03, specific duty on 1 gal.
615
$18.45, duty to be paid.

1. How much is the duty at $2\frac{1}{2}$¢ per pound, on 15 boxes of raisins, 24 lb. to a box, tare $6\frac{1}{2}$ lb. to a box?

2. At 45% how much is the duty on 50 doz. watch crystals, invoiced at $1.80 per dozen?

3. What is the duty on 10 bbl. spirit varnish, 32 gal. to a barrel, invoiced at $6 per gallon, allowance for leakage being 10 %? The duty on varnish is 35 % and $1.32 a gallon.

4. A firm in New York imported from Canada 500 bbl. of apples, containing 2½ bu. each. What was the duty at 25¢ a bushel?

5. A Boston merchant imported 1500 doz. eggs from Nova Scotia. What was the duty at 5¢ per dozen, 10 % being allowed for breakage?

6. 100 doz. table knives costing $6 a dozen were brought into this country from England. Find the amount of duty at 15 % and 16¢ each.

7. A man imported from Italy paintings valued at 125,500 lire, upon which there was an ad valorem duty of 20 %. The freight and insurance charges were $95.60. How much did his paintings cost him? [A lira is $.193.]

8. The duty on macaroni is 1½¢ per pound. Find the entire cost of 1500 lb. of macaroni bought in Italy for 400 lire, with freight charges $9.40.

9. A merchant imported 75 doz. razors valued at $6.50 a dozen. He paid a duty of $1.75 a dozen and 20 % ad valorem. Find entire cost.

10. The duty on plate glass is 35¢ a square foot. Find the duty on 325 plates, each plate being 8½ ft. by 10 ft.

11. Find the duty at 65¢ a cubic foot on a block of Italian marble 1½ ft. by 2½ ft. by 7 ft.

12. At 44¢ a square yard and 40¢ ad valorem, find the duty on 265 yd. of Brussels carpet ¾ of a yard wide, invoiced at 3½ shillings. (A shilling is $.24.)

13. Find the duty on 50 bales of peanuts, invoiced at 80 lb. each, tare being 6 % and the duty ½¢ a pound.

330. A house may be destroyed by fire. Merchandise sent abroad may be wrecked at sea, and the merchandise may be lost. Accidents may cause injuries for life.

The knowledge that people would be glad to pay a certain sum of money yearly to protect themselves against loss in these various ways has led to the formation of companies that agree to make good to individuals all or part of their losses.

The business carried on by these companies is called **insurance**, which is simply security against loss.

NOTE. The security against loss of property by fire is called **fire insurance**; against loss of property at sea, **marine insurance**; against loss by personal injuries, **accident insurance**; against loss by death, **life insurance**. There are also many other forms of insurance.

A **policy** is a written contract between the insurance company and the person who is insured.

The **face** of the policy is the amount of insurance.

The **premium** is the price paid for insurance, and is reckoned at a certain rate per cent on the face of the policy.

Written Exercise

331. Illustrative Example. A house is insured against fire for $4500 for five years. The rate of insurance is $3\frac{1}{2}\%$. Find the premium.

WORK

$4500
$.03\frac{1}{2}$
22 50
135 00
$157.50, premium

EXPLANATION. Since the face of the policy is $4500, and the rate of insurance is $3\frac{1}{2}\%$, the premium will be $3\frac{1}{2}\%$ of $4500, or $157.50.

Ans. $157.50.

1. What is the premium for insuring a house for $6500 at $1\frac{1}{2}\%$?

2. A house worth $8400 is insured for $\frac{7}{8}$ of its value at $\frac{1}{6}\%$. Find the premium.

3. Find the cost of insuring a factory for $16,400 at $\frac{7}{8}\%$.

4. A merchant insured the stock in his store for $2700 for one year at $\frac{2}{3}\%$. Find the premium.

5. A fire insurance company charged $90 for insuring a house for $6000. What was the rate of insurance?

6. A factory worth $10,000, but insured for $8000 at $1\frac{1}{2}\%$, was damaged by fire to the extent of $6000. What was the loss to the owner? What was the loss to the company?

7. A barn valued in the policy at $3000 is insured for 5 years at a premium of 2%. What is the yearly cost of insurance?

8. A steamer carries 10,000 bu. of corn worth 40¢ a bushel. The corn is insured for $\frac{7}{8}$ of its value at a premium of $\frac{1}{5}\%$. Find the loss to the owner in case the corn is lost on account of a shipwreck.

9. A merchant paid $5.50 a year to insure the plate glass windows in his store. He also insured his stock for $15,000 at $1\frac{1}{4}\%$. How much did his insurance cost him?

10. A warehouse valued at $62,500 was insured for $\frac{4}{5}$ of its value. The rate of insurance was $1\frac{1}{3}\%$ for three years. What was the yearly cost of the insurance?

11. A silk manufacturer insured his stock for $36,000; $\frac{1}{2}$ of this amount was insured at $\frac{7}{8}\%$, $\frac{2}{3}$ of the remainder at $\frac{3}{4}\%$, and the rest at $\frac{5}{8}\%$. Find the total premium.

12. A shipment of goods is insured for $7500, and $15 is paid as premium. What is the rate of insurance?

13. For what sum was a ship insured if the rate of insurance was 2% and the premium $16.50?

14. A lot of lumber, which cost $50,000, is insured for $35,000 at $4\frac{1}{2}\%$. If it should be consumed by fire, what would be the loss to the owner?

332. The transaction of business involving the purchase and sale of goods, also the collection of money, is frequently intrusted by one person to another authorized to act for him. The charge for such services is reckoned at a certain per cent on the amount of the sale, purchase, or collection. For example:

A real estate agent collects rents amounting to $500. His charge of 5% on the money collected amounts to $25.

A commission merchant sells produce for $2500. His charge at 4% amounts to $100.

Agents have different names according to the authority given them, or the kind of business they transact.

A **factor** or **commission merchant** buys and sells merchandise for others, but usually has temporary possession of the goods, which he is bound to care for with reasonable prudence (by insuring, preserving from damage, etc.) as if they were his own.

A **broker** is employed to negotiate purchases and sales of merchandise or other property, and usually has no possession of it, but a **stockbroker** has possession and care of the stocks which he buys and sells.

A **collector** is an agent employed to collect debts.

The sum charged by an agent for transacting business for another person is called **commission.**

Commission is reckoned at a certain per cent on the amount of sales, or at a certain per cent on the amount of the purchase, or a certain per cent of money collected.

Oral Exercise

333. Illustration. If an agent sells goods amounting to $300 on a commission of 2%, his commission is 2% of $300, or $6.

If an agent makes a purchase amounting to $425 on a 5% commission, his commission is 5% of $425, or $21.25.

Find the commission on the following articles sold by an agent:

	Article	Selling Price	Rate of Commission
1.	Apples	$400	2 %
2.	Wheat	$1000	$2\frac{1}{2}$ %
3.	Beef	$175	5 %
4.	Eggs	$80	$2\frac{1}{4}$ %
5.	Corn	$500	5 %

6. An agent sells a horse for $250. What is his commission at 3 %?

7. An agent receives 5 % for collecting bills amounting to $500. How much is his commission?

8. If you sell $1000 worth of shoes for a manufacturer on a commission of $4\frac{1}{2}$ %, what is the amount of your commission?

9. A bill of $250 is collected by an agent on a 10 % commission. How much does the agent receive?

334. Illustrative Example. A commission merchant sold 250 bbl. of beef at $14 a barrel at a commission of 2 %. Find his commission.

Work

$14, selling price of 1 bbl.
250
700
28
$3500, selling price of all
.02
$70.00, commission, *Ans.*

The remainder, after the commission and other charges for the care and sale of the goods have been deducted from the gross receipts, is the **net proceeds**, or the amount to be remitted to the employer.

Written Exercise

335. Illustrative Example. A real estate agent sold a house for $2500; he charged a commission of 4% and $8.50 for advertising. What were the net proceeds?

WORK

$2500.00, amount of sale
.04
$100.00, commission
8.50, paid for advertising
$108.50, total expense

$2500.00, amount of sale
108.50, total expenses
$2391.50, net proceeds, *Ans.*

1. My agent sells flour for $250 at 4% commission. Find the commission, also the amount remitted to me.

2. An agent bought goods for $1500, receiving $3\frac{1}{2}$% commission. Find the amount of his commission.

3. What is $6\frac{1}{4}$% commission on a sale of $1500?

4. Find the net proceeds from selling 400 bbl. of flour at $7.50 a barrel on a commission of $4\frac{1}{2}$%.

5. A salesman has a guaranteed salary of $1100 and a 5% commission on his sales, which amount to $42,000. What is his total income?

6. A commission merchant sells flour for $4800 and charges 9%. What are the net proceeds?

7. A factor in Australia purchased for the Saxonville Mills a quantity of wool for which he paid $9500. What was his commission at $1\frac{1}{8}$%?

8. A commission merchant sold 4250 yd. of calico at $3\frac{7}{8}$¢ per yard, 3580 yd. of gingham at $8\frac{5}{8}$¢ per yard, and 2520 yd. of denim at $9\frac{1}{8}$¢ per yard. What was his commission at $2\frac{1}{4}$%?

9. An auctioneer, whose terms are 5% commission, sells 75 yd. carpet at $1.25, 60 yd. carpet at 95¢, 1 rug $28, 1 rug $47.50, and 5 pairs lace curtains at $5.50 per pair. What is the amount of his commission?

10. A collection agent collected $2737.50 on overdue accounts, and he remitted this amount, less his commission of 15%, to his client. What amount did he remit?

11. P. M. Traut receives as commission for collecting rents for the estate of V. M. Damon an average amount of $300 a month on a 5% commission. What is the total amount of rent collected? ($300 is 5% of amount collected.)

12. G. N. Smith collects a bill for A. B. Frost & Son, hardware dealers, and receives a 4% commission amounting to $8.50. Find the amount of the bill.

13. Johnson Bros. pay an agent $54.50 for collecting a bill on a $3\frac{1}{3}$% commission. What is the amount of the bill?

14. A real estate broker, who charges 4% commission, receives $224 for selling a house and preparing the papers necessary for the transfer of the property. What price is paid for the house? How much does the former owner receive?

15. What per cent is an agent charging if he sells a house for $3000 and receives $150 commission?

16. A book agent sells a set of cyclopedias for $45 and receives $15 commission. What per cent does he receive?

17. If a salesman receives $2500 a year as commission on sales amounting to $60,000, what is the rate of commission?

18. A commission house, whose business is to sell the entire product of certain cotton factories, and to guarantee payments for the same, sells, during the year, cotton cloth to the amount of $3,597,846.75. How much does the house receive, the commission for selling being $2\frac{1}{2}$%, and that for guaranteeing payments $2\frac{1}{2}$% more?

336. **1.** A store rents for $50 a month. How much is this for a year?

2. If the store is worth $6000, what per cent is realized yearly on its value?

If this $6000, instead of being invested in the building, were loaned to a man to invest in business, he should pay for the use of it as he pays for the use of the store.

Money paid for the use of money is called **interest.**

Mr. Wood decides to buy a house, but not having sufficient money he hires $2000 from the bank, and agrees to pay 5% a year for the use of it. At the end of the year he pays the bank 5% of $2000, or $100 interest.

At 4% a year, what is the interest on $1000 for 1 year? 2 years? 4 years?

The sum of money for the use of which interest is paid is called the **principal.**

A man borrowed $800 for a year at 6%. At the end of that time he paid the $800 and also the interest. How much was paid in all?

The sum of the principal and the interest is called the **amount.**

The per cent of the principal paid each year for the interest is called the **rate.**

Oral Exercise

337. Find the interest for 1 yr. on:

1. $600 at 5%.
2. $900 at 4%.
3. $550 at 6%.
4. $200 at $5\frac{1}{2}$%.
5. $1000 at 6%.
6. $50 at 7%.
7. $300 at $2\frac{1}{2}$%.
8. $800 at $6\frac{1}{4}$%.
9. $1200 at $3\frac{1}{3}$%.
10. $450 at 1%.
11. $2500 at 2%.
12. $150 at 3%.

Find the interest at 4 % on:

13. $200 for 1 yr.; for 3 yr.
14. $100 for 1 yr.; for 2 yr.
15. $500 for 1 yr.; for 4 yr.
16. $900 for 1 yr.; for 5 yr.
17. $1000 for 1 yr.; for 4 yr.

At 6 % per annum,* what is the interest:

18. Of $500 for 3 yr.?
19. Of $700 for 2 yr.?
20. Of $900 for 4 yr.?
21. Of $1200 for 2 yr.?
22. Of $1500 for 2 yr.?
23. Of $300 for 10 yr.?
24. Of $600 for 5 yr.?
25. Of $800 for 2 yr.?
26. Of $1000 for 3 yr.?
27. Of $2000 for 4 yr.?

Written Exercise

338. Illustrative Example. What is the interest on $850 for 3 yr. at 5 %?

WORK

$850, principal
.05
$42.50, interest for 1 yr.
3
$127.50, interest for 3 yr., *Ans.*

Find:

1. Interest of $1244 for 3 yr. at 6 %?
2. Interest of $478 for 6 yr. at 4 %?
3. Interest of $12,617 for 2 yr. at 5 %?
4. Interest of $2400 for 4 yr. at 4½ %?
5. Interest of $156.50 for 5 yr. at 6 %?
6. Interest of $237.75 for 4 yr. at 3 %?
7. Interest of $327.05 for 4½ yr. at 6 %?

* Per annum is the Latin for "by the year."

Interest for Years and Months

Oral Exercise

339. 1. What part of a year are 6 mo.? 3 mo.? 9 mo.? 4 mo.? 8 mo.? 2 mo.?

2. What is the interest on \$300 for 1 yr. at 5%? What part of this interest is the interest for 6 mo.? for 9 mo.? for 4 mo.? for 8 mo.? for 2 mo.?

3. What is the interest of \$300 at 6% for 6 mo.? 9 mo.? 4 mo.? 8 mo.? 2 mo.? 3 mo.? 10 mo.?

4. Find the interest of \$200 for 1 yr. 4 mo. at 4%.

5. Find the interest of \$300 for 2 yr. 6 mo. at 5%.

6. Find the interest of \$400 for 3 yr. 3 mo. at 6%.

7. Find the interest of \$100 for 5 yr. 8 mo. at 6%.

Find the interest on:

8. \$200 for 6 mo. at 4%.
9. \$100 for 6 mo. at $4\frac{1}{2}$%.
10. \$300 for 4 mo. at 6%.
11. \$200 for 5 mo. at 6%.
12. \$700 for 6 mo. at 3%.
13. \$100 for 6 mo. at 5%.
14. \$120 for 1 mo. at 5%.
15. \$250 for 2 yr. at 6%.
16. \$500 for 9 mo. at 4%.
17. \$125 for 1 yr. at 5%.
18. \$700 for 2 yr. at $4\frac{1}{2}$%.
19. \$300 for 1 yr. at $3\frac{1}{2}$%.
20. \$975 for 9 mo. at 4%.
21. \$800 for 2 yr. at $5\frac{1}{2}$%.
22. \$300 for $2\frac{1}{2}$ yr. at 4%.
23. \$240 for 10 mo. at 5%.
24. \$200 for 6 mo. at $3\frac{1}{2}$%.
25. \$400 for 8 mo. at 3%.
26. \$500 for $1\frac{1}{3}$ yr. at 2%.
27. \$600 for 6 mo. at 7%.
28. \$75 for 1 yr. at 5%.
29. \$50 for $3\frac{1}{2}$ yr. at 6%.
30. \$800 for 4 yr. at 5%.
31. \$20 for 6 mo. at 6%.
32. \$35 for 9 mo. at 4%.
33. \$75 for $1\frac{1}{2}$ yr. at 6%.
34. \$800 for $2\frac{1}{4}$ yr. at 4%.
35. \$900 for $2\frac{1}{2}$ yr. at 3%.

Written Exercise

340. Illustrative Example. Find the interest and amount of \$144 for 2 yr. 8 mo. at 5%.

\$144, principal 2 yr. 8 mo. $= 2\frac{2}{3}$ yr.

.05

\$7.20, interest for 1 yr.

$2\frac{2}{3}$

4 80

14 40

\$19.20, interest for 2 yr. 8 mo., *Ans.*

144 , principal

\$163.20, amount, *Ans.*

To find the interest for years and months, we change the months to the fraction of a year. Then we find the interest for 1 *yr. and multiply by the number of years. The amount is the interest plus the principal.*

What is the interest and amount of:

1. \$126.50 for 1 yr. 3 mo. at 4%?
2. \$925.62 for 2 yr. 5 mo. at 7%?
3. \$1362.56 for 17 mo. at $4\frac{1}{2}$%?
4. \$263.75 for 2 yr. 3 mo. at $2\frac{1}{2}$%?
5. \$562.00 for 3 yr. 6 mo. at 8%?
6. \$916.50 for 4 yr. 2 mo. at $3\frac{1}{2}$%?
7. \$2685.00 for 1 yr. 6 mo. at 5%?
8. \$3480.00 for 8 mo. at 9%?
9. \$3456.75 for 3 yr. 2 mo. at 6%?
10. \$4220.50 for 5 yr. 6 mo. at 5%?
11. \$5680.50 for 4 yr. 9 mo. at 4%?
12. \$7245.25 for 6 yr. 8 mo. at 9%?

Interest for Years, Months, and Days

Written Exercise

341. Illustrative Example. Find the interest of $6280 for 2 yr. 6 mo. and 25 da. at 7%.

Work

$6280, 2 yr. 6 mo. 25 da. $= 2\frac{41}{72}$ yr.

.07

$439.60, interest for 1 yr.

$2\frac{41}{72}$

250.32

879.20

$1129.52, interest for $2\frac{41}{72}$ yr.

Explanation. Reckoning a year as 12 months of 30 days each, we find the interest for one year at the given rate to be $439.60. This we multiply by $2\frac{41}{72}$, the number of years.

Ans. $1129.52.

Find the interest of:

1. $325 for 1 yr. 8 mo. 5 da. at 5%.
2. $560 for 2 yr. 4 mo. 15 da. at 5%.
3. $50 for 3 yr. 5 mo. 10 da. at 6%.
4. $2500 for 2 yr. 6 mo. 12 da. at 4%.
5. $5000 for 3 yr. 8 mo. 11 da. at 6%.
6. $640 for 5 yr. 7 mo. 18 da. at 3%.
7. $350.60 for 4 yr. 9 mo. 10 da. at 4%.
8. $896 for 6 yr. 1 mo. 21 da. at 7%.
9. $1.25 for 10 yr. 4 mo. 27 da. at $2\frac{1}{2}$%.
10. $1500 for 2 yr. 2 mo. 19 da. at $3\frac{1}{2}$%.

Interest from Date to Date

Written Exercise

342. Illustrative Example. What is the time in years, months, and days from May 11, 1897, to August 7, 1899?

Solution. From May 11, 1897, to May 11, 1899, is 2 yr.; to July 11 is 2 mo. more. From July 11 to July 31 is 20 da., and to August 7 is 7 da. more.

Ans. 2 yr. 2 mo. 27 da.

From the preceding illustration may be derived the following:

To find the number of years, months, and days between two dates: First find the number of entire years between the two dates, then the number of full months remaining, and lastly, the remaining days, including the day of the later date.

1. Find the time in years, months, and days from April 19, 1891, to March 4, 1904. *Ans.* 12 yr. 10 mo. 14 da.

NOTE. Observe that the year 1904 is a leap year.

Find the number of years, months, and days:

2. From January 21, 1901, to April 29, 1904.

3. From September 12, 1899, to December 26, 1903.

4. From April 1, 1900, to November 25, 1905.

5. From August 20, 1901, to February 27, 1902.

6. From February 8, 1902, to January 29, 1905.

7. From July 18, 1903, to March 18, 1906.

8. From May 10, 1902, to August 25, 1905.

9. From June 15, 1904, to October 6, 1906.

10. From November 23, 1903, to June 10, 1904.

11. What is the interest on $920 from October 19, 1903, to April 25, 1907, at 6 %?

12. What is the amount of $528.16 on interest at 6 % from August 22, 1904, to September 27, 1906?

13. What is the amount of $600 on interest at 6 % from June 20 to October 2, 1906?

14. What is the amount at 6 % of $12,625 on interest from March 25, 1901, to January 1, 1905?

15. What is the interest on $2500 from November 11, 1899, to August 15, 1905, at 7 %?

16. What is the interest on $130.27 from February 7, 1904, to December 1, 1908, at 8 %?

17. What is the interest on $19.60 from October 25, 1895, to July 28, 1908, at 10 %?

18. What is the interest on $322.65 from August 13, 1902, to April 2, 1904, at $5\frac{1}{2}$ %?

19. What is the interest on $625 from the 20th of January, 1891, to the 1st of May, 1902, at $4\frac{1}{2}$ %?

20. What is the interest on $2340 from March 15, 1904, to February 28, 1905, at 5 %?

21. What is the interest on $955.65 from April 10, 1903, to October 2, 1904, at $2\frac{1}{2}$ %?

22. What is the interest on $244.96 from April 19, 1909, to May 30, 1913, at $7\frac{1}{2}$ %?

23. What is the interest on $564.28 from May 15, 1903, to January 6, 1912, at $6\frac{2}{3}$ %?

24. What is the interest on $628.50 from June 30, 1904, to October 31, 1909, at 4.7 %?

25. What is the interest on $526.80 from January 30, 1903, to March 19, 1906, at $5\frac{3}{10}$ %?

26. On the 12th of October, 1908, I gave two notes, one for $450 with interest at $4\frac{1}{2}$ % and the other for $960 with interest at 4 %. What amount will be required to pay these notes November 5, 1908?

27. Mr. White owes $500 payable October 15, 1909, but is told that interest at 5 % will be deducted if he will pay the debt March 25, 1909. How much will he save by paying at the earlier date?

343. To find the rate.

1. At what rate will $300 yield $18 in 3 yr.?

Since the interest is $9 in 3 yr. at 1%, at what rate will $300 yield $18 interest in the same time? *Ans.* 2%.

2. At what rate will $200 yield $80 in 5 yr.?

3. At what rate will $500 yield $100 in 4 yr.

Written Exercise

344. Illustrative Example. At what rate will $450 give an interest of $81 in 3 yr.?

WORK

$450
.01
$4.50, interest for 1 yr. at 1%
3
$13.50, interest for 3 yr. at 1%
6
$13.50)$81.00

EXPLANATION. Since the interest of $450 for 3 yr. at 1% is $13.50, to gain $81 in the same time, the interest must be as many per cent as $13.50 is contained times in $81, which is 6 times. Therefore the rate is 6%. *Ans.* 6%.

PROOF. The interest of $450 for 3 yr. at 6% = $81.

1. At what rate will $10,000 gain $5000 in 10 yr.?

2. At what rate will $12 gain $4 in 3 yr. 4 mo.?

3. At what rate will $450 gain $49.87½ in 1 yr. 7 mo.?

4. At what rate will $250 gain $35 in 2 yr. 9 mo. 18 da.?

5. At what rate will $57.20 gain $4.719 in 1 yr. 6 mo.?

6. At what rate will $75 gain $23.62½ in 3 yr. 6 mo.?

7. At what rate will $1000 gain $500 in 6 yr. 8 mo.?

8. The amount of $150 for 2 yr. 6 mo. was $157.50. What was the rate?

9. I deposited $600 in a bank and a year later was credited with $30 interest. What rate of interest was paid?

10. A house that cost $3500 was rented for $25 a month. Repairs, taxes, and insurance cost the owner $90. What rate of interest was realized on the investment?

11. A man pays $200 for the use of $2000 for 2 yr. What rate per cent does he pay?

345. To find the time.

What interest will $400 earn in 1 yr. at 5%? How long a time would it take $400 at 5% to earn $40 interest? $80? $100?

Written Exercise

346. Illustrative Example. In what time will $750 at 7% yield $78.75?

WORK

$750
.07
$52.50, interest for 1 yr.

$52.50)$78.75

1½ yr., or 1 yr. 6 mo.

EXPLANATION. Since the interest on $750 at 7% for 1 yr. is $52.50, it will take the principal as long a time to gain $78.75 as $52.50 is contained times in $78.75, or 1½ yr.

Ans. 1 yr. 6 mo.

In what time will

1. $600 gain $30 at 6%?
2. $400 gain $48 at 4%?
3. $800 gain $84 at 7%?
4. $3200 gain $160 at 5%?
5. $816 gain $217.60 at 7½%?
6. $53.50 gain $10.165 at 7½%?
7. In what time will $4280 amount to $5107.46 at 4%?
8. In what time will $326 amount to $385.332 at 7%?
9. How long must a note for $15,000 run, to amount to $20,000 at 8%?
10. In what time will $100 amount to $200 at 6%?
11. In what time will $500 amount to $1000 at 5%?
12. In what time will any sum of money double itself at 5%? at 6%?

Written Exercise

347. To find the principal.

Illustrative Example. What principal invested at 4% a year will give an annual income of $800?

EXPLANATION. Since the interest of $1 for a year at 4% is $.04, as many dollars must be invested to yield an income of $800 as $.04 is contained times in $800, which is 20,000 times; therefore, $20,000 must be invested.

1. What principal at 5% will gain $100 in 5 yr.?

2. What principal at 6% will gain $72 in 2 yr.?

3. How much money must be put on interest at 3% to yield $229.50 in 2 yr. 6 mo.?

4. To gain $341 in 7 mo. 15 da., what sum of money must be put on interest at 4%?

5. What sum of money put on interest at 4% will gain $100,000 in 100 yr.?

6. A man willed a library enough money invested at 4% to yield an income of $2500. How much money was willed?

7. A man borrowed money at 6%. He pays interest of $150 a year. How much money did he borrow?

8. A man insures his life for an amount which, invested at 5%, will give an annual income of $20,000 after his death. For what amount is his life insured?

ACCURATE INTEREST

348. Interest that is computed by taking the exact number of days between dates, and reckoning 365 days to a year, is called **accurate interest.**

Accurate interest is used by the United States government and by some banks. It is used for terms of less than one year.

Written Exercise

349. Find the accurate interest on:

1. \$300 for 60 da. at 6 %.
2. \$500 for 33 da. at 6 %.
3. \$450 for 90 da. at 5 %.
4. \$840 for 45 da. at 3 %.

5. Find the accurate interest on \$7982.50 from January 15 to June 15, 1895, at 6 %.

6. Find the accurate interest on \$9865.00 from January 20, 1894, to March 28, 1895, at 6 %.

7. Find the accurate interest on \$892.75 from June 12 to September 12, 1895, at 6 %.

8. Find the accurate interest on \$4860 from July 1, 1895, to December 1, 1896, at 7 %.

9. Find the accurate interest on \$50,000 from December 15, 1893, to October 25, 1894, at 5 %.

10. Find the difference between the accurate interest and the interest by the common method on \$4,285,629 from April 4 to July 17 of the same year at 6 %.

350. James Wood deposits \$800 in a savings bank with the understanding that he is to receive 3 % a year interest. How much interest is due him at the end of the first year?

He does not withdraw this interest, and it is added to the original sum that he deposited. On what sum should he receive interest for the second year?

When the interest is added to the principal as it becomes due, and the amount draws interest, the owner is said to receive **compound interest.**

Interest is compounded annually, semiannually, quarterly, or for any other regular period as may be agreed upon.

Written Exercise

351. Illustrative Example. Find the compound interest of $800 for 2 yr. at 6 %, interest compounded annually.

WORK

$800.00, principal
.06
$ 48.0000, interest for 1st year
800
$848.00, principal for 2d year
.06
$ 50.88, interest for 2d year
848.00
$898.88, amount
800 , deduct original principal
$98.88, compound interest, *Ans.*

To compute compound interest:

1. *Find the amount of the given principal for the first period of time. With this as a new principal, find the amount for the second period of time, and so continue for the whole time. The last amount is the amount required.*

2. *The last amount, less the original principal, is the compound interest.*

Find the compound interest of:

1. $300 for 3 yr. at 6 %.
2. $520 for 5 yr. at 5 %.
3. $750 for 4 yr. at 3 %.
4. $860 for 2 yr. at 5 %.
5. $970 for 3 yr. at 6 %.
6. $650 for 2 yr. at 4½ %.
7. $1000 for 4 yr. at 3 %.
8. $1630 for 3 yr. at 4 %.
9. $1500 for 2 yr. at 3 %.
10. $1800 for 4 yr. at 2 %.

NOTE. Unless otherwise specified, interest is understood to be compounded annually. If compounded semiannually, the rate must be considered one half the annual rate mentioned; if compounded quarterly, one fourth the annual rate, etc.

Written Exercise

352. Illustrative Example. Find the amount and the compound interest of $2000 for 2 yr. 6 mo. at 4 %; interest compounded semiannually.

Work

$2000, principal
.02
$ 40.00, interest for 1st six months
2000.00
$2040, principal for 2d six months
.02
$ 40.80, interest for 2d six months
2040
$2080.80, principal for 3d six months
.02
$ 41.6160, interest for 3d six months
2080.80
$2122.42, principal for 4th six months
.02
$ 42.4484, interest for 4th six months
2122.42
$2164.87, principal for 5th six months
.02
$ 43.2974, interest for 5th six months
2164.87
$2208.17, amount
2000.00, deduct original principal
$ 208.17, compound interest, *Ans.*

Find the amount and compound interest of:

Compounded Semiannually	Compounded Quarterly
1. $200, 2 yr. 6 mo., 5 %.	4. $900, 2 yr., 8 %.
2. $3500, 2 yr. 6 mo., 7 %.	5. $700, 1 yr. 6 mo., 4 %.
3. $5000, 3 yr. 6 mo., 6 %.	6. $800, 1 yr. 3 mo., 6 %.

353. A **bill** is an itemized account of goods purchased, or of services rendered, containing the names of the buyer and seller, the date and place of sale, or service, the price, kind, and quantity of goods sold or service rendered, and the terms of sale or service.

SANBORN & HALL

FINE IMPORTED AND
DOMESTIC GROCERIES

AUBURN, N. Y. June 10, 1909.

SOLD TO G. K. Nesbit & Co.
Utica, N. Y.

TERMS: 1% 10 DAYS OR 30 DAYS NET

214	lb. Coffee @ 20¢	42	80	
340	lb. Coffee @ 24¢	81	60	
				124 40

Recd Payment,
Sanborn & Hall.
July 10. 1909. per Frey.

This bill of goods has been paid and payment acknowledged by **receipting** the bill, that is, by writing *Received Payment*, the date of payment, and the name of the person or firm to whom it is paid.

354. An **invoice** is a list of goods purchased and the price of each kind, which is given to the buyer by the seller at the time when the goods are delivered. Its form and contents are like those of a *bill* of goods.

355. A **statement of account** is an account of all amounts due by and received from the purchaser, *i.e.* all debits and credits as shown on the books of the *seller*.

The following monthly statement is in the form usually sent out to show the standing of an account the first of each month.

We render statements for bills, DUE AND NOT DUE, as per our Ledger, on the first of each month. This gives opportunity for the correction of errors and is not a demand for bills NOT DUE.

MONTHLY STATEMENT

New York, July 1, 1909.

School Dept. New Britain, Conn.

IN ACCOUNT WITH

AMERICAN BOOK COMPANY

WASHINGTON SQUARE

May	25	Mdse. as per bill	160	40		
"	28	" " " "	50	39		
"	30	" " " "	162	40		
June	20	" " " "	65	54		
					438	73
		Credit				
June	1	By check			150	00
		Balance due			288	73

356. A **receipt** is a written statement of the receipt of money, or other property, stating the name of the place where written, the date, the names of the interested parties, and usually for what the consideration was given.

$37.50

Stratford, Conn., April 1, 1909.

Received of D. W. Hood, thirty-seven and 50/100 dollars for rent of house No.280 Page Street, for the month of March, 1909. Albert J. Bates

A RECEIPT FOR RENT

357. An **order** is a written direction or request sent by one person to another for the delivery of money or property to a third person, usually the one holding the order.

Philadelphia, Pa., June 4,1909.

Mr. Robt. T. White:

Please deliver to C. K. Leonard merchandise to be selected by him to the amount of fifteen dollars,($15.) and charge to my account. E. K. Sawyer

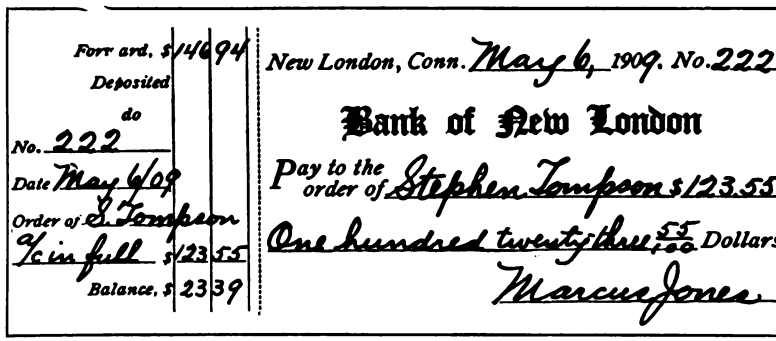
AN ORDER

1. Write a receipt for one month's tuition in French, 9 lessons at $1.50 each.

2. Write an order to a publisher for one dozen copies of "Little Women" to be delivered to bearer.

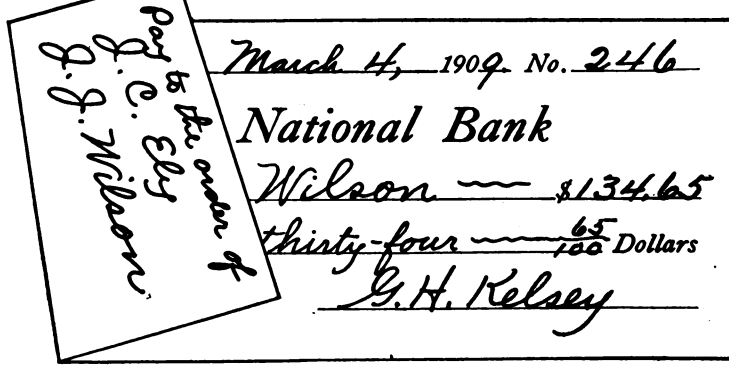
3. Write an order to B. Johnson & Co. to call for suit sent on approval and credit it to your account.

358. A **check** is a written order from a depositor on a bank or trust company for the payment of a specified sum of money to the person or party named therein.



A CHECK

359. The **maker** or **drawer** of a check is the person who signs it, *e.g.* Marcus Jones. The **payee** is the person to whom the check is to be paid, *e.g.* Stephen Tompson. The **face** is the sum to be paid, *e.g.* one hundred twenty-three $\frac{55}{100}$ Dollars.



AN INDORSED CHECK

A check is **negotiable** if it contains the word "Pay to the order of," or the words "or order." Otherwise, it is *not negotiable*, and can be paid only

to the person whose name is mentioned in the check. A non-negotiable check must be indorsed by the holder in order to make it payable, as shown on p. 252. This check, drawn by G. H. Kelsey (the maker), in favor of J. J. Wilson (the payee), for $134.65 (the face), has been indorsed by the payee to be paid to J. C. Ely. To receive payment at the bank or elsewhere J. C. Ely must also indorse it. The indorsements make each indorser in turn responsible for the final payment.

360. A **promissory note** is a written promise made by one person to pay another a definite sum of money at a definite time.

$4400.00 Milton, Mass., Jan. 6, 1909.

Three months after date I promise to pay to the order of Elmer W. Barston

Four thousand four hundred 00/100 Dollars

at The First National Bank, Boston

Value received with interest at the rate of 6 per cent per annum

No 299 Due Apr. 6, 1909. John C. Moody

A TIME NOTE

This note is a time note drawn or made by J. C. Moody (the maker), in favor of E. W. Barston (the payee), for $4400 (the face), to be paid in three months from date (the term of the note), with interest at 6% (the rate), for value received (the consideration). The date of the note is Jan. 6, 1909.

$344.00 Stark, Me., Mar. 8, 1908.

On demand after date I promise to pay to the order of S. C. Goodrich

Three hundred forty-four 00/100 Dollars

at National Bank of Skowhegan

Value received.

No. 46 W. W. Snow

A DEMAND NOTE

Payment on this note may be demanded at any time after date.

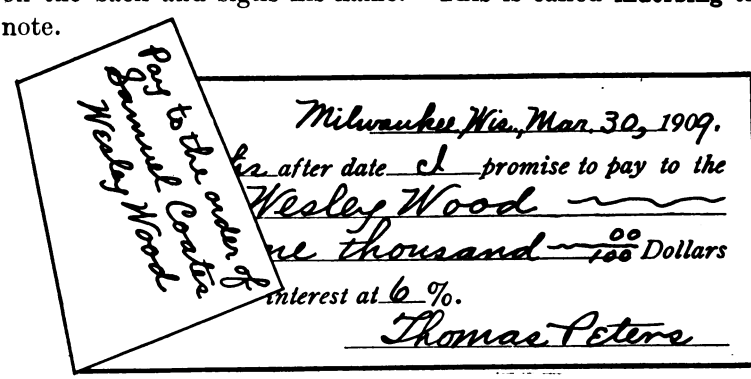
A note *must:*

1. Be signed by the person who promises to pay, *i.e.* the maker or drawer.
2. Tell by name the person or party to whom the note is to be paid, *i.e.* the payee.
3. Tell the date when the note is made and the place where made.
4. Tell the time of payment.
5. Tell the place of payment.
6. Tell the rate of interest, if interest is to be collected.

361. When a note becomes due, it is said to have **matured.** The date when it is due is the **date of maturity.**

A note that contains the words "to bearer" may be sold or transferred by delivery, and may be collected by any person who may hold it at the date of maturity.

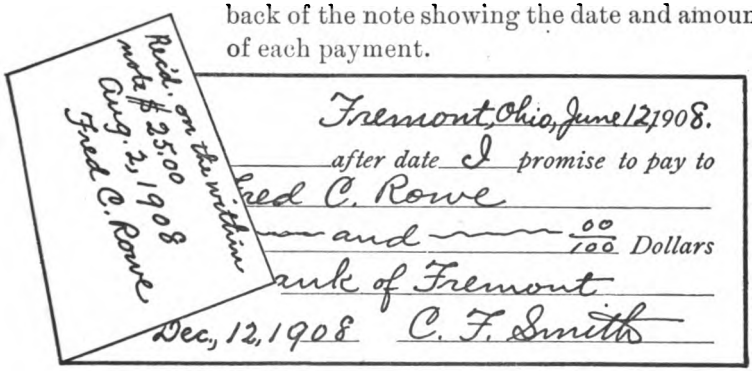
A note that is drawn payable "to order" cannot be sold or transferred unless its payment to another person is ordered in writing by the payee. To do this the payee writes his order on the back and signs his name. This is called **indorsing** the note.



AN INDORSED NOTE

This note has been indorsed by the payee, Wesley Wood, ordering its payment to Samuel Coates.

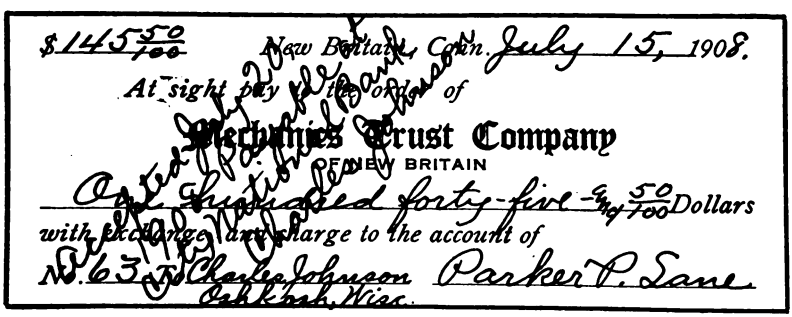
The maker of a note may arrange with the payee to make **partial payments** on the face of the note from time to time. A record of these partial payments is made or **indorsed** on the back of the note showing the date and amount of each payment.


Rec'd. on the within note $25.00
Aug. 2, 1908
Fred C. Rowe
Fremont, Ohio, June 12, 1908.
after date I promise to pay to
Fred C. Rowe
and 00/100 Dollars
Bank of Fremont
Dec., 12, 1908 C. F. Smith

PAYMENT INDORSED ON NOTE

This note shows the form of indorsement for a partial payment on the face of the note.

362. A **draft** is a written order or request by one party to a second party for the payment of a specified sum of money to a third party at a specified time.


$145 50/100
New Britain, Conn. July 15, 1908.
At sight pay to the order of
Mechanics Trust Company
OF NEW BRITAIN
One hundred forty-five 50/100 Dollars
with exchange and charge to the account of
No. 63 To Charles Johnson Parker P. Lane
Oshkosh, Wisc.

A SIGHT DRAFT

The person who signs his name to a draft is the **drawer**, *e.g.* Parker P. Lane as above. The person who is to pay the draft

is the **drawee**, *e.g.* Charles Johnson. The person to whom the draft is to be paid is the **payee**, *e.g.* Mechanics Trust Co. of New Britain.

Business men employ drafts, as on p. 255, very commonly for collecting accounts. The draft (p. 255) drawn by Parker P. Lane on Charles Johnson for the amount of a bill due of one hundred and forty-five $\frac{50}{100}$ dollars, is taken by Mr. Lane to the Mechanics Trust Co. By the Mechanics Trust Co. it is indorsed and sent to some bank in Oshkosh, Wisconsin, for collection. Upon receipt of the draft the Oshkosh bank takes it to Mr. Johnson to see if he is willing to pay it. If not willing, he refuses to accept the draft. If willing, he writes across its face the word "Accepted," and the date and where he will pay it, and signs his name as in the form given.

363. Banks are institutions authorized by law for the following purposes:

(1) To provide places for the safe-keeping of money.
(2) To aid business by means of loans on safe security.
(3) To make collections on commercial paper easier and safer.
(4) To facilitate domestic and foreign exchange.

A **national bank** is authorized by law:

(1) To issue its notes (called bank notes) to an amount equal in value to the par value of the United States bonds which it has purchased.
(2) To receive money on deposit.
(3) To loan money on discount notes.
(4) To furnish exchange.

A **savings bank** is a bank that is organized for the following purposes:

(1) To receive money on deposit, even in small sums.
(2) To return the money to depositors on due notes with a moderate rate of interest.

(3) To loan money on certain classes of securities authorized by law, *e.g.* real estate, municipal bonds, etc.

A **trust company** in addition to executing such trusts as making and caring for investments performs many of the functions of banks.

Deposits

364. Each time a deposit is made the depositor hands in a statement of the deposits made out on a **deposit slip**, and the amount is entered to the depositor's credit in a small book which he brings with him, called a **bank book.**

The usual form of deposit slip is here shown.

DEPOSIT SLIP

Mechanics National Bank

CHICAGO, ILLINOIS

DEPOSITED FOR ACCOUNT OF

Franklin R. Murray

Sept. 23, 1908.

Gold		
Silver		
Currency	200	00
Checks	77	07
"	30	00
"	50	00
Total	357	07

The form of entry for both deposits and withdrawals in the pages of the bank book are shown on p. 258.

Dr. Mechanics National Bank, *in acct.* with James E. Freeman Cr.

1906						1906				
Sept.	7	Dep.		200		9/7/06 to 11/2/06			544	34
Oct.	1	"		233	33	Balance			347	32
"	16	"		125						
"	23	"		50						
"	29	"		283	33					
1906				891	66				891	66
Nov.	2	Balance		347	32					

On the credit side of the bank book the whole amount of money withdrawn is entered, the list of separate checks being kept on a separate slip.

Withdrawals

365. Whenever the depositor wishes to draw money from a national bank, he fills out a blank order for the payment of money called a **check**. This blank he usually tears out from a book of blank checks. It will be seen from the following form that a check is a written order on a bank for the payment of a certain amount of money to some person named on the check, or to his order, and is signed by a depositor.

No. 93 New Britain, Conn. Sept. 30, 1907.

The New Britain Trust Company

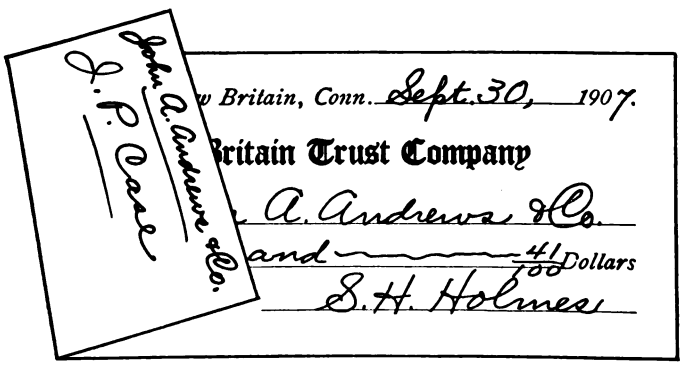
Pay to the order of John A. Andrews & Co.

Eighty ——— and ——— 41/100 Dollars

$80.41 S. H. Holmes

A CHECK

The person in whose favor it is drawn must sign his name across the back of the check, that is, **indorse** it. He may then turn it over as the equivalent of an amount of money equal to its full value either in payment of a debt, or he may deposit it in his own bank and have it credited to his account, or he may have it cashed at any national bank that will accept it.



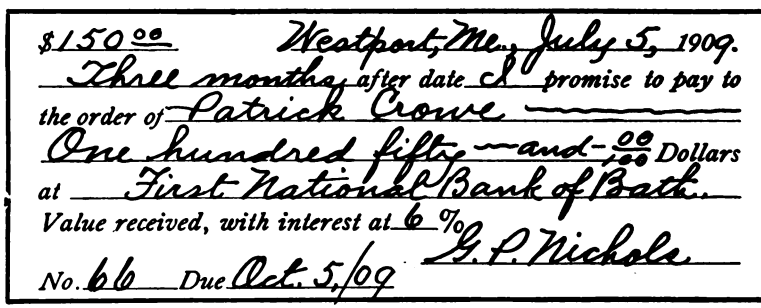
AN INDORSED CHECK

Each successive holder of the check must sign his name across the back until it reaches the bank on which it was drawn.

The above form shows a check correctly indorsed by John A. Andrews & Co. and J. P. Case.

366. An important and profitable part of the business of a bank is the loaning of money on **notes.**

Mr. G. P. Nichols buys a horse of Mr. Patrick Crowe for $150. At the time of purchase Mr. Nichols arranges with Mr. Crowe to give him instead of the money a promissory note for the amount ($150), which is to be paid in three months after date of sale with interest at 6 %.


$150.00 Westport, Me., July 5, 1909.
Three months after date I promise to pay to
the order of Patrick Crowe
One hundred fifty —and-00/100 Dollars
at First National Bank of Bath.
Value received, with interest at 6 %
No. 66 Due Oct. 5/09 G. P. Nichols

A PROMISSORY NOTE (WITH INTEREST)

Mr. Crowe wishes to secure the money on the above note at once, and he is willing to sacrifice a certain per cent of its full value in order to do so. He therefore writes his name across the back of the note (indorses it), thus making himself responsible for its payment at the time when it is due (the date of maturity), Oct. 5, 1909. Then he asks the bank to discount it for him.

If the bank officers think that both Nichols and Crowe are reliable persons who will be sure to pay when the note is due, they reckon the amount due on the note at the date of maturity. The interest on $150 for 3 mo. at 6 % is $2.25. $150 + $2.25 = $152.25, amount of note at maturity. The interest on $152.25 for 3 mo. at 6 % = $2.28+. Therefore this is the amount to be taken off by the bank, *i.e.* the **bank discount** which the bank receives for the use of the money for 3 mo. The

amount actually loaned, called the **proceeds** of the note, is $152.25 − $2.28 = $149.97. This note bears interest at 6 %. When the note bears no interest, the discount is reckoned on the face of the note.

$500.00 Bona, Mo., June 15, 1909.
Six months after date I promise to pay to
the order of Nathaniel Bank
Five hundred 00/100 Dollars
at Bona, Mo.,
Value received.
G. F. Gleason
No. 72 Due Dec. 15/09

A PROMISSORY NOTE (WITHOUT INTEREST)

In 6 mo. from date, *i.e.* Dec. 15, 1909, $500, the face of the above note, will be due with no interest. The bank reckons the interest at 6 % on $500, the face of the note, for 6 mo. and finds the discount to be $15. This $15 is taken out of the $500 and leaves $485, which the bank will pay on the note on June 15, the date when it is made. If this note is not discounted until Aug. 15, the time from the date of discount, Aug 15, to maturity, Dec. 15, 4 mo., is taken as the term of discount, *i.e.* interest on $500 for 4 mo. at 6 % is $10. $500 − $10 = $490, the proceeds of the note if discounted Aug. 15, 1909.

Written Exercise

367. Illustrative Example.

482\frac{50}{100}$ HARTFORD, CONN., June 20, 1904.

Three months after date, I promise to pay to the order of James M. Richey, Four Hundred Eighty-two and $\frac{50}{100}$ Dollars, at the Merchants' National Bank, value received.

WILLIAM CONWAY.

Discounted, at date, at 6 %.

Find the date of maturity, the term of discount, the bank discount, and the proceeds of the note on p. 261.

The date of maturity is 3 mo. after June 20, 1904, which is Sept. 20, 1904.
The term of discount from June 20, 1904, to Sept. 20, 1904, is 92 days.
The bank discount is the interest on $482.50 for 92 days, at 6%, or $7.40.
The proceeds is $482.50 – $7.40, or $475.10.

Find the date of maturity, the term of discount, and proceeds of the following:

1. A 60-day note for $750, without interest, dated May 9, 1905, discounted June 3, 1905, at 6%.

2. A 90-day note for $600, without interest, dated Aug. 4, 1906, discounted at once at 5%.

3. A 15-day note for $1000, with interest at 6%, dated July 28, discounted the same day at 4%.

4. A 4-mo. note for $850, without interest, dated April 30, discounted May 15, at 7%.

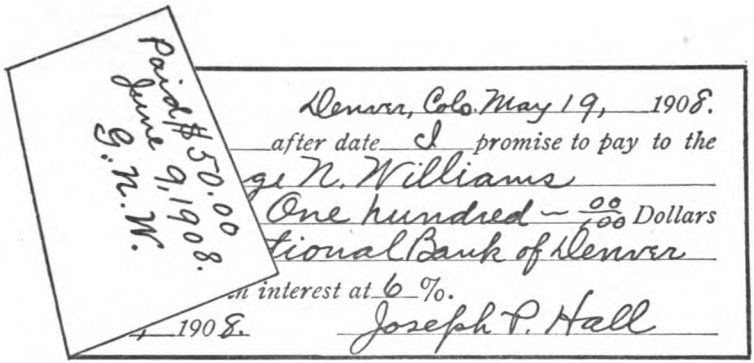
5. A 30-day note for $1000, with interest at 5%, dated Feb. 3, discounted the same day at 6%.

6. A note for $4000 on interest for 2 mo. at 6%, dated Jan. 10, 1907, discounted immediately at 5%.

PARTIAL PAYMENTS ON NOTES OR DEBTS

368. It frequently happens that the person who makes a note (*i.e.* the maker or drawer) prefers to make payments on the note from time to time before the date of maturity. These payments are called **partial payments.**

As soon as such partial payment is received, a record of its receipt (called an indorsement) is made on the back of the note by the holder of the note. See p. 263.



PAYMENT INDORSED ON NOTE

On the above note for $100, to be paid 6 mo. from date, due November 19, 1908, was indorsed a payment of $50 on June 9, 1908.

EXPLANATION. To find what amount is due on November 19, 1908, we find the interest on the face of the note, $100, from May 19, 1908, to the date of the payment, June 9, 1908, *i.e.* for 21 days, to be $.35. We add this to the face of the note, $100. $100 + $.35 = $100.35. From this $100.35 we take the payment of $50 made June 9, 1908, leaving $50.35. On this amount, $50.35, we find that the interest for the time from June 9, 1908, to Nov. 19, 1908, *i.e.* for 163 da. = $1.37. This added to $50.35 gives the amount due at maturity. $50.35 + $1.37 = $51.72, due Nov. 19, 1907.

The above problem is solved by the United States Rule, the legal rule in most states, and made the legal rule, by a decision of the Supreme Court, of the United States. It may be stated as follows:

UNITED STATES RULE FOR PARTIAL PAYMENTS

Find the amount of the principal to the time when the payment or the sum of the payments equals or exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal and proceed as before.

Written Exercise

369. 1. Find the balance due on the following note at maturity:

$650. EMPORIA, KANSAS, May 17, 1908.

One year after date I promise to pay to M. C. Maynard, or order, Six Hundred Fifty and $\frac{00}{100}$ Dollars, at the Emporia National Bank, for value received, with interest at 6 per cent.

Due May 17, 1909. ISAAC JOHNSON.

This note was indorsed by M. C. Maynard as follows:

July 26, 1908, $75
Dec. 15, 1908, $35
Feb. 12, 1909, $175

2. Find the balance due on the following note at maturity:

$800.00. MONTGOMERY, ALA., Dec. 4, 1906.

For value received, on or before the fourth day of December, 1908, I promise to pay to W. C. Bailey, or order, Eight Hundred Dollars, with interest at 7 %.

JAMES P. BELL.

Indorsed as follows:

Feb. 20, 1907, $300
Jan. 14, 1908, $200
June 15, 1908, $150

3. Find the balance due at maturity on a note for $1200 due in 4 yr., with interest at 6 %, dated July 15, 1903, and indorsed as follows:

Sept. 15, 1904, $250
Nov. 15, 1905, $250

4. Find the balance due at maturity on a note for $400, due in 18 mo., with interest at 5 %, dated May 31, 1906, and indorsed as follows:

Aug. 25, 1906, $120
Jan. 1, 1907, $100

370. Savings banks pay the depositors compound interest, compounding it either semiannually or quarterly.

Semiannual interest terms begin Jan. 1 and July 1, or April 1 and Oct. 1. Quarterly interest terms begin Jan. 1, April 1, July 1, and Oct. 1. The interval between the dates at which the interest is paid is called the **interest term.**

The bank books of depositors in savings banks must be presented both when deposits are made and when money is withdrawn, in order that the amounts may be credited or charged as the case may be.

Books should also be presented at the bank to have the interest credited when it is due.

STATEMENT

DATE	DEPOSITED	DRAWN OUT	INTEREST	BALANCE
1904				
Dec. 20	250			250
1905				
Jan. 1				250
Feb. 4	25			275
Mar. 6	10			285
April 1			1 87	286 87
June 10		50		236 87
July 1			1 77	238 64
Aug. 9	75			313 64
Sept. 4		25		288 64
" 18	10			298 64
Oct. 1			1 78	300 42
Nov. 5		15		285 42
Dec. 1	5			290 42
Dec. 10	10			300 42

The above account taken from a savings bank book is an illustrative statement of deposits and withdrawals with interest compounded quarterly at 3% a year.

The statement on p. 265 shows that the deposit of Dec. 20, 1904, did not begin to draw interest until Jan. 1, 1905. The interest for the quarter from Jan. 1 to April 1 was computed on the smallest balance for the quarter, namely, $250. It is the custom with most savings banks to credit interest at the end of every interest term on the smallest balance on deposit during the entire term. Interest is not reckoned on fractional parts of a dollar.

Written Exercise

371. Arrange the following items as a savings bank book and find the balance due Jan. 1, 1906.

1. Interest is reckoned quarterly (Jan. 1, April 1, July 1, and Oct. 1) at 3%.

Deposits: July 5, 1904, $275; Sept. 19, 1904, $100; Jan. 17, 1905, $20; Oct. 4, 1905, $10.

Withdrawals: June 25, 1905, $50; Nov. 20, 1905, $25.

2. Interest semiannually (Jan. 1, July 1) at 4 %.

Deposits: Dec. 20, 1904, $450; Feb. 15, 1905, $40; Aug. 6, 1905, $75.

Withdrawals: Jan. 30, 1905, $35; Sept. 10, 1905, $35.

3. Find the balance on the following, Jan. 1, 1907. Interest quarterly (Jan. 1, April 1, etc.) at 4%.

Deposits: Jan. 1, 1906, $600; May 30, 1906, $125; June 29, 1906, $80; Sept. 15, 1906, $15.

Withdrawals: Feb. 1, 1906, $35; Dec. 23, 1906, $55.

4. Same as exercise 3, except interest semiannually.

5. Find balance due Jan. 1, 1907. Interest semiannually at $3\frac{1}{2}$%.

Deposits: Jan. 1, 1903, $200; March 20, 1903, $50; Feb. 6, 1904, $90; Sept. 10, 1905, $85; April 6, 1906, $110; July 24, 1906, $60; Nov. 10, 1906, $15; Dec. 1, 1906, $30.

Withdrawals: none.

372. Coöperative banks or Building and Loan Associations (as organized under the laws of Massachusetts) are associations formed for the purpose of caring for the regular savings of their members, and of lending money to members only.

No member is allowed to hold more than twenty-five shares. Upon each share a member holds he must pay one dollar a month. If he fails to pay promptly, he must pay a fine of two cents a share for each month or part of a month. At the end of six months no more fines will be charged, but his shares may be declared forfeited.

The shares are generally issued in series. A series is usually open for subscriptions for a term of three months, six months, or twelve months. When a second series is issued, the issues of a previous series cease.

When the money paid in monthly with the dividends amounts to two hundred dollars a share, the share is said to **mature**, and its value is paid to the shareholder after deducting the money he may have borrowed.

A member may borrow, giving real estate as a security, not more than two hundred dollars for each share he holds, paying interest monthly upon the amount borrowed.

Dividends of profits are declared, usually once in six months, at a certain per cent upon the money paid in and the dividends that have been previously declared. Dividends are not paid in cash, but are added to the amounts paid in by the various members.

A member may borrow from the bank as much money as his shares are worth, giving the shares as security if he has not borrowed money upon the same shares, having given real estate as security for the loan.

The rate of interest charged for loans and the rate of the dividends is fixed by the officers of the bank called **directors**. The directors decide as to the value of the security given.

Written Exercise

373. 1. A holds 5 shares in a coöperative bank. How much a month does he pay upon his shares?

2. B fails to pay upon his 10 shares when due. How much does he owe for the first month of such failure?

3. If B fails to pay upon his 10 shares for 2 months, how much does he owe?

4. C borrows $50 upon the 10 shares he holds. How much must he pay to the bank monthly, interest being 6% per year?

5. D, who holds shares on which he has paid $120, and the profits on which are $10, desires to withdraw from the bank. To do so, he gives up one fourth of his profits. How much will he receive?

6. E, who holds 15 shares, has borrowed $2500 upon them, giving his house and land as security and paying interest at the rate of 6% per year. How much does he pay per month?

7. If E fails for three months to pay his dues and interest, how much does he owe to the bank?

8. If E pays in June the dues and interests accrued for March, April, May, and June, what sum of money does he have to pay?

9. F desires to withdraw from the bank. He has paid in $240. His dividends have amounted to $26.80, of which he is compelled to forfeit one fourth. He has borrowed $100 at 6% per year and is in arrears on 5 shares (has failed to pay at the proper time) and interest for one month. How much will he receive?

10. G's 20 shares mature on which he borrowed $3200. If he is not in arrears, what sum will the bank owe him?

374. Paying debts to people living in other places without actually transferring the money is called **exchange.**

There are various methods of exchange such as:

(1) By postal money order.
(2) By express money order.
(3) By telegraphic money order.
(4) By check.
(5) By bank draft.
(6) By foreign bill of exchange.

Domestic Exchange

375. Exchange between two places in the same country is called **domestic exchange.**

A **postal money order** is an order made by a postmaster to pay to the person named therein a specified sum of money.

$100 is the largest sum for which a single order may be issued. When a larger sum is to be paid, additional orders must be obtained.

The cost of postal money orders is:

For orders on sums not exceeding $2.50	3¢
Over $2.50, and not exceeding $5	5¢
Over $5, and not exceeding $10	8¢
Over $10, and not exceeding $20	10¢
Over $20, and not exceeding $30	12¢
Over $30, and not exceeding $40	15¢
Over $40, and not exceeding $50	18¢
Over $50, and not exceeding $60	20¢
Over $60, and not exceeding $75	25¢
Over $75, and not exceeding $100	30¢

All the leading express companies issue money orders payable at any of their offices. The rates are the same as for postal money orders.

Many of the telegraph offices in the United States are money order offices. The charge is 1% on $25 or over, and 25¢ on smaller sums, with a further charge of a sum not to exceed twice the rate for a 10-word message between the two places.

376. A personal check drawn on a bank in which the drawer has money deposited may be sent by mail. The one to whom the check is sent gets it cashed at a bank, and this bank sends it to the bank on which it was drawn for collection, sometimes charging a small fee for collecting.

If a person does not keep money on deposit in a bank, he may buy a **bank draft**, or a cashier's check, which is simply a check drawn by one bank upon another. Thus:

$500.00 Albany, N. Y., May 5, 1909.

First National Bank

Pay to the order of James Howe

Five hundred ——— 00/100 Dollars

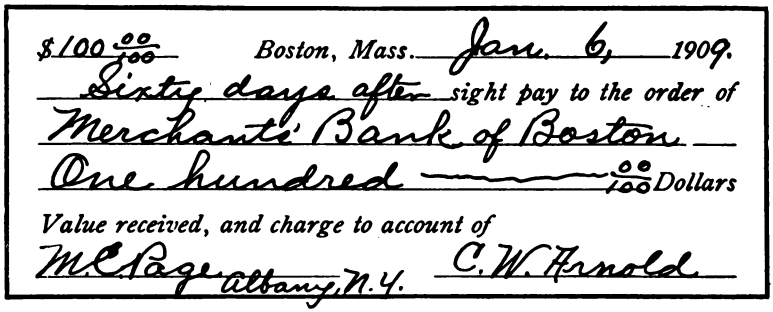
To Commercial Bank, New York — John Mills, Cashier

A Bank Draft

The person who signs a draft is the **drawer**; the one who is directed to pay it is the **drawee**; and the one to whom it is to be paid is the **payee**.

377. The collection of debts is made largely through banks by means of **commercial drafts.**

Suppose C. W. Arnold of Boston has sold leather to M. C. Page of Albany, and that Mr. Page has not paid the bill when due, say 60 days after the goods were bought. Mr. Arnold wishes to have the account settled, so he draws on Mr. Page by means of a commercial draft, thus:


$100.00/100 Boston, Mass. Jan. 6, 1909.
Sixty days after sight pay to the order of
Merchants' Bank of Boston
One hundred 00/100 Dollars
Value received, and charge to account of
M. C. Page Albany, N.Y. C. W. Arnold

A COMMERCIAL DRAFT (TIME DRAFT)

The Merchants' Bank of Boston sends this draft to some bank in Albany for collection. The Albany bank either collects of Mr. Page and remits to the Boston bank; or if payment is refused, returns it to the Boston bank, which in turn notifies Mr. Arnold to take up the draft, and if the money has been advanced on it, to refund it.

The form shown above is called a **time draft.** The form shown on p. 255 is a **sight draft.**

If Mr. Page intends to pay the above draft when presented, he will write across the face "Accepted" with the date, and sign his name underneath, as was done in the sight draft shown on p. 255. When this draft is accepted, it becomes equivalent to a promissory note *due sixty days after date*, and may be discounted at a bank.

In discounting, banks sometimes add several days to the discount term for time to collect; they may also charge for exchange.

Fluctuations in supply and demand sometimes cause a variation from the face, called the **rate of exchange.**

For small sums, perhaps $500 or less, New York, Chicago, or Philadelphia exchange usually sells at a premium, or above 0.1%. On large sums, the rate of exchange varies.

The premium or discount is usually quoted as a certain per cent of the face of the draft, or as a certain amount on $1000.

Written Exercise

378. Find the cost of the following drafts:

1. \$500 at 0.1 % premium.
2. \$1500 at 0.2 % premium.
3. \$3500 at 0.1 % discount.
4. \$2650 at 0.1 % premium.

5. A leather dealer in Chicago sold leather to a Boston firm and drew on them at sight for \$15,000. A Chicago bank bought the draft at a discount of 50 cents on \$1000. How much did the bank pay for the draft?

Foreign Exchange

379. If a man buys foreign goods, he often has to send money abroad. This is done by means of what is called a **foreign bill of exchange.** This is similar to a bank draft and is payable in the money of the country on which it is drawn. Commercial drafts are also drawn and accepted as in domestic exchange.

The rate of foreign exchange varies continually, depending on the demand. Thus, when English exchange is quoted at 4.90 (that is, a draft for £1 costs \$4.90) it is above par, because \$4.8665 is par.

English exchange is quoted at dollars to the pound. Thus, \$4.88 means that a bill for £1 costs \$4.88. **French exchange** is quoted by the number of francs that can be bought for \$1. Thus, 5.14 means that 5.14 francs can be bought for \$1. **German exchange** is quoted by the number of cents that 4 marks cost. Thus, $95\frac{3}{4}$ means that 4 marks cost $95\frac{3}{4}$ cents.

Newspapers usually give exchange rates for cable transfers, demand bills, and sixty-day bills, thus:

	Cable	Demand	60 Days
Sterling	4.87¾	\$4.86¼	4.83
Francs	5.16	5.18	5.20
Marks	95½	95½	94½

Foreign exchange for small amounts is usually effected by postoffice or express money orders.

380. Several men in the town of H. have held a meeting and decided that there is need of an electric road to connect their town with Boston. It is estimated that to build and equip this new road will require a capital of $50,000.

This amount is called the **capital stock**, and is divided into a number of equal parts called **shares**. A share is usually of the value of $100, although it may be more or less than that sum.

The original value of a share is called the **face** or **par value**.

NOTE. Unless otherwise stated, $100 is to be regarded as the par value.

The shares are offered for sale, and are bought by people who think the new road is likely to prove a good investment.

The people who buy the shares are called **shareholders**, or **stockholders**.

After all the shares are sold, or subscribed for, the stockholders meet and vote to obtain a charter from the state, which shall authorize them by law to transact business and to hold property as an individual. The company of stockholders may then be called a **corporation**.

After a charter is obtained, a president, treasurer, secretary, and board of directors are elected by the stockholders. In voting, each stockholder has as many votes as he owns shares.

Papers are prepared according to a fixed form, showing that a given individual owns a certain number of shares in the capital stock of the company. These papers are called **certificates of stock** and are delivered to the individuals entitled to them.

The income of the road referred to above not only pays all the running expenses during the first year, but shows a net profit of $3000. This profit, called a **dividend**, is divided among the stockholders. As the amount of the dividend is usually reckoned on the par value, we find what per cent $3000 is of $50,000, the par value of all the shares. This is 6%, therefore each shareholder will receive a 6% dividend on the par value of each share that he owns.

Stocks may be bought and sold the same as merchandise. As this road pays a good dividend, Mr. B, who sells 5 of his shares, receives for them more than their par value. The stock in this case is said to sell **above par**, or at a **premium**.

If this road had paid no dividend or a very small one, the shares would probably not have sold for their par value. The stock would then be said to sell **below par**, or at a **discount**.

We see from the above that shares of stock are bought and sold in the stock market, the transaction consisting in a delivery of the certificates of stock to the purchaser, and a record of the transfer on the books of the corporation. So the members of a corporation are constantly changing, some going out and others coming in. Persons who make a business of buying and selling stocks for their customers are called **stockbrokers**; and their compensation is called **brokerage**.

Stock brokerage is reckoned on the par value of stocks, *i.e.* a broker who sells a share of stock at $\frac{1}{8}$ brokerage receives $\frac{1}{8}\%$ of \$100, provided \$100 is the original, or par value, of a share.

Stocks are of two kinds, **preferred** and **common**. Preferred stock entitles the owners to dividends out of the net profits before the holders of the common stock are paid any dividend.

Bonds

381. The government of the United States, the governments of the several states, and counties, cities, towns, or school districts frequently borrow money, issuing as evidence of their indebtedness interest-bearing promissory notes called **bonds**.

Government bonds are usually offered by the government publicly, for sale to the highest bidders.

The price obtained depends on the rate of interest offered, on people's confidence in the government's good faith and financial resources, and on the abundance or scarcity of money seeking investment.

When government bonds have passed into private hands, they are bought and sold in the stock market.

Business corporations obtain loans by selling bonds in the same way that governments do; but a corporation usually secures (*i.e.* guarantees) its bonds by a mortgage on its property.

Records of stock sales called *quotations* are made in the newspapers as follows:

16 Old Colony R.R.	185
18 Union Pacific	115½
20 U.S. Coal & Oil	10¾
100 U.S. Steel	25½

These quotations in order mean:

16 shares of Old Colony railroad stock sold at 185 % of their par value.

18 shares Union Pacific stock at 115½ % of their par value.

20 shares U.S. Coal & Oil stock at 10¾ % of their par value.

100 shares U.S. Steel stock at 25½ % of their par value.

BOSTON STOCK EXCHANGE

STOCK SALES MAY 4

50 Boston & Albany	184½
50 Boston & Lowell	210
5 Boston & Maine	137½
19 Edison Elec. Ill.	205
30 Elm River Min.	80
30 Franklin Min.	7½
50 Gen. Electric	114
68 N.E. Tel. & Tel.	103½
62 W. End St. Ry.	31½
5 W. End St. Ry. pr.	98½
10 W. Union Tel.	56¼
350 Winona Min.	4

Oral Exercise

383. 1. Which of the stocks in the quotations given on p. 275 are selling above par? below par? at a premium? at a discount?

2. What is the market value of 5 shares of stock when quoted at par? at 120? at 90? at 50?

3. The capital stock of a company is $50,000. How many shares of $100 par value are there? I own 8 shares; what is the par value of all my shares?

4. A dividend of 5% is declared on a certain railroad stock. If I own 6 shares, what is my dividend?

5. At 1% brokerage, how much is paid on the sale of 1 share of stock? on the sale of 10 shares? (Par value $100.)

6. If the par value of a stock is $100, how much above par is it when a share sells for $150? How much below par when it sells for $60?

7. If there is a net profit of $4000 on a capital of $100,000, what per cent of dividend may be divided?

8. If the semiannual dividend on stock is $2\frac{1}{2}$%, what income do I receive yearly from 1 share? from 10 shares?

9. If a bank stock is quoted at 200, how much must be paid for 1 share? for 6 shares?

Written Exercise

383. Illustrative Example. Find the cost of 500 shares of B. & M. stock selling at 160, brokerage $\frac{1}{8}$%.

WORK

160% of $100 = $160, market price of 1 share.
$\frac{1}{8}\%$ of $100 = $.12$\frac{1}{2}$, brokerage on 1 share.
$160 + $.12$\frac{1}{2}$ = 160.12\frac{1}{2}$, total cost of 1 share.
160.12\frac{1}{2}$ \times 500 = $80,062.50, cost of 500 shares.

1. How much must be paid for 15 shares of Canadian Pacific railroad stock at $175\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

2. If U. S. Steel stock is worth $35\frac{3}{4}$, how much will 100 shares cost me, including brokerage at $\frac{1}{8}\%$?

3. 133 shares of the stock of the United Fruit Co. sold to-day at 115. Find the selling price of the lot.

Written Exercise

384. Illustrative Example. How many shares of stock can be bought for \$800 at $79\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

WORK

$79\frac{7}{8}\%$ of \$100 = \$79.875, market value of 1 share.
$\frac{1}{8}\%$ of \$100 = \$.12½, brokerage on 1 share.
\$79.875 + \$.12½ = \$80, total cost of 1 share.
\$800 ÷ \$80 = 10.

\$800, the money to be spent, divided by \$80, cost of 1 share, gives 10. Therefore, 10 shares can be bought. *Ans.* 10 shares.

1. Massachusetts Gas stock is selling at 49. How many shares can I buy for \$393, brokerage $\frac{1}{8}\%$?

2. A broker has \$5010 to invest in stock at 25% premium. How many shares can he buy if he charges $\frac{1}{4}\%$ brokerage?

3. How many shares of People's Gas Company stock quoted at $116\frac{7}{8}$ can be bought for \$5850, brokerage $\frac{1}{8}\%$?

4. How many shares of the stock of the Pullman Co. can I buy for \$1333.125 when they are selling in the market for 148 and I pay a broker $\frac{1}{8}\%$?

5. The shares of the Phœnix Mine are selling at 40% below par. How many shares can my broker buy for \$601.25, brokerage $\frac{1}{8}\%$?

6. How many U. S. 5% bonds selling at $107\frac{7}{8}$, brokerage $\frac{1}{8}\%$, can I buy for \$1080?

Written Exercise

385. Illustrative Example. A broker sold for me 250 shares of stock at $108\frac{3}{4}$, brokerage $\frac{1}{8}\%$. How much did I receive from the sale?

WORK

$108\frac{3}{4}\%$ of \$100 = \$108.75, market value of 1 share.

$\frac{1}{8}\%$ of \$100 = \$.125, brokerage on 1 share.

\$108.75 − \$.125 = \$108.625, amount I receive on 1 share.

\$108.625 × 250 = \$27,156.25, amount I receive on all shares.

1. B. & O. stock is selling for 75. I own 15 shares which a broker sells for me, charging me $\frac{1}{8}\%$. How much do I receive?

2. I sell through a broker 17 shares of N. Y. Central at $117\frac{3}{4}$, brokerage $\frac{1}{8}\%$. How much does the broker pay me?

3. A man bought 25 shares of Missouri Pacific when it was quoted at $91\frac{5}{8}$, and sold it at a premium of 9%. Find his gain, brokerage $\frac{1}{8}\%$ for each transaction.

Written Exercise

386. Illustrative Example. What income will be realized from investing \$3725 in 5% bonds, bought at 93, brokerage $\frac{1}{8}\%$?

WORK

\$93 + \$.125 = \$93.125, total cost of 1 bond.

\$3725 ÷ \$93.125 = 40, therefore I can buy 40 bonds.

5% of \$100 = \$5, income on 1 bond.

\$5 × 40 = \$200, income on all bonds.

1. Find the income on 39 U. S. bonds of \$1000 each, paying interest at $4\frac{1}{2}\%$.

2. I invest \$4423.25 in Illinois Central R.R. stock at 170, brokerage $\frac{1}{8}\%$. What income do I receive from this at $6\frac{1}{2}\%$?

3. Which is more profitable and how much, to invest \$6000 in 6% stock at 75, or in 5% stock bought at 60?

387. Illustrative Example. When a 6% stock is sold at $95\frac{3}{4}$, brokerage $\frac{1}{4}$%, what rate of income on the investment will the stock yield?

WORK

$95\frac{3}{4}$% of \$100 = \$95.75, market price of 1 share.
$\frac{1}{4}$% of \$100 = \$.25, brokerage on 1 share.
\$95.75 + \$.25 = \$96, total cost of 1 share.
6% of \$100 = \$6, income on 1 share.
\$6 is what per cent of \$96? It is $\frac{6}{96} = \frac{1}{16} = 6\frac{1}{4}$%.

1. What per cent shall I make on my money if I buy a 5% stock at $124\frac{7}{8}$, brokerage $\frac{1}{8}$%?

2. What per cent on the investment will a 4% bond bought at 160 yield?

3. What per cent on the investment will a 5% stock bought at 20% discount yield?

4. How much must I pay for a 5% stock in order to realize 6% on my investment?

SUGGESTION. 5% of \$100 = \$5, dividend on 1 share; \$5 is 6% of the investment.

5. How much must I pay for a 4% stock in order to realize 5% on the investment?

6. How much is paid for a 6% stock which realizes 6% on the investment?

7. Find the cost of a $4\frac{1}{2}$% stock when the money invested in it yields 8%.

8. The cost of 20 shares of Atlantic National Bank stock including $\frac{1}{4}$% brokerage is \$2200; what is the market value per share?

SUGGESTION. \$2200 ÷ 20 = \$110, total value of 1 share. The brokerage on each share is \$.25. Hence, \$110 − \$.25, or \$109.75 is the market value of 1 share.

9. The cost of 16 shares of the General Electric stock is $1764.24, which includes $\frac{1}{8}\%$ brokerage. Find the market value of each share.

10. What is the market value per share when 30 shares of Adams Express Co. stock, brokerage $\frac{1}{8}\%$, cost $4806?

Written Exercise

388. Illustrative Example. How much must be invested in 5% bonds purchased at 103 to yield an income of $800?

WORK

5% of $100 = $5, income on 1 bond.

$800 ÷ $5 = 160, bonds one must own to have the given income.

103% of $100 = $103, cost of 1 bond.

$103 × 160 = $16,480, amount to be invested.

1. What sum must be invested in 6% stock bought at 96 to obtain an income of $1380?

2. An opportunity is offered to invest money in 4% stock selling at 10% discount. How much money must I invest to obtain an income of $1000?

3. My income from a 7% stock is $1400. How many shares do I own, and how much did they cost if the market value was $97\frac{1}{2}\%$?

4. How many shares of American Express Co. stock selling at 195, brokerage $\frac{1}{8}\%$, can I buy for $3122? If the shares pay an 8% dividend, what will be my income from this stock?

5. A man bought 300 shares of railroad stock at $87\frac{1}{2}$ and sold them at $90\frac{1}{4}$, brokerage $\frac{1}{8}\%$ each way. How much did he gain?

6. Find the cost of 25 shares of stock at $69\frac{3}{4}$, brokerage $\frac{1}{8}\%$, if the par value of a share is $50.

389. The decimal system of measures and weights which had its origin in France is called the **metric system.** It has for its *basal unit* the **meter,** and to this all the other units are related.

The use of the metric system is required by law in many countries, and is permitted in the United States and a few other countries. Being a decimal system, its simplicity has brought it into general use in *scientific work.*

The word "meter" means a *measure.* The standard meter is a certain bar of platinum carefully preserved at Paris. The standard meter of the United States is a copy of this bar kept at Washington. The meter sticks made for ordinary use are copies of the standard meter. It was intended that the meter should be one ten-millionth of the distance from the equator to the poles, but later calculations have shown it to be a little less than that.

The *primary unit of length* is the **meter.** It is 39.37+ in. long.

The *primary unit of capacity* is the **liter.** It is the contents of a cube $\frac{1}{10}$ of a meter on each edge.

The *primary unit of weight* is the **gram.** It is the weight of a cube of water $\frac{1}{100}$ of a meter on *each edge.*

To each of the above primary units there are added, to indicate **decimal parts** of the unit, the following *Latin prefixes:*

deci, meaning $\frac{1}{10}$, or .1, as in decimeter = $\frac{1}{10}$ of a meter.
centi, meaning $\frac{1}{100}$, or .01, as in centigram = $\frac{1}{100}$ of a gram.
milli, meaning $\frac{1}{1000}$, or .001, as in milliliter = $\frac{1}{1000}$ of a liter.

To indicate **multiples** of the primary unit, there are added the following *Greek prefixes:*

deka, meaning 10, as in dekameter = 10 meters.
hekto, meaning 100, as in hektoliter = 100 liters.
kilo, meaning 1000, as in kilogram = 1000 grams.
myria, meaning 10,000, as in myriagram = 10,000 grams.

In the picture we have a measure which is $\frac{1}{10}$ of the length of a meter, or a **decimeter.**

ONE DECIMETER

Measure upon the blackboard or on the floor a length ten times as long as the measure shown in the picture. The length ten times as long is a *meter length.*

The decimeter represented in the picture is divided into 10 parts, each of which is $\frac{1}{100}$ part of the meter, and is called a *centimeter.* Each centimeter in the picture is subdivided into 10 parts, each of which is $\frac{1}{1000}$ of a meter, and is called a *millimeter.* 10 meters equal 1 *dekameter;* 100 meters, or 10 dekameters, equal 1 *hektometer;* 1000 meters, or 100 dekameters, or 10 hektometers, equal 1 *kilometer.*

In metric **long measures** 10 times one unit of any denomination equals one unit of the next higher denomination.

In metric **measures of surface** 100 times one unit of any denomination equals one unit of the next higher denomination.

In metric **measures of volume** 1000 times one unit of any denomination equals one unit of the next higher denomination.

390. LONG MEASURES

10 **millimeters (mm.)**	= 1 **centimeter (cm.)**
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter (m.)**
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 **kilometer (Km.)**
10 kilometers	= 1 myriameter (Mm.)

391. Measures of Surface and Land

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.)
100 square centimeters	= 1 square decimeter (sq. dm.)
100 square decimeters	= 1 square meter (sq. m.) = 1 centare (ca.)
100 square meters	= 1 square dekameter (sq. Dm.) = 1 are (a.)
100 square dekameters	= 1 square hektometer (sq. Hm.) = 1 hektare (Ha.)
100 square hektometers	= 1 square kilometer (sq. Km.)

392. Measures of Volume

1000 cubic millimeters (cu. mm.)	= 1 cubic centimeter (cu. cm.)
1000 cubic centimeters	= 1 cubic decimeter (cu. dm.) = 1 liter (l.)
1000 cubic decimeters	= 1 cubic meter (cu. m.) = 1 stere

393. Measures of Wood

10 decisteres (ds.)	= 1 stere (= 1 cu. m.)
10 steres	= 1 dekastere

394. Measures of Capacity

10 milliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 liter (l.) = 1 cu. dm.
10 liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 hektoliter (Hl.)
10 hektoliters	= 1 kiloliter (Kl.) = 1 cu. m.

395. Measures of Weight

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.) = weight of 1 cu. cm. of water
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 kilogram (Kg.) = weight of 1 cu. dm. of water
10 kilograms	= 1 myriagram (Mg.)
10 myriagrams	= 1 metric quintal (Q.)
10 quintals	= 1 Tonneau (T.) = weight of 1 cu. m. of water

Approximate Equivalents

396. The equivalents in United States measures here given are accurate enough for most purposes, and are easy to remember.

A decimeter = 4 in.
A meter = 3 ft. $3\frac{3}{8}$ in., or $1\frac{1}{10}$ yd.
A dekameter = $32\frac{8}{10}$ feet.
A kilometer = $\frac{6}{10}$ of a mile.
An are = 4 sq. rd., or $\frac{1}{40}$ of an acre.
A hektare = $2\frac{1}{2}$ A.
A stere = $\frac{1}{4}$ of a cord.
A liter = 1.06 liquid quart or $\frac{9}{10}$ of a dry quart.
A dekaliter = 1 pk. and 1 qt.
A hektoliter = $2\frac{5}{8}$ bu.
A gram = $15\frac{1}{2}$ grains.
A kilogram = $2\frac{1}{5}$ lb. avoirdupois.
A metric ton = 2200 lb. avoirdupois.

Exercise in Measuring

397. 1. Measure off 4 meters on the blackboard; 6 meters 20 centimeters.

2. Find the length and the breadth in meters of the schoolroom.

3. Measure 5 dekameters in the school yard and pace it off.

4. Measure off 1 kilometer on the sidewalk.

5. Upon the schoolroom floor, measure off a square meter; upon the school yard, measure off the are (10 meters square).

6. In the corner of the schoolroom, measure off the stere, 1 meter each way.

To the Teacher. Have a tin liter and a cubic centiliter made. Fill the cubic centiliter with water and its weight equals 1 gram; or for the gram take a strip of sheet lead whose weight is exactly equal to that of a nickel and divide it into five equal parts. One of these parts weighs a gram.

Written Exercise

398. 1. Express as meters and add 387 cm., 5437 dm.

2. Express as meters and add 268 Dm., 4023 Hm., and 4 Km.

3. Add 38.06 m., 607.23 m., 3708.9 m., 200.6 m., and express the answer in kilometers.

4. Ellen's hoop is 3.6 m. around. How many times will it turn in rolling a distance of 1.08 Km.?

5. Express the following in ares and add them: 2.2 Ha., 143.4 a., 34 Ha., 36 ca.

6. In a piece of land 13 m. long and 12.5 m. wide, how many square meters or centares are there? how many ares?

7. A had 8 Ha. 6 a. 7 ca. of land and sold 0.2 of it at $54 an are. How much did he receive for what he sold?

8. Express the following in cubic meters and add them: 7 cu. m., 40 cu. dm., 19 cu. cm., 25 cu. cm.

9. If I burn 27 steres of wood in the 3 winter months, what must be the length of a pile 1 m. wide and $\frac{3}{4}$ m. high to last a month, and how much will it cost at $2.25 a stere?

10. How many hektoliters of oats can be put into a bin that is 2 m. long, 1.3 m. wide, and 1.5 m. deep?

11. What must be the length of a bin 1 m. wide and 1 m. deep, to contain 4500 l. of grain?

12. At $11 per ton for coal, how much will the coal cost to keep a fire a week if 30 kilos (kilograms) are burned each day?

13. What is the weight of 10 cu. cm. of mercury, mercury being 13.5 times as heavy as water?

14. If marble is 2.7 times as heavy as water, what is the weight of a pedestal 1 m. square at each end and 2 m. high?

15. What weight of water in kilos may be contained in a cistern 4 m. deep, 1.5 m. long, and 1.2 m. wide?

399. 1. What is the net cost of a lot of musical instruments amounting to \$1875.60 on which a discount of 10%, 5%, and $2\frac{1}{2}$% is allowed?

2. I bought a house for \$6240 and sold it for \$8424. What per cent did I gain?

3. An agent bought 3000 bu. of corn at $37\frac{1}{2}$¢, charging 2% commission and \$8.75 insurance. For how much a bushel must it be sold to gain 20% on the entire cost?

4. A merchant sold a quantity of flour for \$282, losing 6%. How much money did he lose?

5. An agent sold 550 bu. of oats at 30¢ a bushel and charged \$4.95 commission. What rate of commission did he charge?

6. How many bushels of wheat can be bought for $\$53\frac{1}{4}$, at the rate of $29\frac{2}{3}$ bu. for $\$22\frac{1}{4}$?

7. How many cords of wood are there in a pile 80 ft. long, 5 ft. high, and 4 ft. wide?

8. How many acres are there in a rectangular piece of land 8450 ft. long and 3580 ft. wide?

9. What is the interest on a note for \$460 for 3 yr. 5 mo. 23 da. at 5%?

10. How many square inches are there in the surface of a cube whose edge is 17 in.?

11. Find the net amount of \$450, discounts 20% and 10%.

12. $6^6 = ?$ $\sqrt{54802.81} = ?$ $(\frac{3}{4})^4 = ?$ $\sqrt{\frac{7}{27}} = ?$

13. Find the cost at 25¢ per foot of fencing a rectangular garden 35 rods wide and three times as long as it is wide, containing 3675 sq. rd.

14. Find the base of a right-angled triangle whose altitude is 36 ft. and the hypotenuse 45 ft.

15. Find the area of a circle 8 ft. in diameter. Find the volume of a sphere of the same diameter.

16. What is the volume of a pyramid 18 ft. high and 4 ft. square at the base?

17. What is the area of a trapezoid whose parallel sides are 42 ft. and 56 ft. in length and 26 ft. apart?

18. A note for $308 was dated Jan. 15, 1904, with interest at 5%. On Apr. 15, 1907, $125 was paid. How much remained due Sept. 12, 1907?

19. If I should buy 2 horses at $150 apiece, and should sell one at a gain of 20% and the other at a loss of 20%, how much should I gain or lose on both?

20.

FACE	DATE	TIME	DATE OF DISCOUNT	INTEREST
$89	July 3, 1905	90 da.	Sept. 10, 1905	6%

Find the bank discount and the proceeds. Rate of discount, 6%.

21. If a ladder 26 ft. long is placed with its foot 10 ft. from a wall, how high up the wall will it reach?

22. Write a note containing the following data: date, Nov. 3, 1906; face, $1050; time, 6 mo.; payee, George Clinton; maker, yourself; interest, 6%. What sum will be received for this note if discounted Jan. 3, 1907, at 7%?

23. How many acres are there in a triangular field whose base is 60 rd. and altitude 35 rd.?

24. A man wishes to settle an income of $700 on his son. How much must he invest in U. S. bonds, paying $3\frac{1}{2}$% and bought at 105, to yield that income?

25. How much will it cost to carpet a floor 16 ft. 6 in. long and 14 ft. wide with carpeting 27 in. wide, at 1.12\frac{1}{2}$ per yard?

26. How much will 600 bbl. of flour cost at $4.80 a barrel with discounts of $16\frac{2}{3}\%$ and 5% for cash?

27. Find the commission at $3\frac{1}{4}\%$ on the sale of 1200 yd. of gingham at $12\frac{1}{2}¢$ a yard.

28. What is the amount due on my note for $175, given June 12, 1903, if paid April 1, 1904, with interest at 5%?

29. What amount should I receive at a bank for my 90-da. note for $480 without interest if discounted at date at 6%?

30. What will be the difference in the cost of fencing 2 fields, each containing 40 A., the first being in the form of a square and the other 160 rd. long and 40 rd. wide, at $2.25 per rod?

31. A man sold his farm for $2400, and lost 25%. If he had sold it for $3600, what per cent would he have gained or lost?

32. A city building lot, rectangular in shape, is advertised as having an area of 2107 sq. ft. The lot has a frontage of $21\frac{1}{2}$ ft. What is the depth of the lot?

33. A man bought 50 shares of Atchison, Topeka, and Santa Fe stock at $65\frac{7}{8}$ and sold it at $69\frac{1}{8}$. How much did he gain, the brokerage on each transaction being $\frac{1}{8}\%$?

34. How many square feet of surface are there on the outside of a smokestack 30 ft. high and 2 ft. in diameter?

35. A pair of skates cost a boy $2.50, but the merchant who sold them made a gain of 25%. How much did they cost the merchant?

36. A gate 3 ft. high and 6 ft. wide is braced by a stick fastened diagonally across it. How long is the stick?

37. Multiply 205 millionths by forty-six thousandths.

38. Find the cost of 2500 ft. of boards at $60 per M, and 785 ft. at $75 per M.

39. A shipment of cotton worth $8640 was insured for $\frac{5}{8}$ of its value at $3\frac{3}{4}\%$. What was the premium?

40. A farm containing 378 A. was bought for $95 per acre. It was sold for $48,837.60. What was the gain per cent?

41. A commission merchant sold 224 bbl. of potatoes at $3\frac{1}{2}$ a barrel and 216 bbl. of apples at $4.75 a barrel, charging $90.50 for his services. What was the rate of his commission?

42. A man made $2.75 by buying and selling goods at a profit of 25%. How much did the goods cost?

43. A house worth $27,000 is insured for $\frac{3}{4}$ of its value at $\frac{1}{2}\%$ premium. What is the amount of the premium? If the house were totally destroyed by fire, how much would the owner lose?

44. A man sold a house for $9500 and gained 10% by so doing. What would have been the gain or loss if he had sold it for $8500?

45. At $4.80 a cord find the value of a pile of wood 180 ft. long, 12 ft. wide, and 8 ft. high.

46. What is the cost of carpeting $\frac{3}{4}$ of a yard wide for a room 22.5 by 13.5 ft. at $.60 per yard?

47. Goods marked at $115 were sold at $6\frac{1}{4}\%$ discount and 5% off for cash. What was the selling price?

48. What is the amount of $1600 for 6 yr. 5 mo. 18 da. at 4%?

49. A rectangular block of marble is 4 ft. 3 in. long, 1 ft. 6 in. wide, and 2 ft. thick. How many square feet are there in the entire surface? How many cubic feet in its volume?

50. Had a certain house lot been sold for $1100, $8\frac{1}{3}\%$ of its value would have been lost; at what price must it be sold to gain $8\frac{1}{3}\%$?

51. How many square feet are there in the surface of a spire that is in the form of a hexagonal pyramid whose slant height is 75 ft. and each side of whose base is 12 ft.?

52. What is the amount of $600 at compound interest for 2 yr. 6 mo., at 6% interest, compounded semiannually?

53. What is the duty at 18% ad valorem on 375 boxes of raisins, each box containing 35 lb., invoiced at $9\frac{1}{2}¢$ a pound?

54. Find the bank discount and proceeds of a 3 mo. 6% note for $500, dated June 5, 1906, discounted August 5, 1906, at 6%.

55. If the tax rate is $14.95 per thousand, what will be the total tax of a man who pays a $2 poll tax and has an estate valued at $12,460?

56. Of the 75,000,000 population of the United States in 1900, there were 5,000,000 foreigners who were not naturalized. What per cent were not naturalized?

57. The cost of our government may be roughly stated as about $1,500,000,000 a year. If a man should work 8 hr. a day and count $60 a minute, how long would it take him to count it?

58. David P. Osgood received a note for $500 from Jonas Macey on July 6, 1909, due in 6 mo., with interest at 5%. Mr. Osgood discounted this note at 6% on Sept. 6, 1909. What were the proceeds?

59. Find the average size of ten lots of land measuring respectively as follows:

100 sq. rd. 200 sq. rd. 150 sq. rd. 250 sq. rd. 120 sq. rd.
350 sq. rd. 75 sq. rd. 350 sq. rd. 220 sq. rd. 10 sq. rd.

60. A house valued at $7500 is mortgaged for $\frac{2}{3}$ of its value. The mortgage bears interest at the rate of 5%. What is the yearly interest? In what time will the total interest payments equal the amount of the mortgage?

NOTE. The teacher should explain the meaning of *mortgage*.

61. A wagon is worth \$49, but is sold for \$60. What per cent is gained above its value?

62. For settling an estate amounting to \$750,000, Mr. Wood receives a commission of $5\frac{1}{2}\%$. To what does the commission amount?

63. The estate of Henry B. Page amounted to \$84,000. Of this the widow received $\frac{1}{3}$, and of the remainder $\frac{1}{2}$ was divided equally between 4 children, and the rest was given to found a scholarship at Amherst College. How much did each beneficiary receive?

64. The estate of Carl Henking inventoried as follows:

ASSETS		
Cash in Banks	\$10,500	
House and Land on East St.	7,800	
Stocks and Bonds	34,000	
Bills Receivable	1,200	
Total assets		\$53,500
LIABILITIES		
Debts Outstanding	\$1,650	
Notes Due	3,400	5,050
Net assets		\$48,450

What is the widow's share if she receives $\frac{2}{3}$ of the estate? If the son receives the remainder, how much does he receive?

65. A bill of goods for \$245.60 is subject to a rebate of \$30 for brokerage and a discount of 2% for cash in 10 da. What is the actual amount paid?

66. A building worth \$2500 is insured for \$1500. The building is injured by fire and water to the extent of \$1000, which the insurance company pays. What is the net amount received for insurance, if $1\frac{1}{2}\%$ premium had been paid?

67. An insurance company has risks amounting to $1,200,000, on which it receives an annual premium averaging $1\frac{1}{4}\%$. What is the net income, if its expenses amount to $\frac{1}{2}\%$ on the risks?

68. A man's life is insured for $5000. On this he pays a yearly premium of $150. What is the per cent of premium?

69. Penney & Ellis enter into a partnership with a capital of $35,000, of which Penney puts in $20,000 and Ellis $15,000. Their stock of goods is completely destroyed by fire, and they receive insurance for it to the amount of $32,000. They then decide to discontinue the business. What is each one's share of the insurance?

70. How many shares of Chicago and Northwestern R.R. stock, $299.50 per share, should be exchanged for 90 shares of American Smelting Co. stock at $98\frac{75}{100}$ per share, and how much cash should be paid besides to balance the transaction?

71. Quicksilver stock, par value $100 per share, sells at $4 per share. How many per cent below par has it fallen?

72. How much above par has a stock risen if 2 shares sell for $399?

73. If the above stock (Ex. 72) pays a 5% quarterly dividend, what is the per cent of income on the investment?

74. C. W. Jones rents the house at 240 Arch St., Bennington, Vt., to W. D. Stokes at $18 per month. If the annual expense for taxes is $20, for insurance $7.50, and repairs $35, what is the net annual income from the property? This is 5% on what amount?

75. Samuel N. Wood agreed to build a business block for Francis Ward for a contract price of $47,800. Before completing the contract Mr. Wood died. He had already received payments on account amounting to $18,700. If it was estimated that he should have received $62\frac{1}{2}\%$ of the contract price for work done, what balance is due his estate?

76. A farmer received \$135 for $\frac{9}{10}$ of his oats at \$.60 a bushel. How many bushels did he have in all?

77. A dealer purchased $9\frac{1}{2}$ reams of paper at \$.80 a ream, and sold it at \$.$12\frac{1}{2}$ a quire. Find the gain.

78. A farmer sold $37\frac{1}{2}$ bu. of rye from a lot of $187\frac{1}{2}$ bu. What part of the whole was left?

79. How many postage stamps, each containing $\frac{18}{25}$ of a square inch, can be cut from a sheet of $172\frac{4}{5}$ sq. in.?

80. A real estate dealer bought 140.75 A. of land. He sold $18\frac{1}{2}$ A. to W, 14.7 A. to X, $27\frac{9}{10}$ A. to Y, and the remainder to Z. How many acres did he sell to Z?

81. Change to decimals and add: $\frac{1}{5}$, $\frac{7}{8}$, $\frac{11}{20}$, $9\frac{3}{4}$, $6\frac{1}{5}$, $5\frac{1}{4}$, $14\frac{1}{2}$, $\frac{9}{16}$.

82. The Lynn Institute for Savings loaned \$4200 from April 15, 1909, to July 27, 1910, at 6%. What sum was received in payment?

83. Find the amount due on a note for \$360 dated Oct. 6, 1908, if paid Oct. 12, 1909, the rate of interest being $4\frac{1}{2}$%.

84. My net annual income on a house is \$400. If I sell the house for \$8500 and loan the money at 5%, how much shall I increase my annual income?

85. A capitalist endowed a professorship with a salary of \$2100. What sum did he invest at $3\frac{1}{2}$%?

86. Freight weighing 3500 lb. is sent from A to B at the rate of \$50 per ton. The railroad company allows a claim for damage for \$24. This will make the net amount due the railroad equal to what?

87. Emerson & Barnes receive from T. P. Wainwright 440 doz. eggs, 235 lb. butter, and $\frac{1}{2}$ T. of cheese to sell on commission. The prices received are 24¢ per dozen for 100 doz. eggs, 22¢ per dozen for 240 doz. eggs, and 23¢ per dozen

for 100 doz. eggs; 25¢ per pound for the butter; and 15¢ per pound for the cheese. After taking out a commission of 4% for selling and $21 for expense, what amount is returned to Mr. Wainwright?

88. What is the freight bill for 6 boxes of hardware, weighing 1450 lb., at 23¢ per hundred, plus advance charges of $4.60?

89. J. C. Miller owes F. C. Dean & Co. $375 on account. He sends on Jan. 1, 1909, a check for $125, and Dean accepts a 3-mo. note, dated Jan. 1, 1909, for the remainder, at 6%. What are the net proceeds to Dean if he discounts it Feb. 1, 1909, at 6%? What amount does Miller pay at maturity?

90. E. C. Adamson sends a check to settle an account, but has deducted a discount of 15%, amounting to $24, to which he is entitled. What was the full amount due before the discount was taken out?

91. H. A. Hall buys 100 bbl. of flour at $5.35 per barrel, with discount of 6% if paid within 10 da., and an additional trade discount of 5%. What amount should be paid if the bill is settled within the 10 da.?

92. F. A. Smith gave his note to the Central Bank of Lynn for $150, dated Oct. 15, 1909, payable in 4 mo. What was the discount at 6%? What were the proceeds?

93. A real estate dealer divided $4\frac{1}{2}$ A. of land into 5 equal square lots. What were the dimensions of each lot?

94. At $5 a rod it cost $320 to fence a square field. How many rods were there on each side? How many acres were there in the lot?

95. Find the edge of a cube whose entire surface is 1176 sq. in.

96. The base of a right-angled triangle is 20 ft., the altitude is 15 ft. What is the hypotenuse?

97. What is the diagonal of a square field containing 5 A.?

98. How much is gained by crossing a field 84 rd. long and 63 rd. wide diagonally, instead of walking along the side and end?

99. What is the diameter of the largest circular table top that can be carried through a door 6 ft. by 8 ft.?

100. A dealer in hardware bought iron at 50 % off the list price, and sold it at 25 % off the list price. What was the gain per cent?

101. The tax rate in Brookline is $10.50 on a thousand dollars. What is the value of Mr. Cooper's property, whose tax is $199.50?

Foot these ledger columns:

102.		**103.**		**104.**		**105.**	
1344	46	8642	21	422	80	150	
5365	80	843		146	40	382	15
126		464	10	22	25	822	29
3246	96	324	81	6	85	464	10
438	20	425	65	24	23	327	
268	65	5846	90	426	42	62	91
392	22	418	26	5	60	73	59
442	25	24	48	3	25	92	60
344	28	22	60	11	29	8	55
2346	42	6329	24	64	20	34	90
225	52	545		233	09	7	85
346	80	435		86	33	23	94
425		389	60	2	49	44	64
648	45	241	40	4	30	73	83
123	35	58	84	23	35	648	20
8480	12	234	63	226	48	25	
2122	60	3484	36	88	24	745	42

106. I paid \$45 for insuring a stock of goods at $\frac{5}{9}$ of 1%. For how much was it insured?

107. Find the accurate interest on \$238.40 for 65 days at 6% per annum.

108. Find the interest on \$353 for 7 mo. 10 da. at 5% per annum.

109. Find the interest on \$124 for 6 mo. 15 da. at 5%.

110. Find the interest on \$167.58 for 2 yr. 4 mo. at $4\frac{1}{2}$%.

Add:

111.	**112.**	**113.**	**114.**
$15\frac{3}{4}$	$46\frac{1}{8}$	$112\frac{1}{2}$	$100\frac{1}{3}$
$12\frac{3}{5}$	$12\frac{11}{12}$	$134\frac{3}{4}$	$297\frac{1}{4}$
$24\frac{3}{4}$	$23\frac{9}{16}$	$23\frac{3}{4}$	$32\frac{1}{6}$
$13\frac{9}{20}$	$14\frac{3}{8}$	$25\frac{5}{8}$	$105\frac{1}{12}$
$14\frac{7}{8}$	$3\frac{5}{6}$	$123\frac{3}{16}$	$72\frac{5}{6}$
$11\frac{9}{10}$	$11\frac{3}{8}$	$41\frac{1}{2}$	$999\frac{3}{4}$

Multiply:

115.	**116.**	**117.**	**118.**
$250.2\frac{3}{5}$	$31.25\frac{1}{6}$	$12.16\frac{1}{2}$	$1339\frac{5}{6}$
$4.4\frac{1}{3}$	$.022\frac{2}{3}$	$.162\frac{3}{4}$	$.32\frac{2}{3}$

119. How many shares of railroad stock quoted at 112 will pay for 80 shares of telephone stock at 140?

120. The income on 90 shares of telegraph stock is \$405. What rate of dividend is paid?

121. I gained \$480 by selling wool at $16\frac{2}{3}$% above cost. Find the cost and the selling price.

122. A broker bought stock at 97 and sold at 10% premium, gaining \$182. What sum was received for the stock?

123. The list price of a lot of hardware is \$1400. If the discounts are $14\frac{2}{7}$% and 8%, what is the actual price? What should be the selling price to make a net gain of 20%?

124. Find the difference between the simple and compound interest of \$600 for 3 yr. and 6 mo. at 4%, interest compounded semiannually.

125. The sum of \$780 is invested at $3\frac{1}{2}$%, and has yielded \$60.06. How long has it been on interest?

126. The product of 3 factors is 113,400. Two of the factors are 84 and 75. What is the third factor?

127. Find $4.8 \div 6$; $.048 \div 6$; $4.8 \div .06$; $4.8 \div .006$; $48 \div .006$.

128. Find the net amount of the following bill:

13 pcs. Linen Crash, $42\frac{3}{4}$, 44, $46\frac{1}{4}$, 43, 45, $43\frac{1}{2}$, 41, $46\frac{1}{2}$, 45, 49, $48\frac{1}{2}$, 42, $41\frac{1}{2}$ yd.	@ $10\frac{1}{2}$¢		
Less 20 — 10 — 5%.			

Find the bank discount and net proceeds on:

	Date of Note	Date of Discount	Time	Face	Int. Rate	Disc. Rate
129.	Oct. 3, 1907	Dec. 19, 1907	90 da.	\$136.00	5%	6%
130.	Nov. 16, 1907	Dec. 16, 1907	3 mo.	154.21	6%	5%
131.	Jan. 23, 1908	Mar. 1, 1908	6 mo.	941.12	$5\frac{1}{2}$%	6%
132.	Aug. 21, 1909	Aug. 21, 1909	60 da.	450.00	6%	6%

Multiply and carry out results:

	a		*b*		*c*	
133.	$36 \times .05\frac{3}{4}$		$25 \times .35$		14×2.25	
134.	$28 \times .26$		$112 \times .04$		44×8.30	
135.	$34 \times .75$		$18 \times .18\frac{1}{4}$		35×7.15	
136.	$140 \times .58\frac{1}{2}$		$62 \times 1.6\frac{2}{3}$		$32 \times .18\frac{1}{8}$	
137.	$62 \times .65$		$42 \times .48$		$27 \times .45$	

Find the commission, gross charges, and net proceeds for each consignor.

Consignor	Sales May 4, 1907	Rate Com.	Commission	Other Charges	Gross Charges	Net Proceeds
138. Chas. Weeks	$325 80	4½%		$82 60		
139. C. J. Blair	1600 00	5%		22 50		
140. Isaac Pitt	112 00	5%		24 20		
141. John Wells	240 00	4½%		57 30		
142. E. M. Shedd	420 00	4%		22 40		

Find the interest:

Principal	Time	Rate
143. $224.25	92 days	4%
144. $345.50	6 months	6%
145. $2480.00	2 weeks	5%

146. An attorney who collected 90% of a debt at 10% commission received $108 for his services. What was the debt?

147. A manufacturer reduced the wages of his employees 10%. What would be the weekly wages of a man who had been receiving $5 a day?

Subtract:

148. $222\frac{2}{3} - 158\frac{3}{5}$ **149.** $218\frac{3}{5} - 71\frac{3}{4}$ **150.** $112\frac{1}{5} - 15\frac{1}{2}$ **151.** $132\frac{1}{2} - 18\frac{9}{16}$

Divide:

152. $8)2876\frac{3}{4}$ **153.** $5)25\frac{5}{7}$ **154.** $5\frac{1}{2})187\frac{1}{8}$ **155.** $7)2074\frac{1}{4}$

156. Hatch & Co. sell $64,200 worth of real estate at a commission of $2\frac{1}{4}$%. To what amount are Hatch & Co. entitled as commission? What are the net proceeds of the sale?

157. Complete the following pay roll:

NAME	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTAL HOURS	RATE ¢	AMOUNT
Edwin Irving	8½	8	8¼	8	8½	8		30	
Allen Gray	8	8¼	8	8½	8	8		25	
Frank Chase	7½	7	8	7	8	8		20	
Jack O'Neil	8¼	8	8½	8	7	8		25	
James Ford	8	6	8	8¼	8	9		22	
Henry Ward	8½	8	8¼	8	8	6		24	
John Young	8	8		5	9	8		20	
Mary Hall	7½	8¼	8	6	8	8		22	
Daniel Long	8	8	9	7	8	7		22	
Mary Colby	8½	8	7¼	8	8	8		20	
Gilbert Weeks	8	8	8		7½	8		20	
Erastus Dorr	8	9	8	8	8	8		17½	
Jane Fitz	8	7	7½	8	8	7		18	
Will Collins	8	8	8	8¼	8	8		18	
Kate Clark	6	8	6½	8	8	8		12½	
Fred Smith	8	8	8	6	8	7		12½	
Jack Mason	8	8	8	8	7½	8		12½	
Daniel Healy	8	8	8	8	8	8		40	
Edwin Gale	8	7	8	8	6	8		30	
Robert Gale	8	7	8	8	6	8		25	
Annie Fox	7½	7	7½	8	8	8		27½	
John Dodge	8	8	8	7	8	7½		17½	
Milton Page	8	9	8	8	7	8		15	
Mary Beecher	8	8		7	8	8		15	
Mary Clark	6	6½	8	8	8	8		25	
Jane Coleman	8½	8	8		8	8		14	
Fred Howe	7½	8	8	7	7	8		14	
Allen Files	9	8½	8	8	7	7½		12½	
Caleb Gordon	8	8	8	7	8	8		12	
Emily Colby	7	8	8½	3	6½	8		12	
Fred Brooks	8	8	8	8	8	8		37½	
George Foote	6½	8	5	8	9	5½		27½	
Addie Hart	8	8	7	8	8	8		30	
Total									

Find the premium, agent's commission, and amount due the company on each of the following insurance policies :

Policy No.	Company	Amount	Rate per $1000	Premium	Agent's Rate	Agent's Com.	Bal. Due Co.
158. 2,080	Mutual . .	$2,500	$1 20		18%		
159. 1,901	New England	3,750	2 00		20%		
160. 1,304	Germania .	22,500	1 45		25%		
161. 44,607	Household .	25,000	1 25		10%		
162. 2,221	Royal . . .	1,800	1 75		25%		
163. 36,215	Phœnix . .	3,500	1 25		15%		

Note. The agent's commission is a certain per cent on the premium collected. The agent deducts his commission from the premium and returns the balance to the company.

Find the selling price :

	a Cost Price	Gain %	Sell. Price	*b* Cost Price	Gain %	Sell. Price
164.	$133 45	45		$220 00	$16\frac{2}{3}$	
165.	287 60	$62\frac{1}{2}$		188 75	$27\frac{1}{2}$	
166.	125 80	$16\frac{2}{3}$		104 20	25	

Find the selling price :

	a List Price	Discount	Sell. Price	*b* List Price	Discount	Sell. Price
167.	$1280 00	$12\frac{1}{2}-5$		$8820 00	$16\frac{2}{3}-10$	
168.	1826 00	$33\frac{1}{3}-10$		866 00	40–10	
169.	225 00	20–2		112 00	20–15	

170. I bought 800 magazines at 15¢ each. I sold them so as to gain 50%. How much did I pay for the magazines and how much did they sell for? What was the profit?

Find the loss or gain, and the per cent of loss or gain:

	Cost Price	Selling Price	Loss	Gain	Loss %	Gain %
171.	$45 00	$120 00				
172.	100 00	150 00				
173.	225 00	275 00				
174.	25 00	38 75				
175.	37 50	25 00				

Find the selling price, and the per cent of gain or loss:

	Cost Price	Selling Price	Loss	Gain	Loss %	Gain %
176.	$255 00			$150 00		
177.	125 00			144 00		
178.	350 00		$126 00			
179.	560 00			150 00		
180.	347 00		47 00			

Find the marked price, the discount, and the selling price:

	Cost Price	Gain %	Marked Price	Disc. Rate	Discount	Selling Price
181.	$860 00	30		10		
182.	275 50	60		20		
183.	125 00	25		12½		
184.	36 00	65		25		
185.	22 00	30		15		

186. The First National Bank, Springfield, Mass., charges 1½% for collecting a draft for $550 drawn by Mason Bird & Co. of Chicago, Ill., on J. E. Ashton & Co. of Springfield, Mass. What amount is sent to Mason Bird & Co. after deducting the collector's charge?

187. Find the total sales for each clerk for the week as given below, and the total sales for each day and the entire week :

No.	Name	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total for Week
1	Block	$10 25	$23 14	$22 27	$24 65	$22 15	$23 62	$
2	Breen	17 86	16 80	13 61	33 14	61 15	52 13	
3	Brown	24 60	9 89	14 20	56 41	25 25	27 86	
4	Cooper	13 90	22 40	21 62	65 22	14 87	34 64	
5	Hanshaw	22 14	13 29	38 41	30 12	75 69	25 25	
6	Haskins	16 10	21 22	15 19	25 09	20 23	36 42	
7	Hawkins	17 42	25 95	16 84	13 45	14 61	25 33	
8	Holmes	13 31	43 63	22 12	61 13	12 25	13 54	
9	Hovey	15 25	21 46	14 60	34 15	54 10	22 33	
10	Howe	22 52	52 34	30 12	62 18	62 35	25 63	
11	Paddock	16 62	81 72	19 42	40 38	17 81	74 25	
12	Shay	24 25	56 49	23 67	35 91	15 69	38 20	
13	Snow	16 39	21 25	17 89	51 12	32 12	64 65	
14	White	14 21	30 32	12 45	22 35	14 21	35 15	
	Totals	$	$	$	$	$	$	$

Add :

188.

$$23\tfrac{2}{3}$$
$$15\tfrac{1}{8}$$
$$16\tfrac{1}{2}$$
$$42\tfrac{4}{5}$$
$$\underline{21\tfrac{3}{4}}$$

189.

$$24\tfrac{5}{9}$$
$$16\tfrac{4}{5}$$
$$22\tfrac{3}{8}$$
$$45\tfrac{5}{6}$$
$$\underline{13\tfrac{7}{8}}$$

190.

$$11\tfrac{1}{2}$$
$$23\tfrac{5}{6}$$
$$32\tfrac{1}{3}$$
$$45\tfrac{1}{2}$$
$$\underline{58\tfrac{3}{4}}$$

191.

$$51\tfrac{4}{9}$$
$$33\tfrac{5}{6}$$
$$12\tfrac{1}{3}$$
$$35\tfrac{1}{5}$$
$$\underline{56\tfrac{1}{2}}$$

192. Goods valued at $26.84 are sent C.O.D. by the American Express Co. from Chas. B. Folk, Bowling Green, Ky., to F. C. Plaisted, Dover, N.H. The Express Co. charges 2% for collection. What is its collection fee? What amount is remitted to C. B. Folk?

193. Find the total weekly sales for each department in the following record of sales of a department store:

DEPARTMENTS	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTAL FOR WEEK
Dress Goods	$123 42	$135 23	$213 40	$119 20	$216 45	$225 19	$
Hosiery .	26 98	91 22	82 26	23 65	29 02	225 14	
Gloves . .	121 10	73 44	125 90	210 14	136 45	136 44	
Jewelry .	113 95	83 65	134 38	256 15	124 25	542 27	
Toys . .	63 02	3 22	124 27	28 12	62 10	22 15	
Books . .	120	214 94	28 15	334 15	343 15	219 05	
Carpets .	544	353 62	262 20	167 13	446 45	644	
Art Dept.	115 20	128 18	139 40	125 15	125 16	122 56	
Notions .	223 05	134 15	405 25	393 42	496 80	116 25	
Crockery .	46 72	269 13	150	118 24	13 95	42 24	
Kitchen Ware	25	112 25	188 10	162 15	321 18	234 64	
Glass . .	25	5 60	32 84	38 45	34 12	55 82	
Leather .	61 95	120	43 19	616 23	145 13	23 40	
Hair Goods	96 44	25 17	31 17	34 12	22 40	35 64	
Suits . .	125 25	118 34	164 39	262 15	193 17	124 50	
Millinery .	224 35	44 23	19 21	251 10	127 24	123 64	
Underwear	61 35	15 15	22 45	28 13	244 12	46 79	
Groceries	281 19	121 64	113 24	314 20	65 63	27 10	
Totals	$	$	$	$	$	$	$

194. Find the total sales for all departments each day. The total sales for the week of all departments. At 35 % what is the profit on each day's sales? On the week's total sales?

Subtract:

195. $234\frac{3}{4} - 132\frac{2}{5}$

196. $610\frac{7}{9} - 238\frac{11}{12}$

197. $151\frac{3}{10} - 23\frac{1}{2}$

198. $47\frac{3}{5} - 12\frac{1}{5}$

Multiply:

199. $2741\frac{11}{12} \times 14$

200. $1275 \times 39\frac{3}{8}$

201. $2346\frac{11}{12} \times 18$

202. $1875 \times 43\frac{4}{75}$

203. A $5000 automobile was bought for cash at 10% off. It was used a year and sold for $2300. What was the per cent of loss on the purchase price?

204. A typewriter is sold for $100 with 30% off for schools. What will be the cost of 5 for a school, if an allowance of $20 each is made for 5 old machines taken in exchange?

205. A $16 set of books is sold for $12. What is the per cent of discount?

206. 8 wood lathes are sold at $60 each. This is a reduction of 20% from the list price. What is the list price?

207. A set of cyclopedias that cost $19 was sold for $9.50. What per cent was lost?

208. The gross earnings of the P. & U. C. Traction Co. for the year ending Dec. 31, 1906, were $236,690, an increase of $47,378. What was the approximate per cent of increase?

209. J. Vilas & Co., brokers, sell for Mrs. T. H. Sloan 40 shares Boston & Albany Railroad stock at $174 a share. What is the charge for brokerage at $\frac{1}{4}$%?

210. F. S. Gardiner ships to E. J. Merriam, commission merchant, 640 doz. eggs and 225 lb. of butter. The eggs are sold at 24¢ a dozen and the butter at 21¢ a pound. After taking out the commission at $2\frac{1}{2}$%, what amount does the commission merchant send to Mr. Gardiner?

211. Ball & Co., commission merchants, Boston, Mass., receive from S. S. Gray, Winthrop, Maine, 230 bbl. Baldwin apples and 430 bbl. potatoes. The apples sell at $2.10 per barrel and the potatoes at $1.85 per barrel. How much does Mr. Gray realize on the transaction after paying Ball & Co.'s commission at $3\frac{1}{4}$%?

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